|  |  |
| --- | --- |
| **University of Science and Technology of Hanoi**\*\*\***Retake Examination (Open Book)****Subject: Algebraic Structure****Sheet**: 02 **No of pages**: 01 | ***Intake****: BI10-ICT* ***Academic year****: 2020 - 2021****Date****: 30/06/2021* ***Time****: 60 minutes****Important instructions****(according to lecturer’s decision)*Copying from any source leads to 0 for the final mark. |
| **Pathway coordinator** |  | **Lecturer (or Head of Subject)**  | **Dr. Đoàn Nhật Quang** |
| **Student name** |  | **Student’s ID** |  |

**Problem 1** (5pts)

Solve the following linear system where x, y $\in Z\_{11}$

 $\left\{\begin{array}{c}2x+8y ≡8 (mod 11)\\3x-7y ≡10 \left(mod 11\right)\end{array}\right.$

Specify your algebra development to justify answer.

**Problem 2** (5pts)

Give an example of a cyclic group U $⊂ Z\_{13}$. Justify the generator of the cyclic group.

**Problem 3** (5pts)

Using induction to prove that the following statement is true for all positive integers n:

$$\sum\_{k=1}^{n}k^{3}= \left(\frac{(n+1)^{2}n^{2}}{4}\right)$$

**Problem** 4 (5pts)

A function f is defined by f: $Z\_{8} \rightarrow Z\_{11}$

f(x) = 3x (mod 12)

Is the function **homomorphic**? **isomorphic**? Justify your answer.

-----END----