ELECTRICAL CIRCUITS I

BACHELOR – SECOND YEAR B2

Subject	**Course name in English: Electrical Circuits I**		
	Course name in Vietnamese: Lý thuyết Mạch điện I		
Instructor (s)	Dr. Nguyen Xuan Truong mail: nguyen-xuan.truong@usth.edu.vn Dr. Hoang Trung Kien mail: hoang-trung.kien@usth.edu.vn		
Code		Credit points (ECTS)	04
Required	Compulsory		
Prerequisites	Mathematical analysis, Physic (electricity); General mathematics (Differential Equations)		
Time Commitment	Lecture	30 hrs	
	Tutorial/Exercises	08 hrs	
	Practical/Lab-work	12 hrs	
	Total	50 hrs	

DR. NGUYEN XUAN TRUONG DR. HOANG TRUNG KIEN

Academic year
2021 - 2022

ELECTRICAL CIRCUITS
BACHELOR 2 - COURSES

❶ **COURSE 1: ELECTRICAL CIRCUITS I (EC I)**

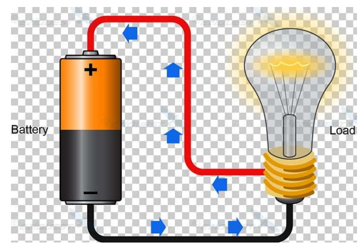
➡ **DIRECT CURRENT (DC) CIRCUITS ANALYSIS**

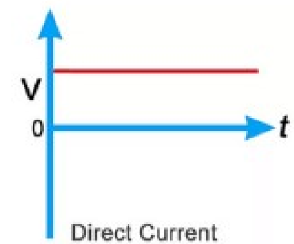
Battery 12V, 24V…
Solar Module convert light energy to DC current..
Operation – Amplifier…

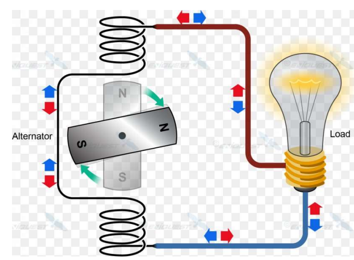
❷ **COURSE 2: ELECTRICAL CIRCUITS II (EC II)**

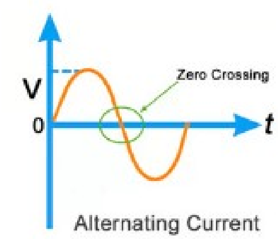
➡ **ALTERNATING CURRENT (AC) CIRCUITS ANALYSIS**

220V/ 50Hz, one phase
380V/50Hz, three-phase









FIRST SEMESTER: EC I - COURSE OUTLINE

❶ **MODULE 1: INTRODUCTION**

- Basic concept & quantities → Dr. Hoang Trung Kien
- Circuit elements → Dr. Nguyen Xuan Truong
- Basic laws in resistive circuit → Dr. Nguyen Xuan Truong

❷ **MODULE 2: DIRECT CURRENT CIRCUITS ANALYSIS**

- Introduction → Dr. Nguyen Xuan Truong
- Nodal & Loop analysis → Dr. Nguyen Xuan Truong
- Matlab DC analysis & Summary → Dr. Nguyen Xuan Truong

❸ **MODULE 3: NETWORK THEOREMS**

- The concepts of linearity and equivalence → Dr. Nguyen Xuan Truong
- Thevenin's and Norton's theorems → Dr. Nguyen Xuan Truong
- Max Power transfert & Summary → Dr. Nguyen Xuan Truong

❹ **MODULE 4: OPERATIONAL AMPLIFIER**

- Introduction → Dr. Nguyen Xuan Truong
- Op-Amp operation & Applications → Dr. Nguyen Xuan Truong
- Circuit analysis & Summary → Dr. Nguyen Xuan Truong

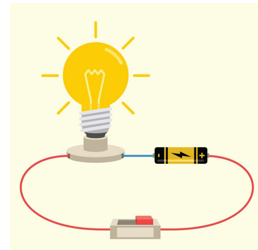
❺ **MODULE 5: FIRST AND SECOND ORDER TRANSIENT CIRCUITS**

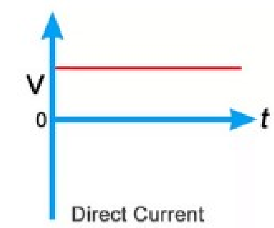
- Introduction: steady-state & transient regimes → Dr. Nguyen Xuan Truong
- First and Secon-Order transient circuits → Dr. Nguyen Xuan Truong
- Matlab for DC transient analysis & Summary → Dr. Nguyen Xuan Truong

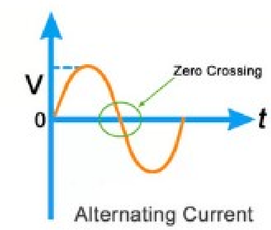
LECTURE SCHEDULE

N°	Content	Schedule
1	**MODULE 1:** INTRODUCTION	3-Hours Lect
2	**PRACTICAL:** N°1	3-Hours Lab
3	**MODULE 2**: DIRECT CURRENT CIRCUITS ANALYSIS	6-Hours Lect & 2-Hours Tutorial
4	**PRACTICAL:** N°2 & 3	6-Hours Lab
5	**MODULE 3**: NETWORK THEOREMS	6-Hours Lect & 2-Hours Tutorial
6	**MID-TERM EXAM**	1.5-Hours
7	**MODULE 4**: OPERATIONAL AMPLIFIER	6-Hours Lect & 2-Hours Tutorial
8	**MODULE 5**: FIRST AND SECOND ORDER TRANSIENT CIRCUITS	9-Hours Lect & 2-Hours Tutorial
9	**PRACTICAL:** N°4	3-Hours Lab
10	**FINAL EXAM**	2-Hours

MODULE 1 : INTRODUCTION – 3 HRS



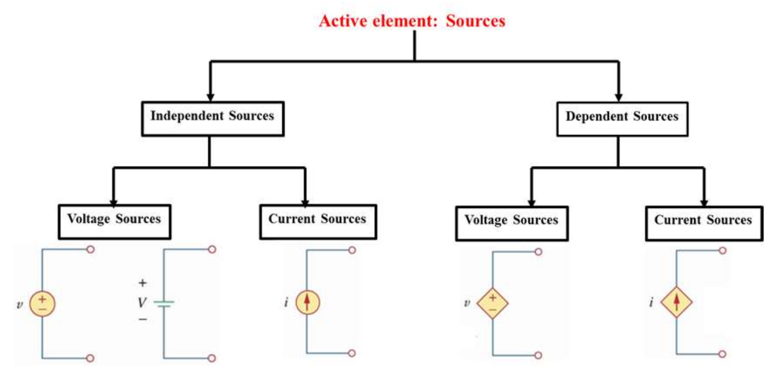




❶ BASIC CONCEPTS AND QUANTITIES

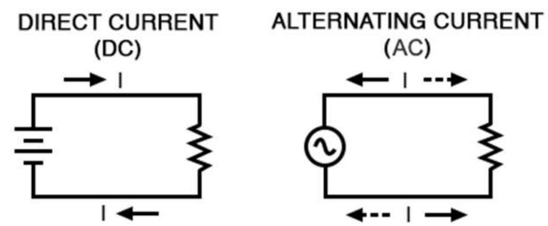
Know the definitions of basic electrical quantities: DC & AC voltage, DC & AC current, and AC & DC power

Basic characteristics of sinusoidal functions (AC current, cosine waveform, frequency, magnitude, AC power…), phasor



❷ CIRCUIT ELEMENTS

Active element: voltage and current source (independent, dependent)
Passive element: resistor, inductor, capacitor
Measuring devices: Ohmmeter, Ammeter, Voltmeter



