Algebraic Structure

Tutorial # 7: Rings and Integral Domains

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Exercise 1:
Show that the matrices of the size $n \times n$ is not an integral domain.

Exercise 2:

- Let R be a ring. Show that every nilpotent element of R is either zero or a zero-divisor.
  An element $s \in R$ is nilpotent if $s^n = 0$ for some positive $n \in \mathbb{N}$
- If R is an integral domain show that no nonzero element of R is nilpotent.

Exercise 3:
Let R be an integral domain. Prove that every subring of R with identity is an integral domain.

Exercise 4:
Let R be a commutative ring with identity element 1. Suppose that a is an element of R for which $a^2 = a$.

Show that if R is an integral domain, then either a=0 or a=1.

Exercise 5:
Let R, S be rings. Define addition and mulplication on the Cartesian product $R \times S$ as follows:

- (r,s) + (r',s') = (r+r', s+s')
- (r,s) . (r',s') = (r.r', s.s')

Show that

- $R \times S$ is a ring.
- If both R, S are commutative, show $R \times S$ is commutative.
- If both R, S have an identity, show $R \times S$ has an identity.