

Algebraic Structure

Tutorial # 3: Cyclic Groups

Exercise 1:

Find all cyclic subgroups of the group \mathbb{Z}_7

Exercise 2:

Let G be a cyclic group of order 6. How many of its elements generate G ? Same question for the cyclic groups of order 5, 8, and 10.

Exercise 3:

Determine whether G is cyclic: $G = \mathbb{Z}_7^*$, $G = \mathbb{Z}_{12}^*$

Exercise 4:

- Find subgroup generated by 5 in $U(18) \in \mathbb{Z}_{18}$
- Find subgroup generated by 3 in $U(20) \in \mathbb{Z}_{20}$

Exercise 5:

Let $G = \mathbb{Z}_{20}^*$ be the group of invertible elements in \mathbb{Z}_{20} . Find two subgroups of order 4 in G , one that is cyclic and one that is not cyclic.

Exercise 6:

Show that if $G = \langle a \rangle$ is a cyclic group and $b \in G$ an element such that $a = b^k$ for an integer k , then b is a generator for G .