❸ BASIC LAWS IN ELECTRIC CIRCUIT

Ohm's Law
Kirchhoff's Current Law
Kirchhoff's Voltage Law

→ **to solve electric circuits**

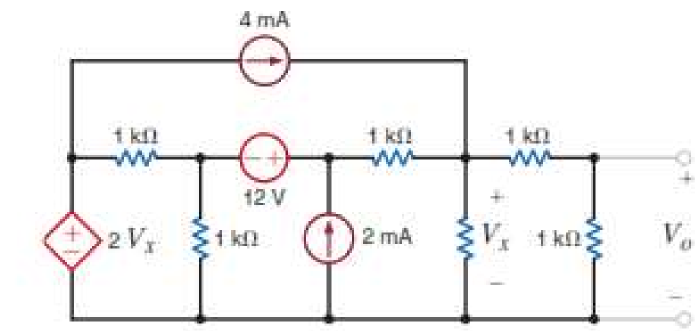
MODULE 2: DC CIRCUIT ANALYSIS – 6 HRS

❶ INTRODUCTION – CIRCUIT ANALYSIS

Circuit analysis: to derive the smallest set of simultaneous equations (parameter: current, voltage) that completely define the operating characteristics of a circuit

To calculate all currents and voltages in circuits that contain multiple nodes and loops

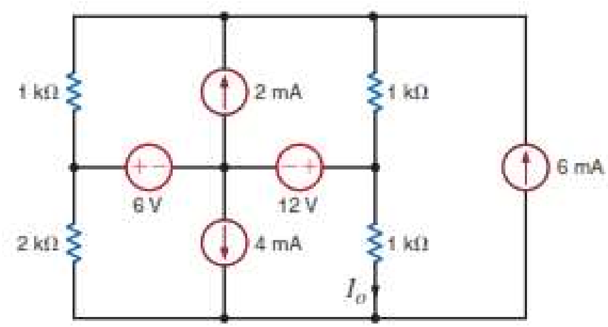
➡ Apply: Ohm's Law & Kirchhoff's Laws



How to determine: voltage, current ?

❷ NODAL ANALYSIS TECHNIQUE

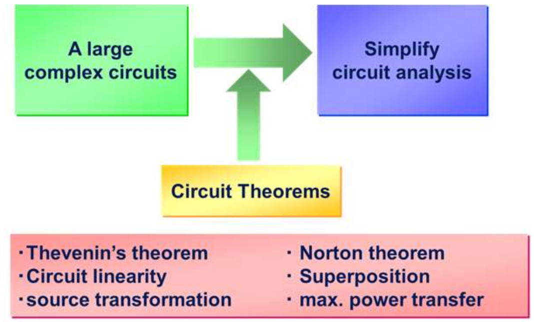
Based: Kirchhoff's Current Law (KCL) to find all circuit variables
General procedure: use of node voltages in circuit analysis as key solutions.



❸ LOOP ANALYSIS TECHNIQUE

Based: Kirchhoff's Voltage Law (KVL) to find all circuit variables
General procedure: use of loop currents in circuit analysis as key solutions.

MODULE 3: NETWORK THEOREMS – 6 HRS



❶ **INTRODUCTION – NETWORK**

Concepts of <u>linearity</u> and <u>equivalence</u>

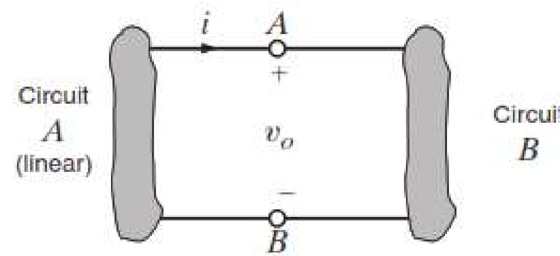
❷ **SUPERPOSITION, THEVENIN & NORTON THEOREMS**

How to analyze electric circuits using the principle of superposition

Calculate a Thévenin equivalent circuit for a linear circuit

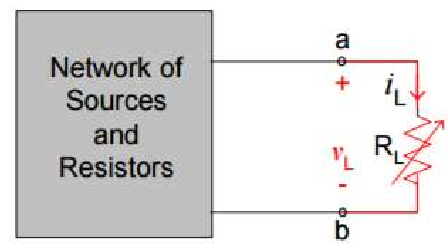
Calculate a Norton equivalent circuit for a linear circuit

When and how to use a source transformation



❸ **MAXIMUM POWER TRANSFERT**

Use the maximum power transfer theorem

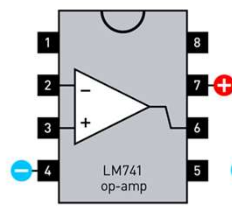


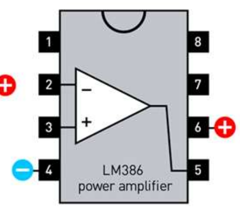
Find the maximum power delivered to the load R_L in the given circuit !!!

MODULE 4: OPERATIONAL AMPLIFIER– 6 HRS

❶ OP-AMP DEVICES

Model the op-amp device; analysing a variety of circuits that employ op-amps
Amplifier impedance
Ideal Op-Amp Circuit
Negative Feedback





❷ OP-AMP OPERATION

How to analyze a variety of circuits that employ op-amps

❸ POPULAR OP-AMP CIRCUITS

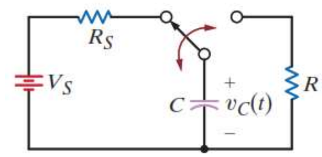
Op amp circuit 1: Voltage follower

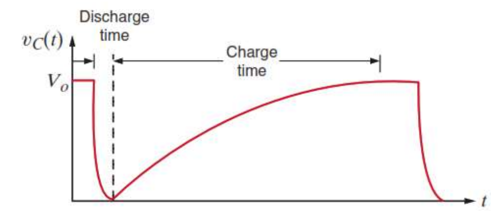
Op amp circuit 2: Inverting and Non-Inverting Amplifier

Op amp circuit 3: Summing Amplifier

Op amp circuit 5: Differential Amplifier (subtractor)

MODULE 5: FIRST & SECOND-ORDER TRANSIENT CIRCUITS– 9 HRS



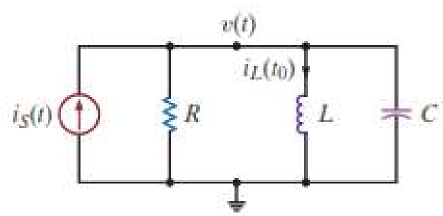


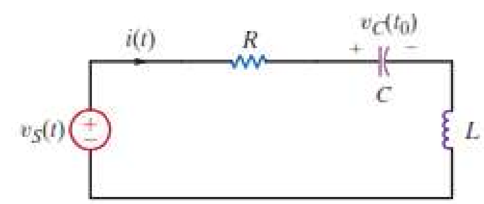
❶ INTRODUCTION

Description of the behavior of a circuit as a function of time after a sudden change in the network occurs due to switches opening or closing

Presence of one or more storage elements, the circuit response to a sudden change: transition period prior → a steady-state

➡ Transient Circuit Analysis: natural response and step response



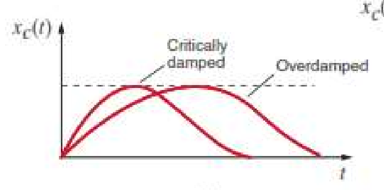


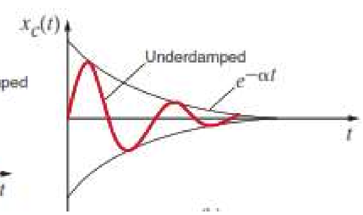
❷ FIRST-ORDER (RC, RL) CIRCUITS

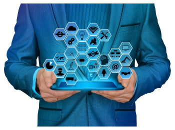
- Characterized by a first- order differential equation.
  + Resistive capacitive, called RC
  + Resistive inductive, called RL

