Introduction to Earth System

Dynamics of the atmosphere

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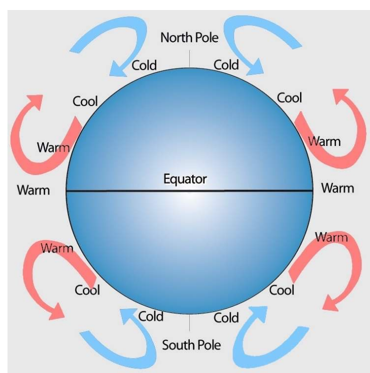
Redistribution of the energy

Transfer of energy: from low to high latitudes via atmospheric and oceanic circulations



Energy redistribution occurs through:

- Conduction
- Convection
- Advection
- Latent heat

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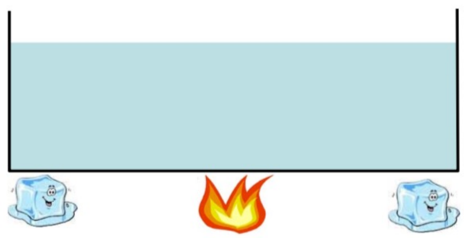
Observe the movements and global structures of the atmosphere



Example: the container has the two ends constantly cooled and the center heated

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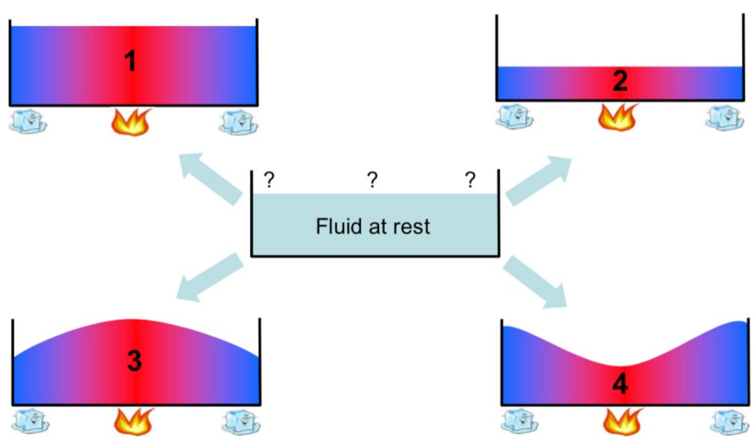
Q: What change can be observed?



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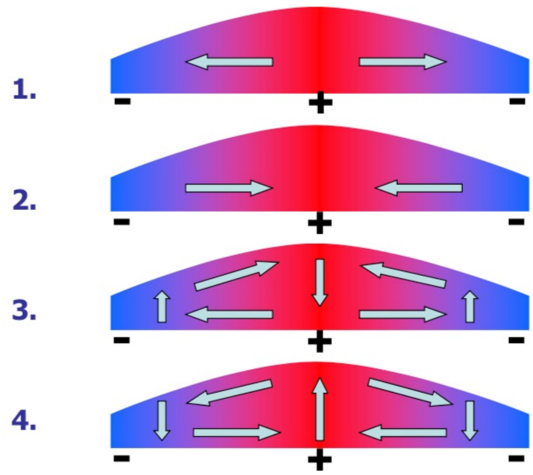
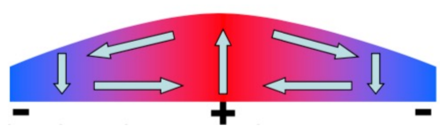
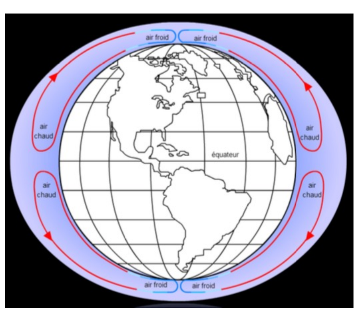
Q: Which sketch best illustrates the movements within the fluid?



