

Labwork 3

Exercise 1: Z score table

- Level of confidence is 85% $\Rightarrow z_{\alpha/2} = 1.44$
- $\frac{\alpha}{2} = \frac{1-0.85}{2} = 0.075$
- $1 - 0.075 = 0.925$
- $Z_{\frac{\alpha}{2}} = 2.17$
- Level of confidence is 97% $\Rightarrow z_{\alpha/2} = 2.17$
- $\frac{\alpha}{2} = \frac{1-0.97}{2} = 0.015$
- $1 - 0.015 = 0.985$
- $Z_{\frac{\alpha}{2}} = 2.17$

Exercise 2: T-distribution table

- Level of confidence is 85% $\Rightarrow t_{\alpha/2} = 1.44$
- Level of confidence is 97% $\Rightarrow t_{\alpha/2} = 2.77$

Exercise 3

Calculate the sample size, sample mean, and sample standard deviation of the variable “tbc” in the dataset `SinhVienCaoDang.xlsx`. Then based on the calculated values, construct a 90% confidence interval for the population mean. Explain the meaning of the result.

- Sample size: $n = 2432$ (large \Rightarrow nearly Normal distribution), $df = 2431$
- Sample mean: $\bar{x} = 6.66$
- Sample standard deviation: $s = 0.5$
- Confidence level = 0.9 $\Rightarrow \alpha = 0.1 \Rightarrow \alpha/2 = 0.05$
- $Z_{0.05} = 1.645$
- Confidence interval: $\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 6.66 \pm 1.645 \cdot \frac{0.5}{\sqrt{2432}} = 6.66 \pm 0.0167$

Exercise 4

A sample of 16 students from a big school gives a mean GPA 2.84 with sample standard deviation 0.48. Based on the values, construct a 90% confidence interval for the mean GPA of all students in the school. Assume that GPA has a normal distribution.

- Degrees of freedom: $df = n - 1 = 15$
- Sample mean: 2.84
- Sample standard deviation: $\sigma = 0.48$
- Confidence level = 0.9 $\Rightarrow \alpha/2 = 0.05$
- Confidence interval: $\bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}$
- Standard error: $\frac{s}{\sqrt{n}} = \frac{0.48}{\sqrt{16}} = 0.12$
- Margin of error: $1.753 \cdot 0.12 = 0.210$
- Final interval: 2.84 ± 0.21

Exercise 6

Given a margin of error $E = 0.25$, find the minimum sample size n to construct a 98% confidence interval for the population mean μ , assuming $\sigma = 1.35$.

- $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
- $\alpha = 0.02 \Rightarrow \alpha/2 = 0.01 \Rightarrow Z_{0.01} = 2.33$
- Solve: $0.25 = 2.33 \cdot \frac{1.35}{\sqrt{n}}$
- Result: $n = 156$

Exercise 7

Given a 90% confidence interval, the yearly income of Hanoi people with at least 3 years of experience is to be estimated within \$2000. The income range is no more than \$32,000, and σ is estimated as \$8000. Find the minimum sample size n .

- $\alpha/2 = 0.05 \Rightarrow z = 1.64$
- $E = 2000 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
- Solve: $2000 = 1.64 \cdot \frac{8000}{\sqrt{n}}$
- Result: $n = 44$

Exercise 8

Given a margin of error $E = 0.26$, find a minimum sample size n to construct a 95% confidence interval for the population proportion p :

- Confidence level = 0.95 $\Rightarrow \alpha/2 = 0.025 \Rightarrow z = 1.96$
- Prior knowledge: $p = 0.15$

- $E = 0.26 = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- Solve: $0.26 = 1.96 \cdot \sqrt{\frac{0.15(1-0.15)}{n}}$
- Result: $n = 8$