❸ SECOND-ORDER (RLC) CIRCUITS

- Characterized by a second-order differential equation.
  + Series RLC circuits
  + Parallel RLC circuits







ASSESSMENT/EVALUATION

	Percentage	Type
Attendance/Attitude	10%	
Practical	20%	Group report & presentation
Mid-term	20%	Written exam
Final exam	50%	Written exam

❶ MID-TERM WRITTEN EXAM

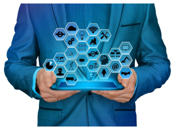
+ Open - book
+ After finishing : module 1, module 2, module 3

❷ FINAL WRITTEN EXAM

+ Open - book
+ After finishing : module 1 – module 5; practical work and have to pass the mid-term exam

❸ PRACTICAL WORK

Item	Content	Hours	Ref./Resources	Related Modules
1	APPLYING THE WHEATSTONE BRIDGE CIRCUIT	3	Wheatstone bridge – PHYWE	Module 1
2	AC TECHNOLOGY I	3	COM3LAB Course	Module 1 & 2
3	DC TECHNOLOGY I	3	COM3LAB Course	Module 1 & 2
4	DC TECHNOLOGY II	3	COM3LAB Course	Module 1, 2 & 5



REFERENCE/ SOURCE/ COURSE MATERIAL

1. **Course material**
   - ✓ **Lecture notes** Nguyen Xuan Truong, https://moodle.usth.edu.vn/course/view.php?id=333#section-1
   - ✓ **Textbook:**

     [1] J. David Irwin, R. Mark Nelms, **"Basic Engineering Circuit Analysis"**, 2008 John Wiley & Sons Inc.

     [2] John O'Malley, **"Schaum's Outline of Theory and Problems of Basic Circuit Analysis"**, Second edition, McGraw-Hill

THANK YOU

MODULE 1 : INTRODUCTION – 3 HRS

1. **BASIC CONCEPTS AND QUANTITIES**
2. **CIRCUIT ELEMENTS**

} **Dr. HT Kien**

3. **BASIC LAWS IN ELECTRIC CIRCUIT**

- ✓ Resistive circuits
- ✓ Ohm's Law
- ✓ Kirchhoff's Current Law
- ✓ Kirchhoff's Voltage Law

→ **to solve: electric circuits**

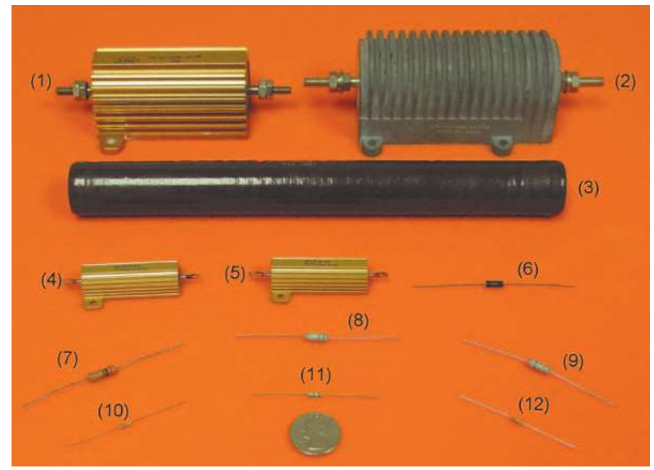
Objectives:

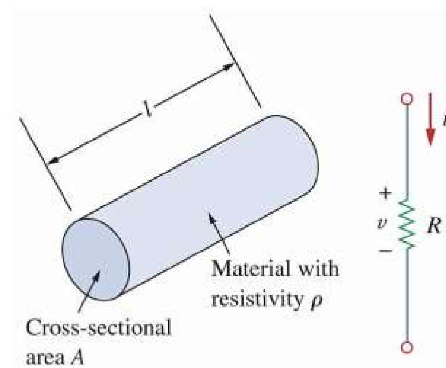
- to learn about resistance and Ohm's Law
- to learn how to apply Kirchhoff's laws to resistive circuits
- to learn how to analyze single-loop and single node-pair circuits
- to learn how to analyze circuits with series and/or parallel connections
- to learn how to analyze circuits that have wye or delta connections

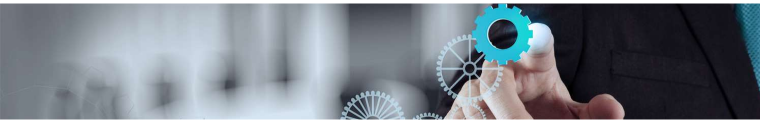
Resistive circuit: resistance concept

- *Resistor:* the ability of a material to resist the flow charge is called its resistivity. It is represented by the letter R
- *Resistance:* is an intrinsic property of matter and is a measure of how much a device impedes the flow of current, measured in Ohms (Ω)
- The greater the resistance of an object, the smaller the amount of current that will flow for a given applied voltage.
- The resistance of an object depends on the material used to construct the object (copper has less resistance than plastic), the geometry of the object (size and shape), and the temperature of the object. ($R = \rho L/A$)

Resistors







Resistive circuit: applications

- to minimize the resistance of an object (in a conductor, for instance).
- to maximize the resistance (in an insulator).
- to relate the resistance of the object to some physical parameter (such as a photoresistor or RTD).
- to precisely control the resistance of an element in order to influence the behavior of a circuit such as an amplifier.

Resistance - characteristic

Several common parameters are used to characterize resistors:

- Ohmic value (nominal) measured in Ohms (Ω),
- Maximum power rating measured in Watts (W), dissipates electrical energy (usually as heat)
- Precision (or tolerance) measured as a percentage of the Ohmic value.

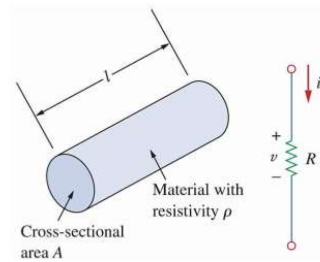
Ohm's Law

Ohm's Law - describes the relationship between the current through and the voltage across a resistor.

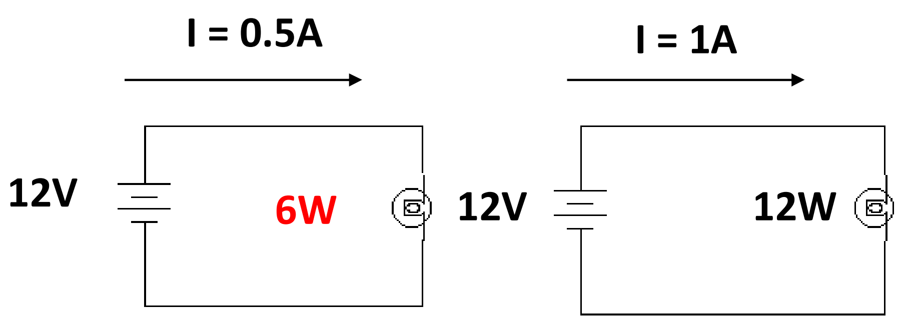
Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it

v(t)=R*i(t), where R ≥ 0

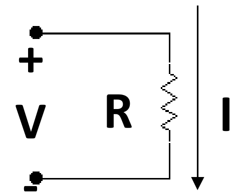
1Ω = 1V/A



Different devices connected to a power source *demand* different amounts of power from that source. That is, different devices present differing amounts of *loading*.