Hadley's cell

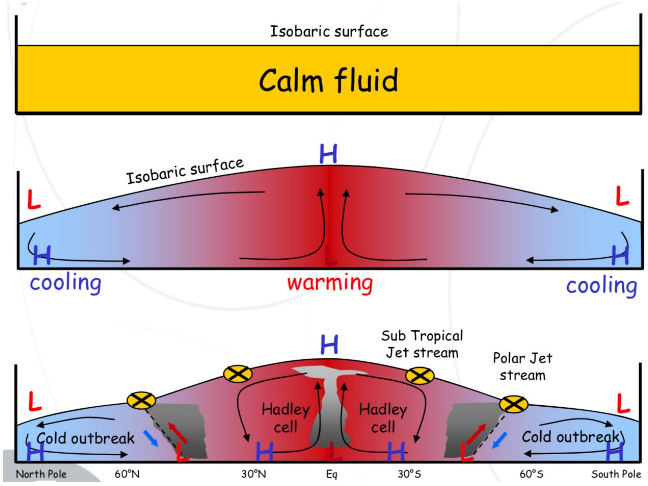


- **Georges HADLEY** in 1735 → a simple model of atmospheric circulation → Hadley's cell.
- Troposphere



The Hadley cell works only for a non-rotating earth

Origin of the General Circulation



Without energy exchange

With energy exchanges

The Earth is round and turns on itself

Q: Why is the Hadley's descending branch at about 30ºN

Q: Why does sub-tropical jet stream exist?

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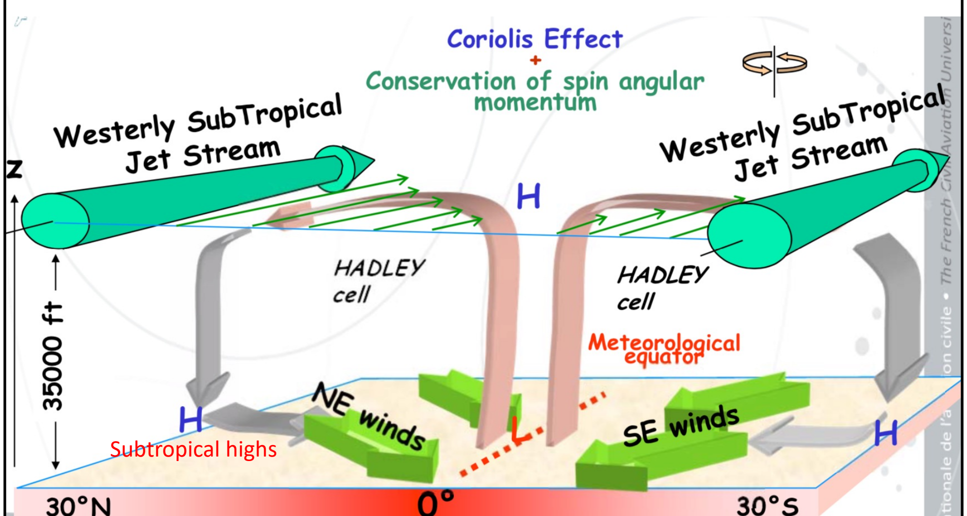
Zoom in tropical region (30ºN-30ºS)



JET = quasi-horizontal, flattened tubular air-current generally located near the tropopause, very unstable and characterized by high speeds (> 60 kt) and by strong vertical and horizontal sharing of the wind

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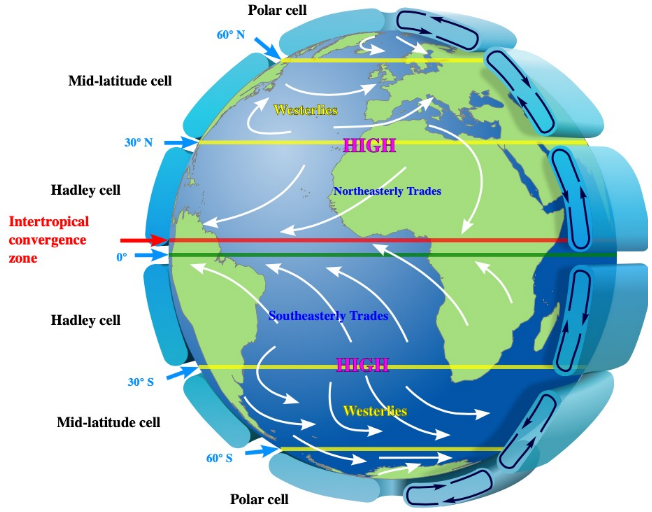

NASA

Global circulation of Earth's atmosphere displaying Hadley cell, Ferrell cell and polar cell.

