Labwork 3

Exercise 1:

Confidence level	α	$\alpha/2$	Table look-up area	$z_{\alpha/2}$
85%	0.15	0.075	0.925	1.44
97%	0.03	0.015	0.985	2.17

Exercise 2:

Confidence level	α	$\alpha/2$	Table look-up area	$t_{\alpha/2}$
85%	0.15	0.075	t 925	1.58
97%	0.03	0.015	t 985	2.72

$n = 12 \rightarrow df = n - 1 = 12 - 1 = 11$

Exercise 3:

Sample size: $n = 2407$

Sample mean: Average (tbc) $= \bar{x} = 6.67$

Sample standard deviation:

$$s = \sqrt{\frac{1}{2407-1} \cdot \sum_{i=1}^{2407} (x_i - \bar{x})^2} = 0.503$$

Confidence interval: 90%

$\alpha = 0.1 \Rightarrow \alpha/2 = 0.05$

$n = 2407 \Rightarrow df = n - 1 = 2406$

$\Rightarrow t_{\alpha/2} = 1.645$

$$CI = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 1.645$$

$$= 6.67 \pm 1.645 \cdot \frac{0.503}{\sqrt{2407}} \Rightarrow [6.65; 6.69]$$

Exercise 4:

$n = 16$; $\bar{x} = 2.84$; $s = 0.48$

90% confidence interval

$\alpha = 0.1 \Rightarrow \alpha/2 = 0.05$

$n = 16 \Rightarrow df = 15 \Rightarrow t_{\alpha/2} = 1.753$

$CI = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 2.84 \pm 1.753 \frac{0.48}{\sqrt{16}}$

$\Rightarrow [2.63, 3.05]$

Exercise 5:

$n = 2407$

$x = 1988$ (number of female students)

$p = \frac{x}{n} = \frac{1988}{2407} = 0.83$

Confidence level: 95%

$\Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$\Rightarrow z_{\alpha/2} = 1.96$

$CI = p \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} = 0.83 \pm 1.96 \cdot \sqrt{\frac{0.83(1-0.83)}{2407}}$

$= 0.83 \pm 0.015$

$= [0.815, 0.845]$

Exercise 6:

$E = 0.25$, 98% confidence interval, $\delta = 1.35$

$\Rightarrow \alpha = 0.02 \Rightarrow \alpha/2 = 0.01$

$\Rightarrow z_{\alpha/2} = 2.33$

$E = z_{\alpha/2} \cdot \frac{\delta}{\sqrt{n}} \Leftrightarrow n = \left(\frac{z_{\alpha/2} \cdot \delta}{E}\right)^2$

$n = \left(\frac{2.33 \times 1.35}{0.25}\right)^2 \approx 159$

Exercise 7:

$E = \$1000$, $\delta = 8000$

90% confidence interval

$\Rightarrow \alpha = 0.1 \Rightarrow \alpha/2 = 0.05$

$\Rightarrow z_{\alpha/2} = 1.64$

$n = \left(\frac{z_{\alpha/2} \cdot \delta}{E}\right)^2 = \left(\frac{1.64 \times 8000}{1000}\right)^2 \approx 44$

Exercise 8:

$E = 0.26$

95% confidence interval

$\Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$\Rightarrow 1 - \frac{\alpha}{2} = 0.975 \Rightarrow z_{\alpha/2} = 1.96$

$$n = \frac{p(1-p)\,z^2_{\alpha/2}}{E^2}$$

- No prior knowledge p

Use the most conservative estimate

$p = 0.5$

(to maximize $p(1-p)$ → maximize n)

$$n = \frac{0.5(1-0.5) \times 1.96^2}{0.26^2} \approx 15$$

→ minimum sample size $n = 15$

- $p = 0.15$

$$n = \frac{0.15(1-0.15) \times 1.96^2}{0.26^2} \simeq 8$$