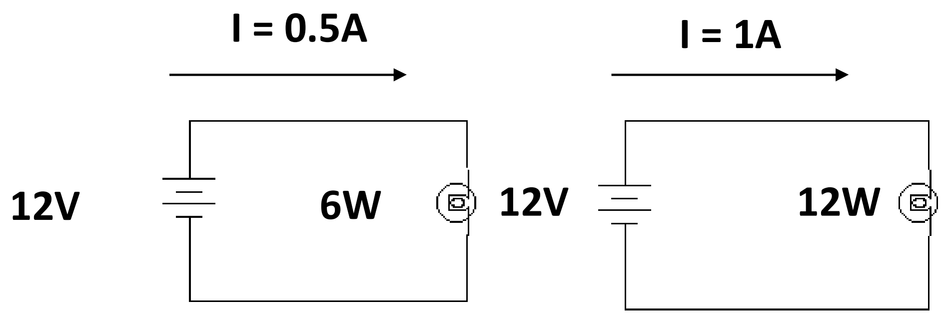


Ohm's Law – Mathematical Definition



R = V/I
I = V/R
V = IR

- Rather than specify the load that a device represents in terms of its voltage/power rating, we can specify that load in terms of its resistance.
- The smaller the resistance, the greater the load (the greater the power demand).



R = 12V/0.5A = 24Ω   **R = 12V1A = 12Ω**

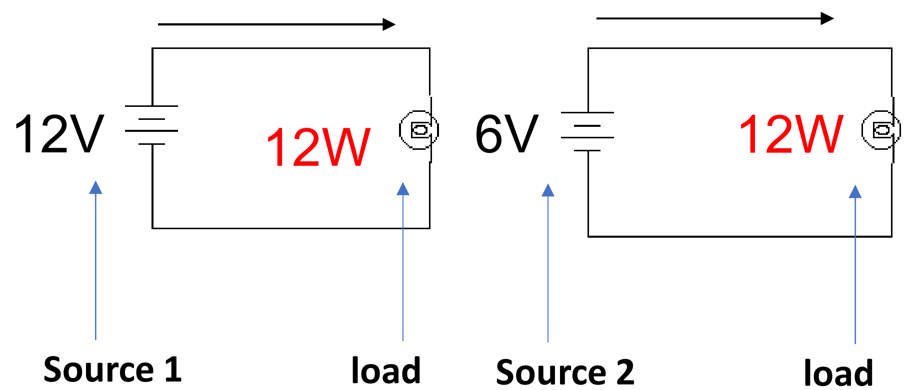
Example

- How much current will a 12V/12W lamp demand if 6V is applied to it?
- How much power is demanded?

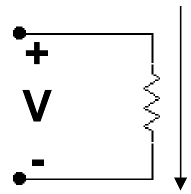
Solution:

A 12W/12V lamp will draw 1A of current:

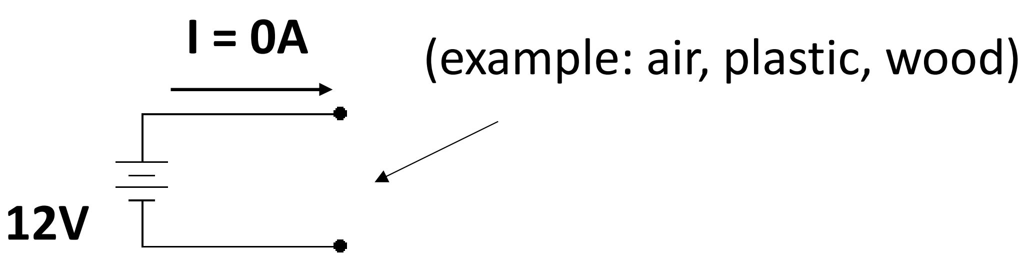
- $P = VI \Rightarrow 12W = 12V * I \Rightarrow I = 1A$
- $V = I*R$ (**Ohm's Law**) $\Rightarrow R = 12V/1A = 12\Omega$
- Therefore, if $V = 6V \Rightarrow I = 6V/1A = 12\Omega$
- $P = 6V * 0.5A = 3W = 0.25 * 12W$.

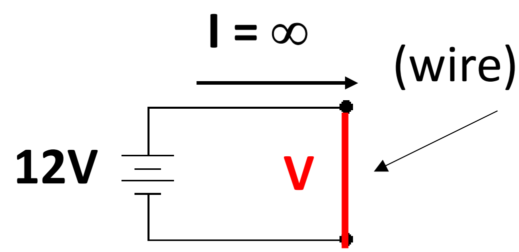


Short & Open Circuits



R = V/I



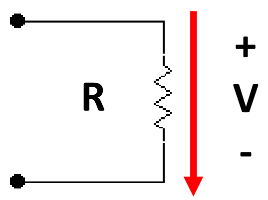


Open Circuit, R = ∞
I = 0 regardless of the value of V (NO LOAD)

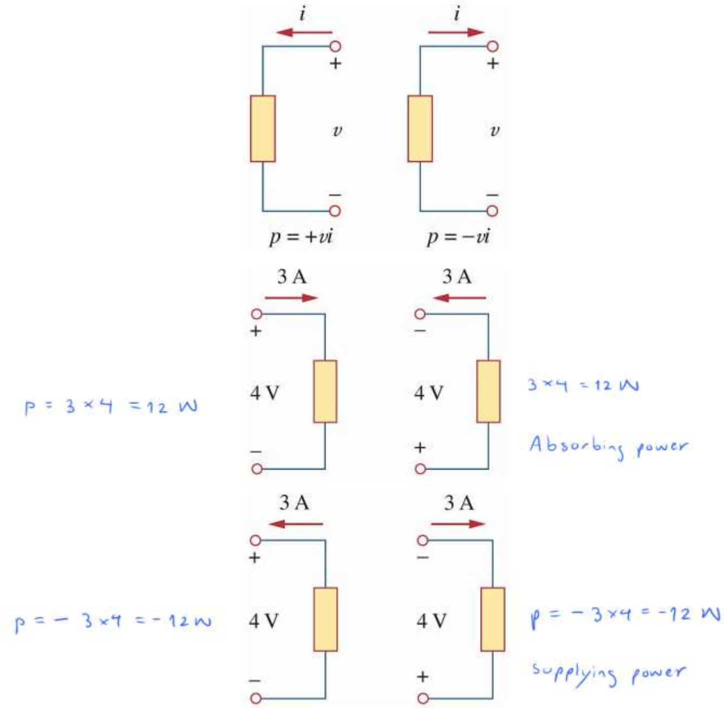
Short Circuit, R =0
V = 0 regardless of the value of I

Ohm’s Law – Voltage Polarity & Current Direction

- Ohms’ law relates the magnitude of the voltage with the magnitude of the current AND
- The polarity of the voltage to the direction of the curre

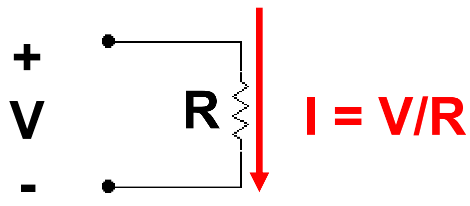


I = V/R



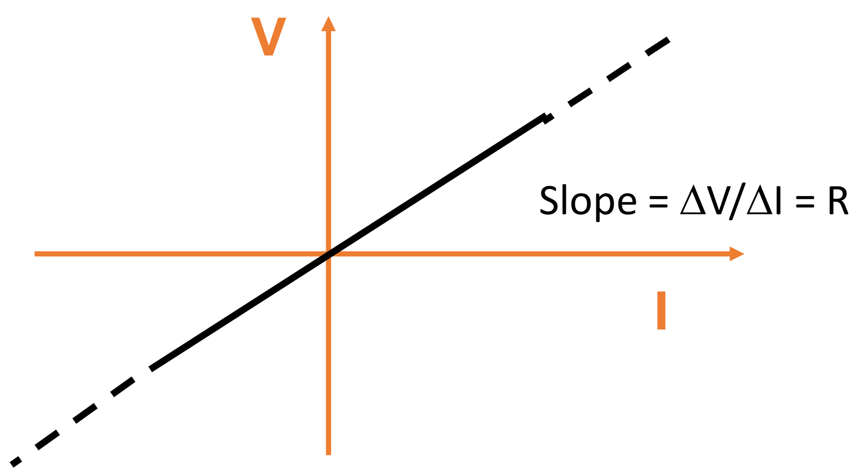
- Resistors always absorb power, so resistor current always flows through a voltage drop.