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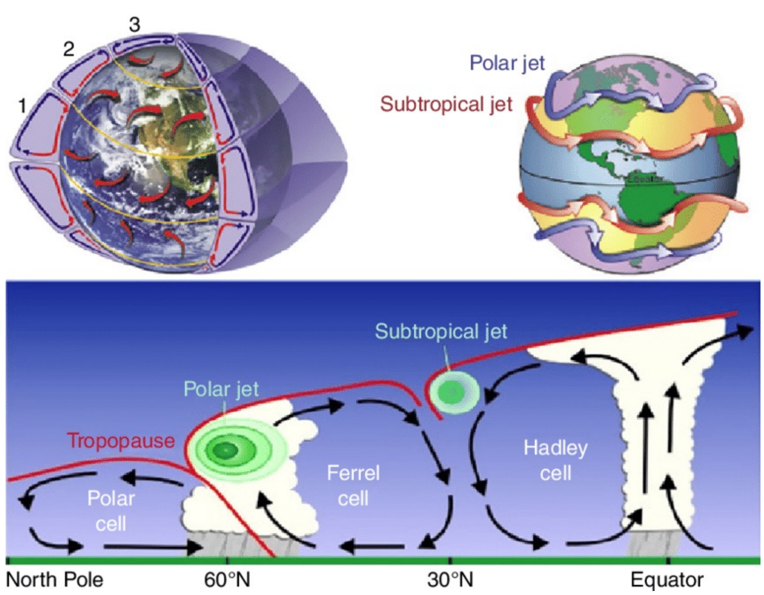

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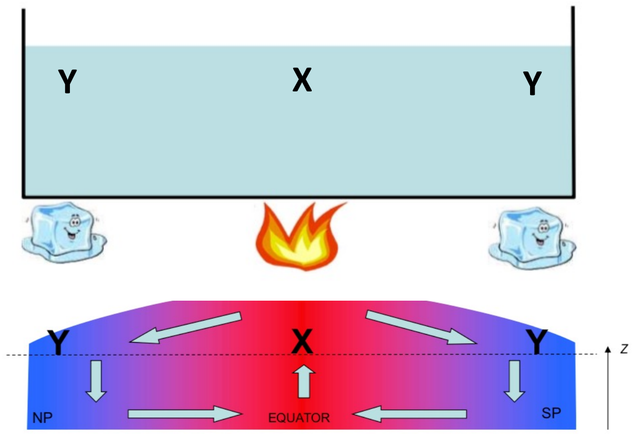
Description at upper level

Q: What can you say about the pressure field observed in X and Y, 2 points located at the same altitude in the troposphere?

1. $P_x=P_y$
2. $P_x>P_y$
3. $P_x<P_y$



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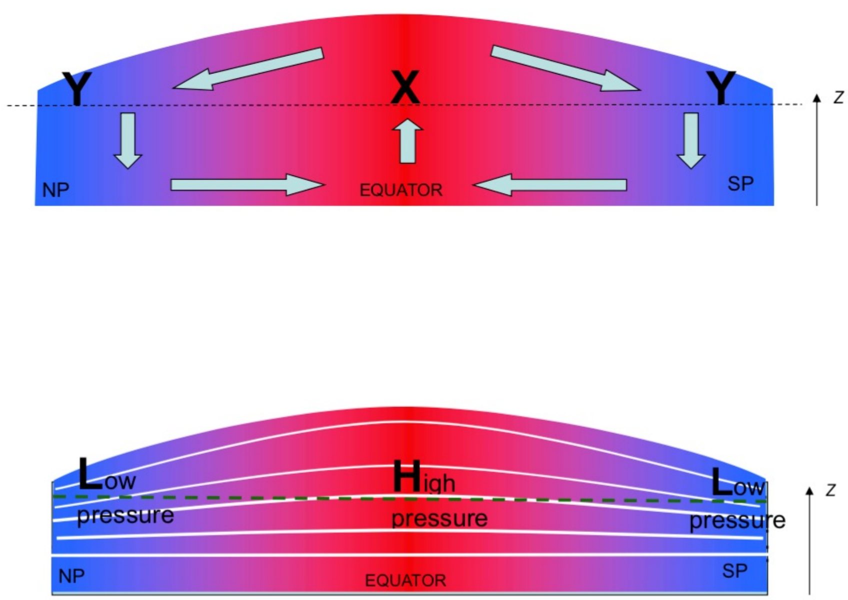

