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Algebraic Structure

Tutorial # 1: Introduction to Algebra

Exercise 1:
Prove that

$$1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \in \mathbb{N}$

Exercise 2:
Prove that $n! > 2^n$ for $n \geq 4$

Exercise 3:
Prove that $10^{n+1} + 10^n + 1$ is divisible by 3 for all $n \in \mathbb{N}$

Exercise 4:
Use induction to prove that $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$ for $n \in \mathbb{N}$.

Exercise 5:
Find all $x \in \mathbb{Z}$ satisfying each of the following equations.

$$3x \equiv 2 \pmod 7$$

$$5x + 1 \equiv 13 \pmod{23}$$

$$2x \equiv 1 \pmod 6$$

Exercise 6:
Solve the following system

$$3x + 7y \equiv 4 \pmod{11}$$

$$8x + 6y \equiv 1 \pmod{11}$$

Hint: Use Cayley's table to solve the system

Exercise 7:
Solve the following system

$$3x + 5y - 7z \equiv 8 \pmod{83}$$

$$8x - 9y + 13z \equiv 13 \pmod{83}$$

$$7x + 4y + 5z \equiv 15 \pmod{83}$$

Exercise 8:

Find the inverse of the followin matrix, whose matrices are elements of $\mathbb{Z}_7$

$$A = \begin{bmatrix} 5 & 2 \\ 6 & 3 \end{bmatrix}$$