Ohm's Law - Graphically

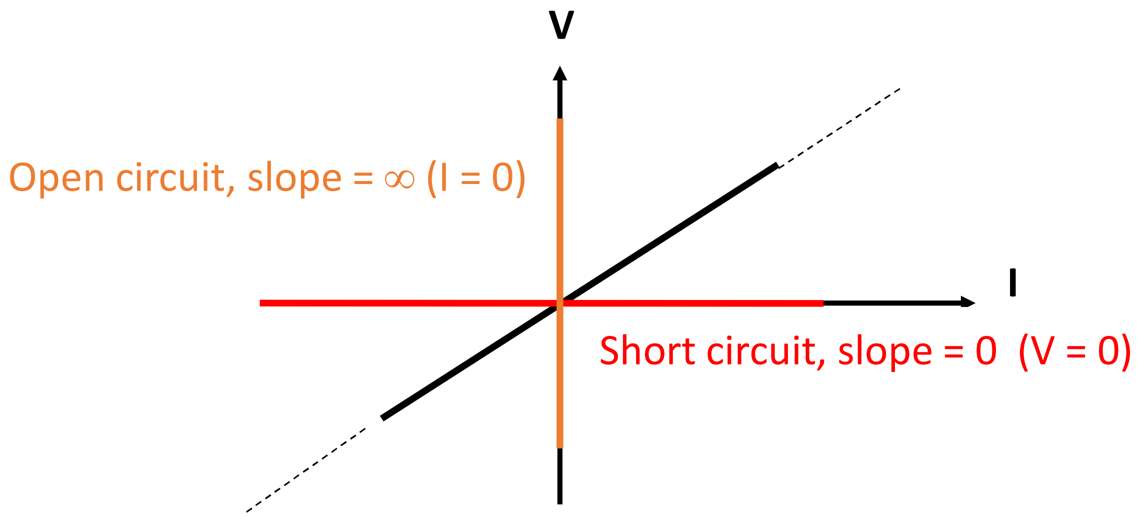


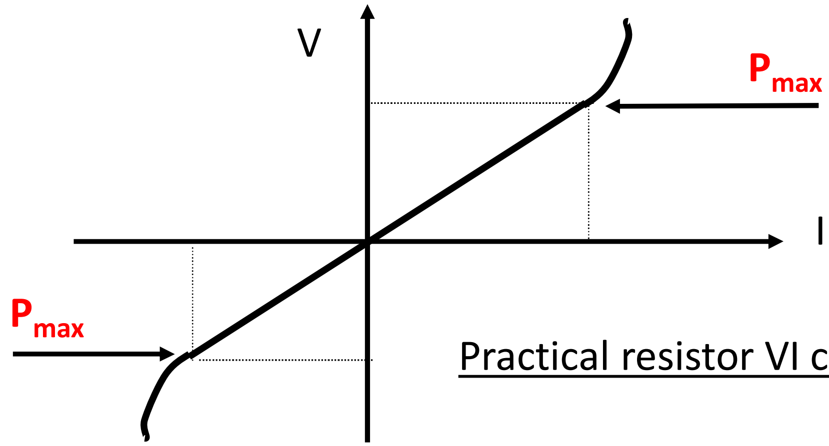
- Ohms' Law can be represented graphically – called a VI characteristic:

Ideal resistor, VI characteristic



Non-ideal Resistors





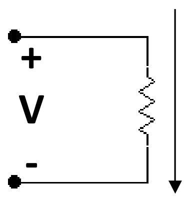
Practical resistor VI characteristic

Conductance

- Resistance is a measure of <u>how much</u> a device impedes the flow of current. Conductance is a measure of <u>how little</u> a device impedes the flow of current.
- Resistance and conductance are simply two different ways to describe the voltage-current characteristic of a device.
- At times, especially in electronic circuits, it is advantageous to work in terms of conductance rather than resistance

$P = VI; P = V^2/R; P = I^2R$

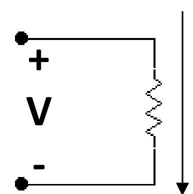
Resistance:



R = V/I,
Ω (ohms)

P = VI (any device)
for a resistor:
$P = V*(V/R) = V^2/R$
or
$P = (IR)*I = I^2R$

Conductance:



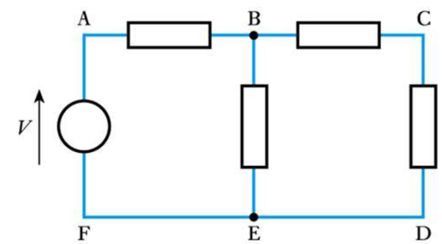
G = I/V,
S (siemens)

$(G = 1/R = R^{-1})$

Kirchhoff's Laws

- Basic concept
- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)

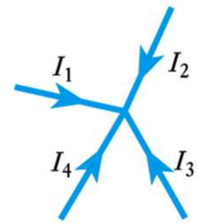
Concept: Node, Branch and Loop



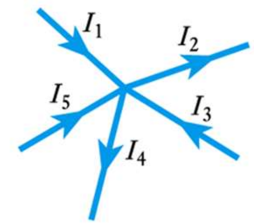
- **Branch:** (b) represents a single element such as a voltage source or a resistor; any two-terminal element
  Branches: AF, AB, BE, BC, CD
- **Node:** (n) is the point of connection between two or more branches.
  Nodes: A, B, C and E
  If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.
- **Loop:** (l) is any closed path in a circuit. A closed path is formed by starting at a node, passing through a set of nodes and return.
  Paths ABEFA, BCDEB And ABCDEFA are loops
- **Mesh:** a loop that contains no other loop → *a loop is said to be independent*. Independent loops or paths result in independent sets of equations
- A network with (b) branches, (n) nodes, and (l) or (m) independent loops will satisfy the fundamental theorem of network topology: b = l + n −1.

Kirchhoff's Current law (KCL)

- KCL is based on the *law of conservation of charge*
- The algebraic sum of all currents at any node in a circuit is exactly zero.
- The sum of all *currents entering* = sum of all *currents leaving*
- We neither gain nor lose current at a node.



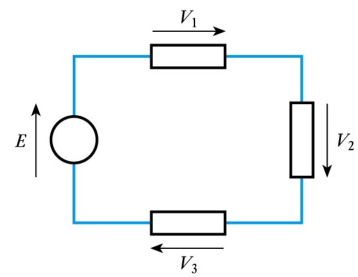
$I_1 + I_2 + I_3 + I_4 = 0$



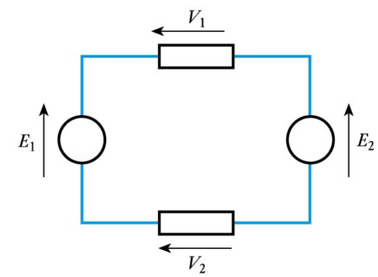
$I_1 - I_2 + I_3 - I_4 + I_5 = 0$

Kirchhoff's Voltage law (KVL)

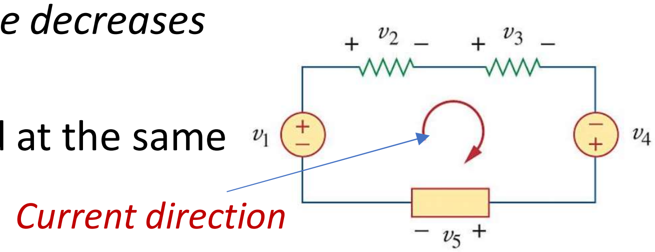
- KVL is based on the *law of conservation of energy*
- The *algebraic sum* of all voltages about any loop in a circuit is exactly zero.
- The sum of all *increases (rises)* = sum of all *voltage decreases (drops)*
- We do not gain or lose voltage if we start and end at the same node.