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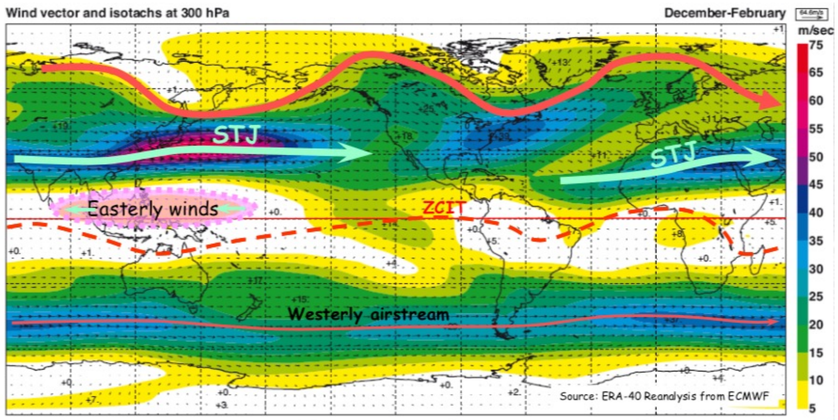
At upper level from December to February

Q: What can you see?



- Fast wave circulation in mid latitudes mainly in Northern hemisphere
- SubTropical Jet (STJ) mainly in Northern hemisphere (can reach 300kts in the East of Asia)
- Light easterly winds near 0º.

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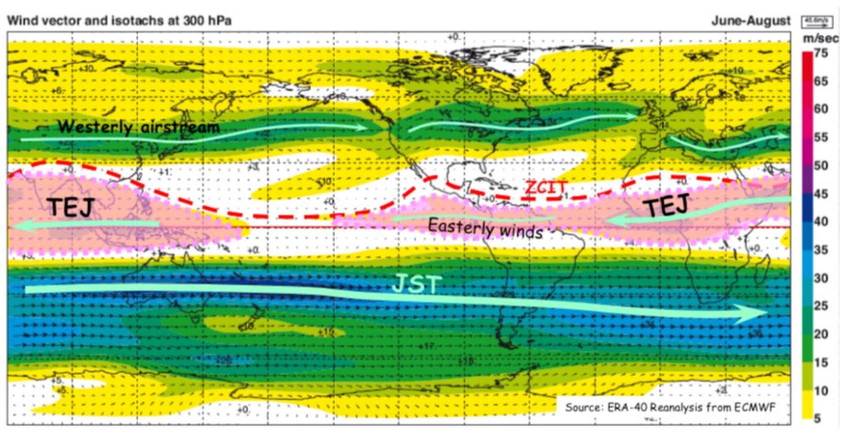
At upper level from June to August

Q: What can you see?



- Weakening of the westerly winds in northern hemisphere
- SubTropical Jet (STJ) mainly in Southern hemisphere
- Tropical Easterly Jet between 20º˜and 0º, from Asia to Africa

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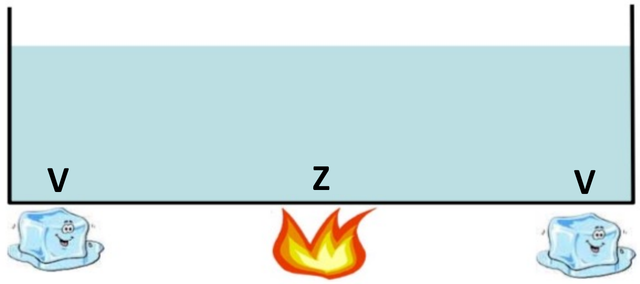
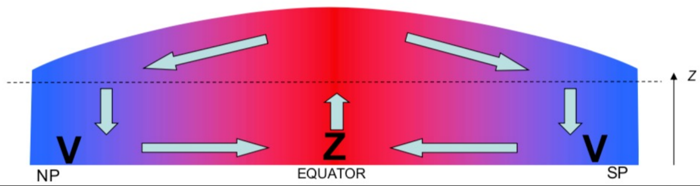
At mean sea level

Q: What can you say about the pressure field observed in Z and V, 2 points located at sea level?

