

Algebraic Structure

Tutorial # 2: Sets, Mapping and Functions

Exercise 0:

Which relation is NOT a function?

$$\{(1, -5), (3, 1), (-5, 4), (4, -2)\}$$

$$\{(2, 7), (3, 7), (4, 7), (5, 8)\}$$

$$\{(1, -5), (-1, 6), (1, 5), (6, -3)\}$$

$$\{(3, -2), (5, -6), (7, 7), (8, 8)\}$$

Exercise 1:

Which of the following relations $f : \mathbb{Q} \rightarrow \mathbb{Q}$ define a function? In each case, supply a reason why f is or is not a function.

$$f(p/q) = \frac{p+1}{p-2}$$

$$f(p/q) = \frac{p}{p+q}$$

Exercise 2:

Determine which of the following functions are one-to-one and which are onto. If the function is not onto, determine its range

$$f : \mathbb{R} \rightarrow \mathbb{R} \text{ defined by } f(x) = e^x$$

$$f : \mathbb{Z} \rightarrow \mathbb{Z} \text{ defined by } f(n) = n^2 + 5$$

$$f : \mathbb{R} \rightarrow \mathbb{R} \text{ defined by } f(x) = \sin x$$

$$f : \mathbb{R} \rightarrow \mathbb{R} \text{ defined by } f(x) = |x| + 2$$

Exercise 3:

Find the inverse of the following functions:

$$f(x) = \frac{1}{2}x + 7$$

$$f(x) = (x - 2)^3 + 1$$

$$f(x) = \frac{1 + 2x}{7 + x}$$

Exercise 4:

Suppose that $f(x) = x^2 - 4x + 3$ and $g(x) = \sqrt{x+1}$. Determine the domain of $f+g$, $f-g$, fg , and f/g .

Exercise 5:

Define a function on real numbers by

$$f(x) = \frac{x+1}{x-1}$$

What are the domain and range of f ? What is the inverse of f ? compute $f \circ f^{-1}$ and $f^{-1} \circ f$

Exercise 6:

What is the inverse of $f(x) = \ln\left(\frac{e^x}{e^x-1}\right)$? what is the domain of f^{-1} ?