$E + V_1 + V_2 + V_3 = 0$



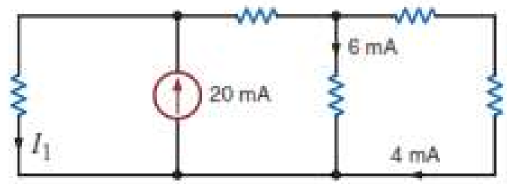
$E_1 - V_1 - E_2 + V_2 = 0$



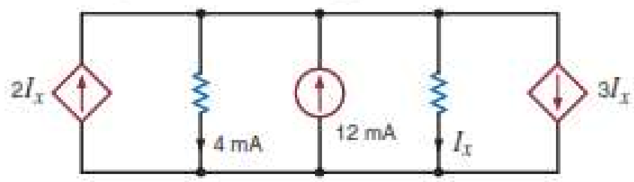
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Problems

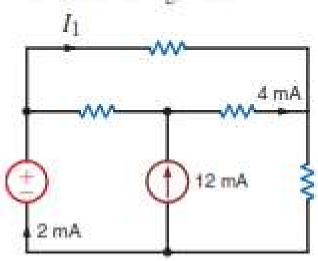
Find I_1 in the network in Fig. P2.10.

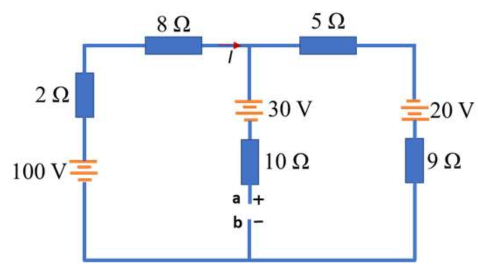


Find I_x in the network in Fig. P2.14.



Find I_1 in the circuit in Fig. P2.13.

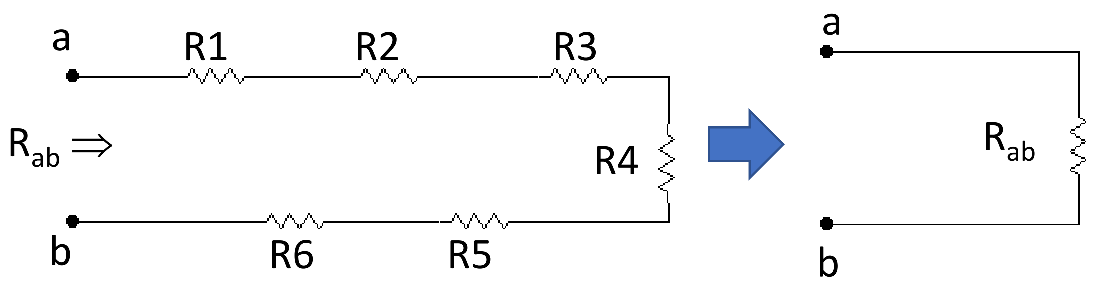




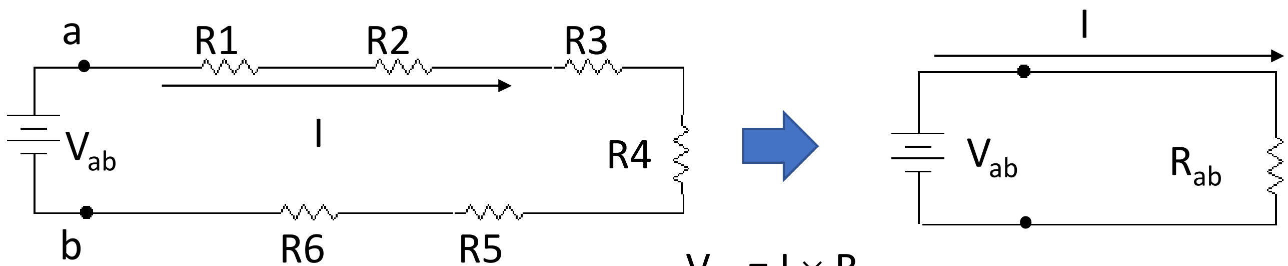
Calculate I and V_{ab}

Series Resistance connection

<u>Series connection</u> (all elements have the <u>*same current*</u>)



The equivalent resistance of two or more series-connected resistors is the sum of the individual resistors.



$V_{ab} = I \times R_{ab}$

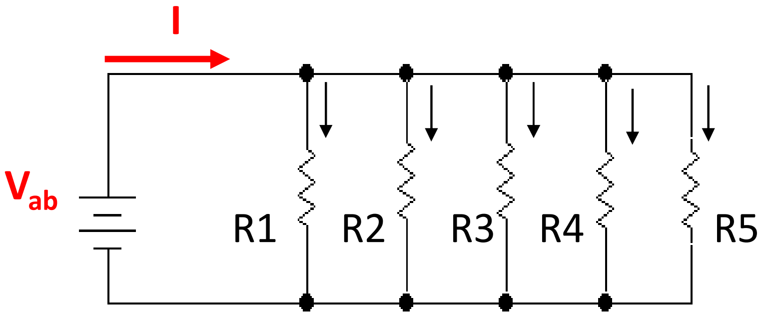
<u>By KCL</u>: $I_{R1} = I_{R2} = \ldots = I_{R6} = I$

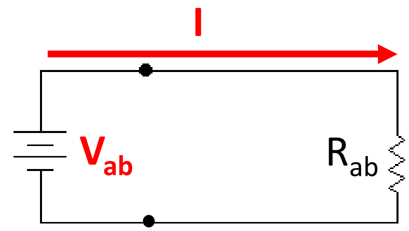
<u>By KVL</u>: $V_{ab} = I*R1 + I*R2 + I*R3 + I*R4 + I*R5 + I*R6$

$V_{ab}/I = (R1 + R2 + R3 + R4 + R5 + R6) = R_{ab}$

Parallel Resistance

Parallel connection (all the elements have the same voltage)





$V_{ab}/I = R_{ab}$

by KCL:

$I = I1 + I2 + I3 + I4 + I5$

$I = V_{ab}/R1 + V_{ab}/R2 + V_{ab}/R3 + V_{ab}/R4 + V_{ab}/R5$

$I = V_{ab}[R1^{-1} + R2^{-1} + R3^{-1} + R4^{-1} + R5^{-1}]$

$V_{ab}/I = [R1^{-1} + R2^{-1} + R3^{-1} + R4^{-1} + R5^{-1}]^{-1}$

$R_{ab} = [R1^{-1} + R2^{-1} + R3^{-1} + R4^{-1} + R4^{-1}]^{-1}$

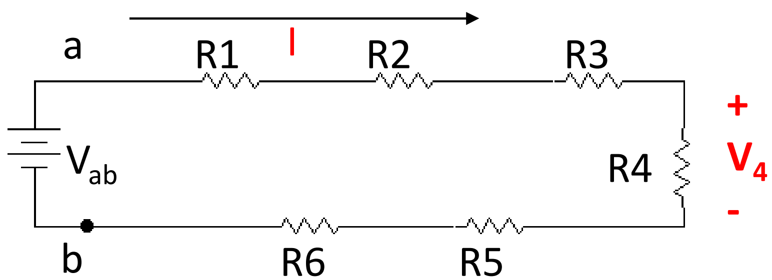
Since $G = 1/R = R^{-1}$

$R_{ab} = [G1 + G2 + G3 + G4 + G5]^{-1}$

$G_{ab} = G1 + G2 + G3 + G4 + G5$

The Voltage Divider Rule (VDR)

The total voltage applied to a group of series-connected resistors will be divided among the resistors. The fraction of the total voltage across any single resistor depends on what fraction that resistor is of the total resistance.