1. $P_Z = P_V$
2. $P_Z > P_V$
3. $P_Z < P_V$





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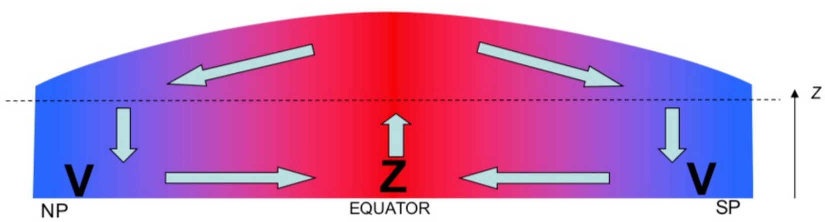
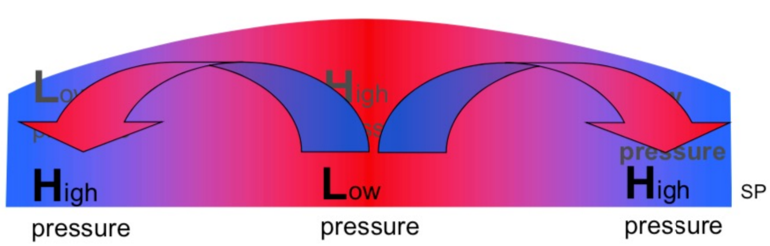
- Cold air having a higher density than hot air → a higher pressure.
- Vertical motion brings mass on polar side, and releases mass on equatorial side.





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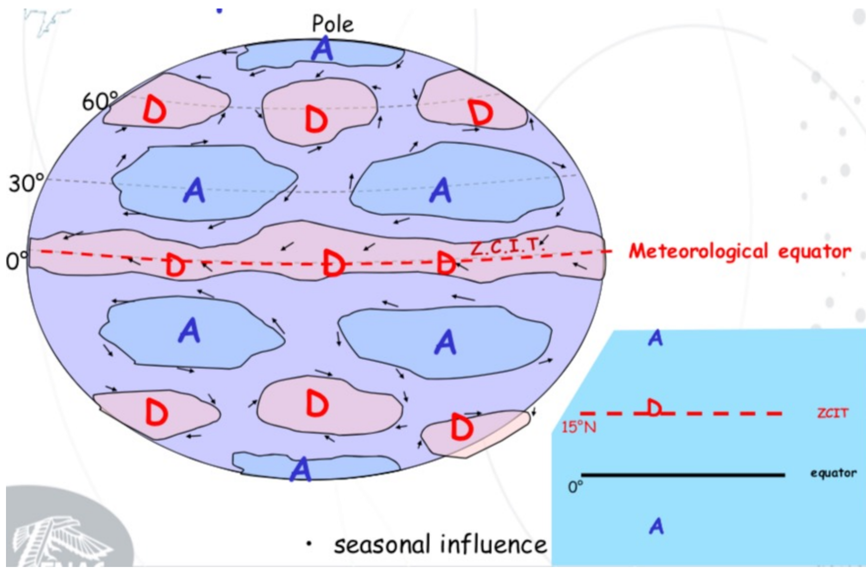
At mean sea level



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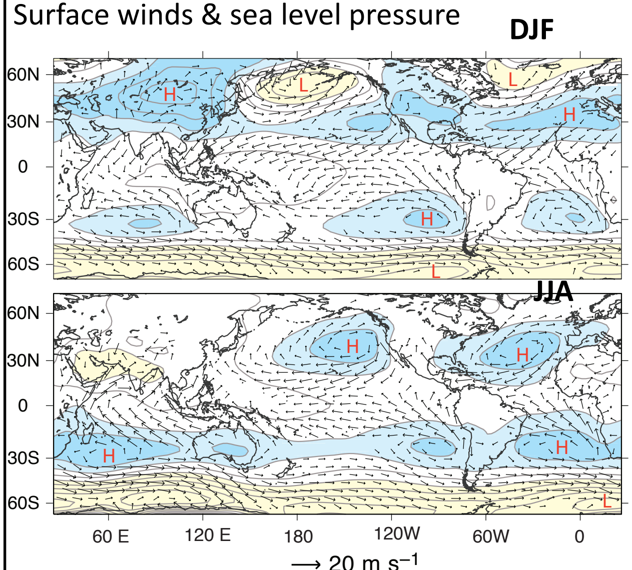
Real world

