Exercise 1:
Show that $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ is a group under the operation of multiplication.

Exercise 2:
Given the groups $\mathbb{R}^*$ and $\mathbb{Z}$, let $G = \mathbb{R}^* \times \mathbb{Z}$. Define a binary operation $\circ$ on G by $(a, m) \circ (b, n) = (ab, m + n)$. Show that G is a group under this operation.

Exercise 3:
Show that addition and multiplication mod n are associative operations in $\mathbb{Z}$.

Exercise 4:
Let G be a group and suppose that $(ab)^2 = a^2b^2$ for all a and b in G. Prove that G is an abelian group.

Exercise 5:
Let $S = \mathbb{R} \setminus \{-1\}$ and define a binary operation on S by $a * b = a + b + ab$. Prove that $(S, *)$ is an abelian group.

Exercise 6:
Let a and b be elements of a group G. If $a^4b = ba$ and $a^3 = e$, prove that ab = ba.

Exercise 7:
If $xy = x^{-1}y^{-1}$ for all x and y in a group G, prove that G must be abelian.

Exercise 8:
Let a be an element in a group. Prove that $(a^n)^{-1} = (a^{-1})^n$.

Exercise 9:
Let a, x be elements in a group G. Prove that ax = xa if and only if $a^{-1}x = xa^{-1}$

Exercise 10:
Let a, b be elements of a group G. Assume that a has order 5 and $a^3b = ba^3$. Prove that ab = ba.

Exercise 11:
Let a and b be integers. Prove that the subset $a\mathbb{Z} + b\mathbb{Z} = \{ak + bl | l, k \in \mathbb{Z}\}$ is a subgroup of $\mathbb{Z}$