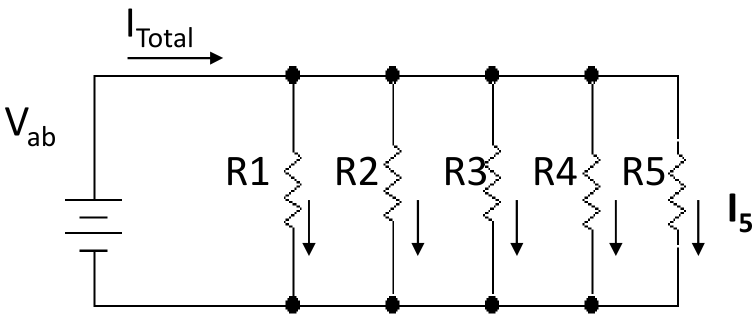


$R_{TOTAL} = R_{ab} = R1 + R2 + R3 + R4 + R5 + R6$

$$V_4 = V_{ab}\left[\frac{R4}{R1 + R2 + R3 + R4 + R5 + R6}\right]$$

The Current Divider Rule (CDR)

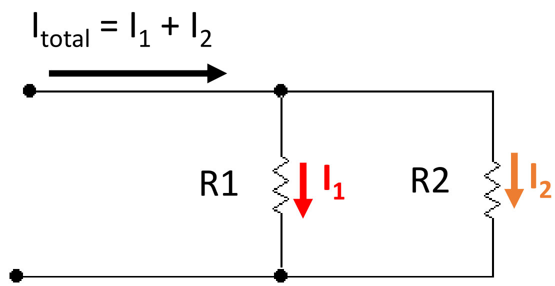
The total current applied to a group of resistors connected in parallel will be divided among the resistors. The fraction of the total current through any single resistor depends on what fraction that resistor is of the total conductance.



$G_{TOTAL} = R1^{-1} + R2^{-1} + R3^{-1} + R4^{-1} + R5^{-1}$

$$I_5 = I_{Total}\left[\frac{G5}{G_{Total}}\right] = I_{Total}\left[\frac{R5^{-1}}{R1^{-1}+R2^{-1}+R3^{-1}+R4^{-1}+R5^{-1}}\right]$$

For example, two resistors:



$$I_1 = I_{Total}\left[\frac{R2}{R1+R2}\right]$$

$$I_2 = I_{Total}\left[\frac{R1}{R1+R2}\right]$$

Observations:

- The smaller resistor will have the larger current.
- If $R_1 = R_2$, then $I_1 = I_2$
- If $R_1 = nR_2$, then $I_2 = nI_1$

MODULE 2: DC CIRCUIT ANALYSIS – 6 HRS

Objectives:

- to learn how to calculate all currents and voltages in circuits that contain <u>multiple nodes and loops</u>
- to learn how to employ Kirchhoff's current law (KCL) to perform a nodal analysis to determine all the node voltages in a circuit
- to learn how to employ Kirchhoff's voltage law (KVL) to perform a loop analysis to determine all the loop currents in a network

1 INTRODUCTION – CIRCUIT ANALYSIS

Circuit analysis: to derive the smallest set of simultaneous equations (parameter: <u>current, voltage</u>) that completely define the operating characteristics of a circuit

To calculate all currents and voltages in circuits that contain multiple nodes and loops

Apply: Ohm's Law & Kirchhoff's Laws

❷ NODAL ANALYSIS TECHNIQUE

- **Concept:**

→ Approach of Kirchhoff's Current Law (KCL) to find all circuit variables without having to sacrifice any of the elements.

→ General procedure which is making use of node voltages in circuit analysis as key solutions.

- **Importance terms**

Node Voltage: Potential difference between a marked node and the selected reference node.

Element Voltage: Potential difference across any element or branch in the circuit.

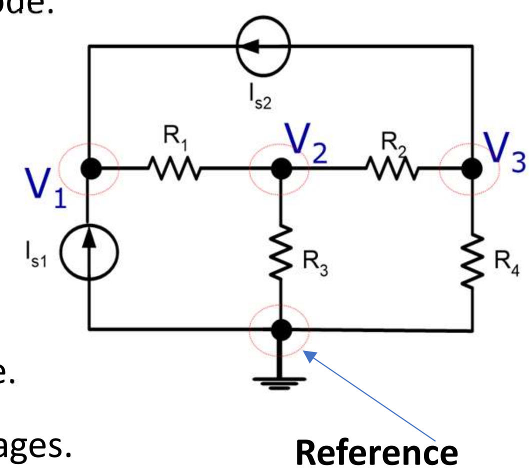
When Node Voltage = Element Voltage?

- **Why use Node Voltage?**

→ Further reduce the number of equations to be solved simultaneously.

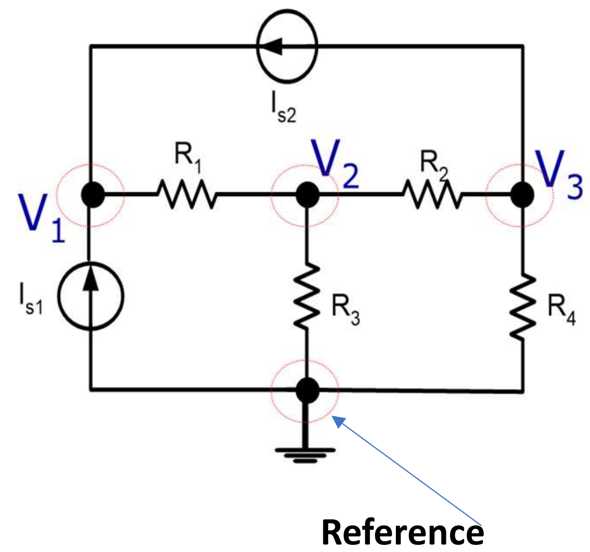
→ N° of independent equations = N° of the marked nodes exclusive of the reference node.

→ Element voltages and currents can be obtained in few steps using the solved node voltages.



❷ NODAL ANALYSIS TECHNIQUE → STEP ?

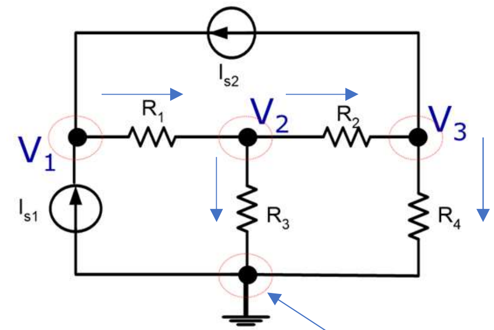
1) Clearly label all circuit parameters and distinguish the *unknown parameters from the known*.
2) Identify *all nodes* of the circuit
3) Select a *node as the reference node* also called the ground and assign to it a potential of 0 Volts. All other voltages in the circuit are measured with respect to the reference node.
4) Label the *voltages at all other nodes*.
5) Assign and *label polarities*.
6) Apply KCL at each non-reference nodes and express the branch currents in terms of the node voltages.
7) Solve the resulting *simultaneous equations for the node voltages*.
8) Now that the *node voltages are known*, the *branch currents* may be obtained from Ohm's law.



❷ NODAL ANALYSIS TECHNIQUE → STEP ?