Surface winds & sea level pressure



The contour interval for s.l.p is 5hPa. Pressures above 1015hPa are shaded blue, and below 1000hPa are shaded yellow

- **Atlantic & Pacific Oceans**: surface winds depict well the features shown in the aqua-planet.
- **Southern hemisphere**: ~ aqua-planet
- NH: surface winds strongly influenced by the presence of high latitude continents
- Difference of DJF & JJA

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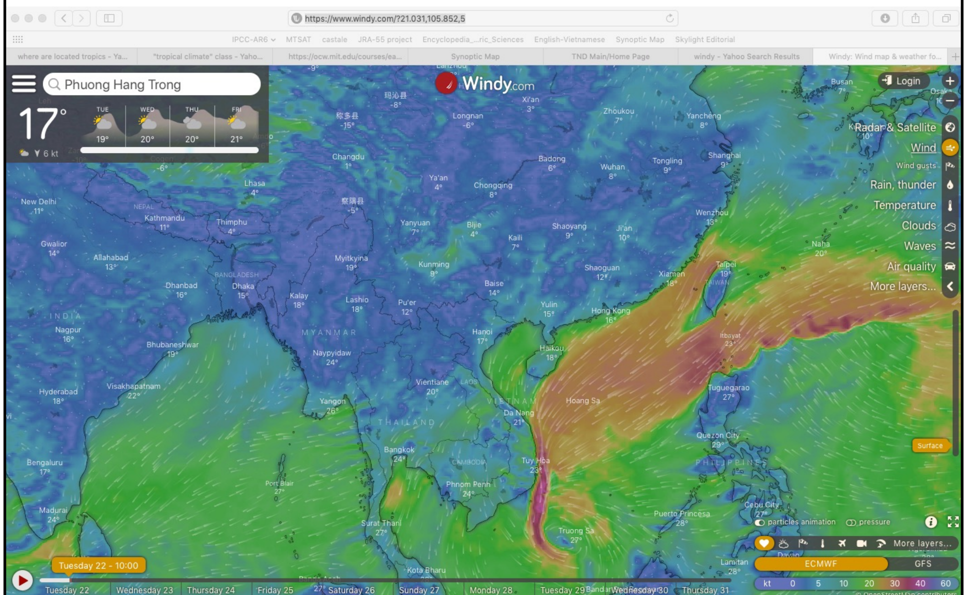
Windy.com

Today's weather



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Dynamics of the atmosphere: The equations

- The dynamics of the atmosphere → in the principles of conservation of momentum, mass, and energy
  - The Newton's equations of motion
  - The equation of continuity
  - The thermodynamic energy equation

$$\frac{du}{dt} - (f + \frac{u\tan\phi}{a})v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + (f + \frac{u\tan\phi}{a})u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$p = R\rho T$$

$$\frac{\partial p}{\partial z} + g\rho = 0$$

$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial \rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [sources - sinks]$$

- **Independent variables:** space & time coordinates (x,y,z,t)
- **Dependent variables:** velocity, pressure, density, temperature

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The hydrostatic equation (see Lecture #2)

$$\frac{du}{dt}-(f+\frac{u\tan\phi}{a})v+\frac{1}{\rho}\frac{\partial p}{\partial x}+F_x=0$$

$$\frac{dv}{dt}+(f+\frac{u\tan\phi}{a})u+\frac{1}{\rho}\frac{\partial p}{\partial y}+F_y=0$$

$$p=R\rho T$$

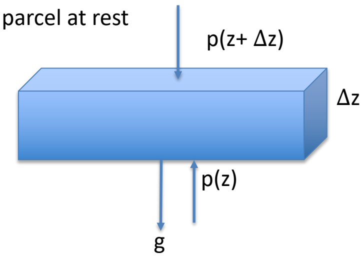
$$\frac{\partial p}{\partial z}+g\rho=0$$

$$\frac{dT}{dt}+(\gamma-1)T\nabla\cdot\mathbf{V}=\frac{Q}{c_p}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\mathbf{V}=0$$

$$\frac{\partial\rho_w}{\partial t}+\nabla\cdot\rho_w\mathbf{V}=[sources-sinks]$$

