

# Algebraic Structure

## Tutorial # 4: Cosets

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### Exercise 1:

List the left and right cosets of the subgroups in each of the following.

- $\langle 8 \rangle$  in  $\mathbb{Z}_{24}$
- $\langle 3 \rangle$  in  $U(8)$

### Exercise 2:

Suppose that  $g^n = e$ , where  $g$  is the generator of  $G$ . Show that the order of  $G$  divides  $n$ .

### Exercise 3:

Let  $H = \{0, \pm 3, \pm 6, \pm 9, \dots\}$ . Find all the left cosets of  $H$  in  $\mathbb{Z}$ . Decide whether or not the following cosets of  $H$  are the same.

- $11 + H$  and  $17 + H$
- $-1 + H$  and  $5 + H$
- $7 + H$  and  $23 + H$

### Exercise 4:

Suppose that  $G$  generated by  $a$  has order 15. Find all of the left cosets of  $\langle a^5 \rangle$  in  $\langle a \rangle$ .

### Exercise 5:

Let  $G$  be a group of order 60. What are the possible orders for the subgroups of  $G$ ?