Algebraic Structure
Tutorial # 8: Fields

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Exercise 1:
Let's consider the set $R = \mathbb{Z}[x]$, which is the set of all polynomials with integer coefficients. Do you agree or disagree that R is a ring/field?

Exercise 2:
Let $Q(x) = \frac{x-3}{x^2+1}$, a polynomial function with rational coefficients. Is this a field?

Exercise 3:
Let $\phi(x) = x^2 - 2$ a mapping from $\mathbb{R} \to \mathbb{R}$. Is ϕ a homomorphism between fields?

Exercise 4:
Let $\mathbb{F} = \{0, 1, 2\}$ be a field. We define $\mathbb{F}_9 = \mathbb{F}_3(\alpha) = \alpha^2 + 1$ where $\alpha \in \mathbb{F}_3$. Show that $\mathbb{F}_9$ is a field.

Exercise 5:
Let $\mathbb{F} = \{0, 1, 2\}$ be a field. We define $\mathbb{F}_9 = \mathbb{F}_3(\alpha) = a\alpha^2 + b$ where $a, b \in \mathbb{F}_3$. List the possible elements in $\mathbb{F}_9$.