- In an air parcel at rest



- Force downward: $-[p(z+\Delta z)\Delta x\Delta y+mg]$
- Force upward: $+[p(z)\Delta x\Delta y]$
- Equilibrium:

$$\frac{p(z+\Delta z)-p(z)}{\Delta z}\Delta x\Delta y\Delta z+mg=0$$

$$\frac{\partial p}{\partial z}+\rho g=0$$

→ **Hydrostatic balance equation:** balance between the **vertical pressure gradient** and **gravity**

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Preliminary notions

❖ **Eulerian** vs. **Lagrangian**

Eulerian: watch the flow

- Denote the change of a variable with time at **a fixed point** by the Eulerian (or partial) derivative:

$$\frac{\partial p}{\partial t}$$ at a fixed point (x, y, z)

Lagrangian: drift along the flow

- Denote the change of a variable with time **following the flow** by the Lagrangian (or total or material) derivative:

$$\frac{\mathrm{d}p}{\mathrm{d}t}$$ & the fluid parcel is fixed

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Connection between partial and total derivatives

- For the pressure p(x(t),y(t),z(t),t)

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial p}{\partial y} \times \frac{\partial y}{\partial t} + \frac{\partial p}{\partial z} \times \frac{\partial z}{\partial t}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \mathrm{v}\frac{\partial p}{\partial y} + \mathrm{w}\frac{\partial p}{\partial z}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \boldsymbol{V}.\boldsymbol{\nabla} p \quad (*)$$

Eq (*) is true for all variables

$$\frac{d(\quad)}{dt} = \frac{\partial(\quad)}{\partial t} + \boldsymbol{V}.\boldsymbol{\nabla}(\quad)$$

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Exercise #1

- Given the flow with the speed *u* in the *x*-direction

$u = \mathrm{a}\sin(kx - wt)$ with the constants
a: amplitude
k: wave number
w: frequency

1. Calculate the local and total time derivative of *u*
2. If $u = 2\mathrm{a}\sin(kx - wt)$, how do $\partial u/\partial t$ and $\mathrm{d}u/\mathrm{d}t$ change?

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Continuity equation (conservation of mass)

$$\frac{du}{dt} - (f + \frac{u\tan\phi}{a})v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + (f + \frac{u\tan\phi}{a})u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$p = R\rho T$$

$$\frac{\partial p}{\partial z} + g\rho = 0$$

$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial \rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [sources - sinks]$$

- Mass conservation: the mass of an air parcel remains unchanged with time
- Continuity equation is the mathematical expression of mass conservation
- **Eulerian** & **Lagrangian** forms of the continuity equation

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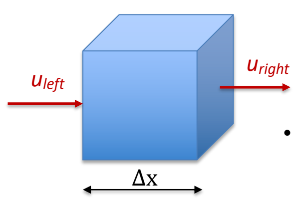
Continuity equation: Eulerian formulation



- Consider a cube fixed in space, of dimension $\Delta x = \Delta y = \Delta z$
- The change of mass of the air in the cube

  $\Delta M = mass_{in} - mass_{out}$

- For simplicity, consider flow in the x-direction

Total mass in the cube: $M = \rho\Delta x\Delta y\Delta z$

Mass change in time Δt: $\Delta M = \frac{\partial M}{\partial t}\Delta t = \frac{\partial \rho}{\partial t}\Delta t.V$

Influx from the left: $\rho_{\mathrm{left}}(u_{\mathrm{left}}\Delta t)\Delta y\Delta z = (\rho u)_{\mathrm{left}}\Delta t\Delta y\Delta z$

Outflux on the right: $\rho_{\mathrm{right}}(u_{\mathrm{right}}\Delta t)\Delta y\Delta z = (\rho u)_{\mathrm{right}}\Delta t\Delta y\Delta z$

Net flow into the cube: $F = [(\rho u)_{left} - (\rho u)_{right}]\Delta t\Delta y\Delta z$

$$F = \frac{[(\rho u)_{left} - (\rho u)_{right}]}{\Delta x}\Delta t\Delta x\Delta y\Delta z$$

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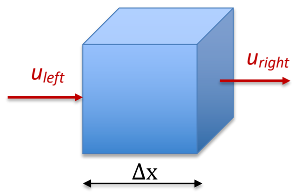
Continuity equation: Eulerian formulation



