Algebraic Structure

Tutorial # 1: Introduction to Algebra

Exercise 1:
Using induction to prove that: $6^n - 1$ is divisible by 5, for n any positive integer

Exercise 2:
Prove that $n! > 2^n$ for $n \geq 4$

Exercise 3:
Using induction to prove that $3.n! > n^2$ for all natural numbers n.

Exercise 4:
The Fibonacci numbers are:

$$1, 1, 2, 3, 5, 8, 13, 21, ...$$

We can define them inductively by $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for $n \in N$.
Using induction, prove that:

- $f_{2n} = f_1 + f_3 + f_5 + ... + f_{2n-1}$ for all n ≥ 1.
- $f_n < 2^n$.
- f_{n+1} and f_n are relatively prime.

Exercise 5:
Find all $x \in \mathbb{Z}$ satisfying each of the following equations.

- $3x \equiv 2 \pmod 7$
- $5x + 1 \equiv 13 \pmod{23}$
- $2x \equiv 1 \pmod 6$

Exercise 6:
Solve the following system with integer solutions:

$$\begin{cases} 3x + 7y \equiv 4 \pmod{11} \\ 8x + 6y \equiv 1 \pmod{11} \end{cases}$$

Hint: Use Cayley's table to solve the system

Exercise 7:

Solve the following system with integer solutions:

$$\begin{cases} 3x - 2y + 6z \equiv 12(mod 13) \\ 2x - 5y + 7z \equiv 11(mod 13) \\ 9x + 2y + 5z \equiv 3(mod 13) \end{cases}$$