Algebraic Structure

Tutorial # 6: Rings and Fields

Exercise 0:
Prove the properties in Proposition 1

Exercise 1:
Which of the following sets are rings with respect to the usual operations of addition and multiplication?

- $7\mathbb{Z}$
- $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2}, a, b \in \mathbb{Q}\}$

Exercise 2:
Let R be a ring . Suppose $(xy)^2 = xy$ for every $x, y \in R$. Prove that R is a commutative ring.

Exercise 3:
If R is a ring with the identity element 1 and ϕ is a homomorphism of R onto R' , then prove that $\phi(1)$ is the identity element of R'.

Exercise 4:
If R is a ring, let $Z(R) = \{x \in R | xy = yx, \forall y \in R\}$. Prove that Z(R) is a subring of R.

Exercise 5:
An element x of a ring R is called nilpotent if some power is zero. Prove that if x is nilpotent, then 1 + x is a unit (an unit in a ring is any element u that has an inverse element v in the multiplicative monoid of R such that $uv = vu = 1_R$)

Exercise 6:
Prove that if $f : A \to B$ and $g : B \to C$ are both ring isomorphisms, then so is their composition $(f \circ g) : A \to C$.