- ΔM = F

$$\rightarrow \frac{\partial \rho}{\partial t} = \frac{[(\rho u)_{left} - (\rho u)_{right}]}{\Delta x} \approx -\frac{\partial (\rho u)}{\partial x}$$

- Similarly for the two other dimensions y and z

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z}\right)$$

- Using the divergence operator

$$\frac{\partial \rho}{\partial t} + \mathbf{\nabla}.(\rho \mathbf{V}) = 0$$

→ The Eulerian form of the continuity equation

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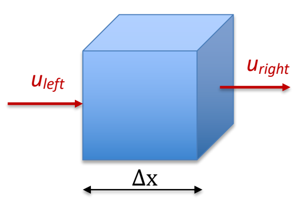
Continuity equation: Lagrangian formulation



- Now, consider an air parcel of mass M, contained in a cube moving with the flow. M is unchanged with time.

$$M = \rho \Delta x \Delta y \Delta z = \rho V$$

→ Mass change in time Δt: $\Delta M = \frac{dM}{dt}\Delta t = 0 \rightarrow \frac{d\log M}{dt} = 0$

We have: $\log M = \log \rho + \log \Delta x + \log \Delta y + \log \Delta z$

$$\rightarrow \frac{1}{\rho}\frac{d\rho}{dt} + \left(\frac{1}{\Delta x}\frac{d\Delta x}{dt} + \frac{1}{\Delta y}\frac{d\Delta y}{dt} + \frac{1}{\Delta z}\frac{d\Delta z}{dt}\right) = 0$$

Note that $\Delta x = x_{right} - x_{left}$, we have

$$\frac{1}{\Delta x}\frac{d\Delta x}{dt} = \frac{1}{\Delta x}\left(\frac{dx_{right}}{dt} - \frac{dx_{left}}{dt}\right) = \frac{u_{right} - u_{left}}{\Delta x} \approx \frac{\partial u}{\partial x}$$

Therefore, we get:

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \leftrightarrow \frac{d\rho}{dt} + \rho \mathbf{\nabla}.\mathbf{V} = 0$$

→ The Lagrangian form of the continuity equation

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Exercise #2

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0$$

Eulerian form

$$\frac{d\rho}{dt} + \rho \nabla.\mathbf{V} = 0$$

Lagrangian form

- Although the two forms look different, but they are equivalent. **Why?**

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Incompressibility

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0$$

Eulerian form

$$\frac{d\rho}{dt} + \rho \nabla.\mathbf{V} = 0$$

Lagrangian form

- If the fluid is incompressible (e.g. ocean)

→ the volume of the parcel is unchanged
→ the material density is constant following the flow
→ the continuity equation becomes

$$\nabla.\mathbf{V} = 0$$

- **Advantages** of assuming incompressibility → a closed system without having to consider the thermodynamics explicitly.
- **For compressible flow**
  → need another equation, i.e. ***the equation of state***, for ρ
  → but this introduces another equation, i.e. ***the thermodynamic equation***, for temperature T

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Practice 5: EBM-0D & Feedback (cont.)

- The planetary albedo depends on the surface temperature. Let's assume the following relations:
  - If $T_{sf} < T_{ice}, \quad \alpha = \alpha_{ice}$
  - If $T_{sf} > T_{land}, \alpha = \alpha_{land}$
  - If $T_{land} \geq T_{sf} \geq T_{ice}$, $\alpha = \alpha_{ice} + (\alpha_{land} - \alpha_{ice}) \times \dfrac{T_{sf} - T_{ice}}{T_{land} - T_{ice}}$

  where $\alpha_{ice} = 0,6$; $\alpha_{land} = 0,32$, $\varepsilon\tau_a$=0.62, $T_{ice} = -10°C$ is the temperature where the Earth becomes a Snowball; $T_{land} = 10°C$ is the temperature where the Earth remains in the nowadays state.

1. Write a program to estimate the equilibrium T_{sf} with the initial temperature varying from 13˚C to 3˚C with a step of 1˚C. Plot a figure showing the dependency of the equilibrium temperature on the initial one. The solar constant remains unchanged at S_0=1368W/m^2.
2. Write a program to estimate the equilibrium surface temperature with varying solar flux S (S/S_0 ranges from 0.2 to 2 with a step of 0.05). The initial temperature varies from 15˚C down to -15˚C with a step of 1˚C.

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