Example 1:

Applying Nodal analysis on simple circuit, find the power dissipated by the R_1=10kΩ resistor, I_{s2} = 3mA; R_1=10kΩ; R_2=5k Ω; R_3=4k Ω; R_4=2k Ω



Assuming – current direction
I_{R1}; I_{R2}; I_{R3}; I_{R4}

Reference: zero volt

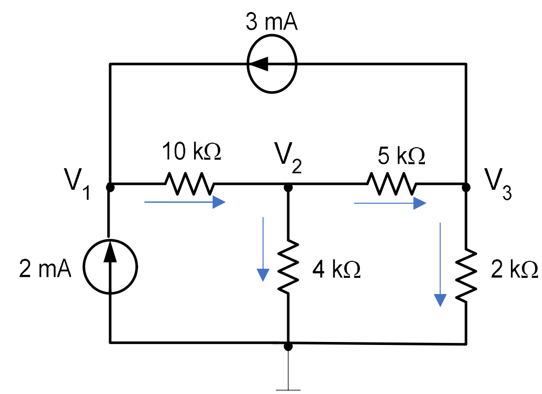
Solution:

→ Mark all essential nodes
→ Assign unknown node voltages
→ Indicate the reference node
→ Identify all polarities
→ Perform KCL at each marked non-reference nodes using Ohm's law to formulate the equations in terms of the node voltages.

- KCL at V_1: $I_{s1} + I_{s2} = \frac{V_1 - V_2}{R_1}$ (1)
- KCL at V_2: $\frac{V_2 - 0}{R_3} + \frac{V_2 - V_3}{R_2} = \frac{V_1 - V_2}{R_1}$ (2)
- KCL at V_3: $I_{s2} + \frac{V_3 - 0}{R_4} = \frac{V_2 - V_3}{R_2}$ (3)

→ The node voltages:
V_1 = 60.55 [V];
V_2 = 10.55 [V];
V_3 = -1.27 [V]
→ The element voltage:

$$P_{R1} = \frac{(V1 - V2)^2}{R1} = \frac{(60.55 - 10.55)^2}{10} = 0.25[W]$$

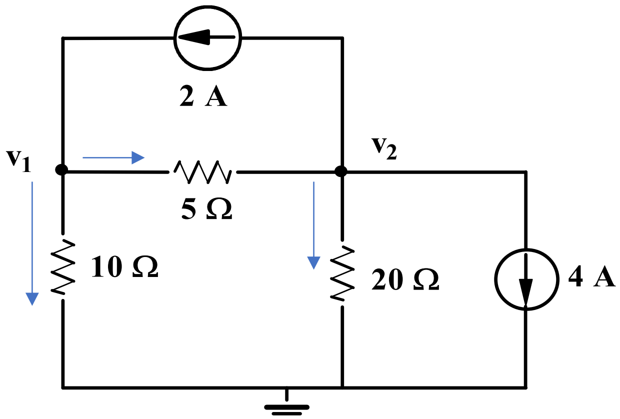


Positive sign → absorbed

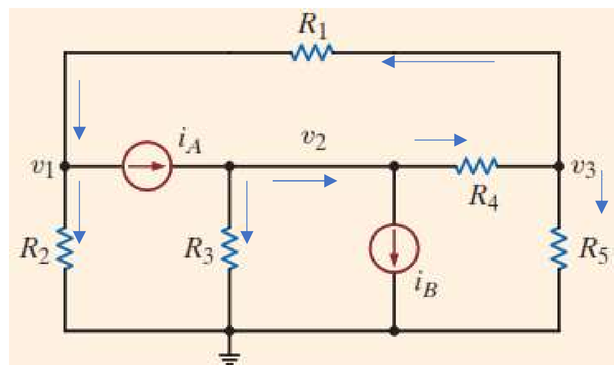
❷ NODAL ANALYSIS TECHNIQUE → STEP ?

Problems:

Calculate the node voltages and the power dissipated in each resistors, in the circuit below



Determine the node voltages using nodal analysis technique (can use Matlab for solving characteristic's equation)
And power supplied by current sources

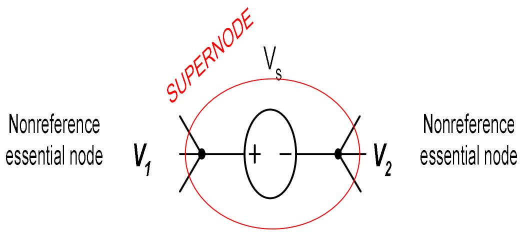


$R_1 = R_2 = 2\ k\Omega$, $R_3 = R_4 = 4\ k\Omega$, $R_5 = 1\ k\Omega$, $i_A = 4\ mA$, and $i_B = 2\ mA$.

❷ NODAL ANALYSIS TECHNIQUE → STEP ?

APPLYING NODAL ANALYSIS ON CIRCUIT WITH VOLTAGE SOURCES

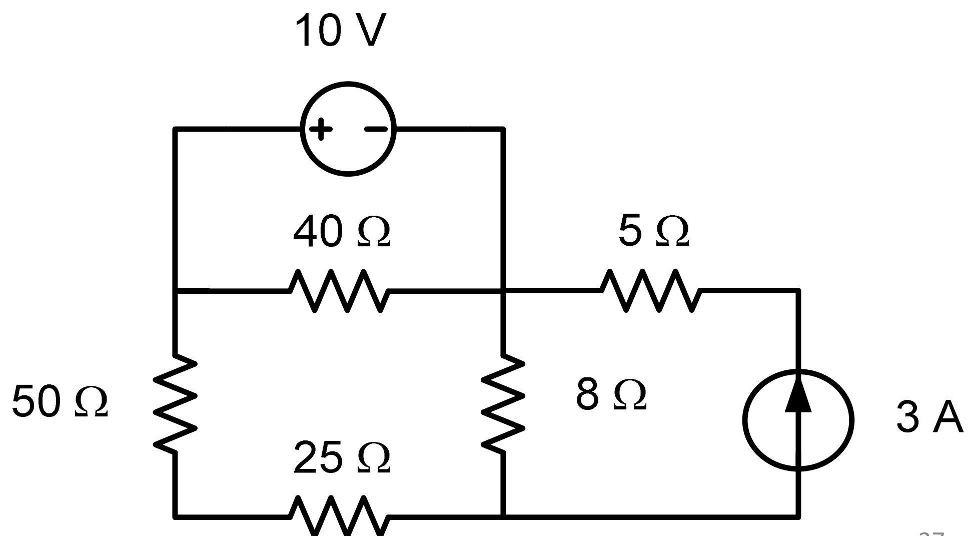
- **Case 1: Voltage source between two non-reference essential nodes**



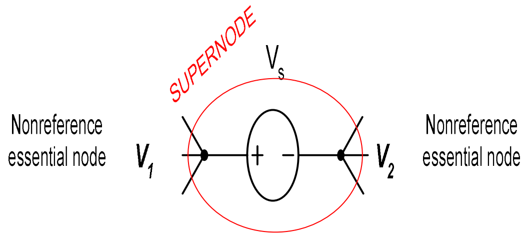
Supernode Equation: $V_s = V_1 - V_2$

Example 1:

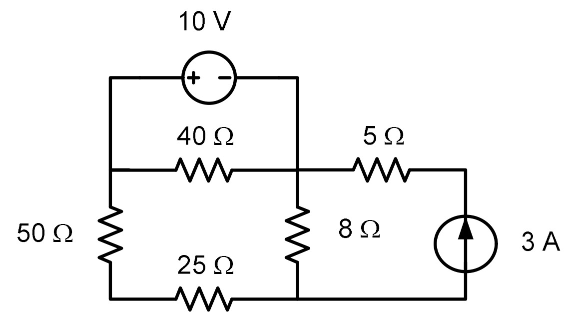
- Find the power of the 10-V voltage source?
- Is it supplying energy to the circuit or absorbing energy from the circuit?
- Show your work according to the nodal analysis procedure.



❷ NODAL ANALYSIS TECHNIQUE → STEP ?

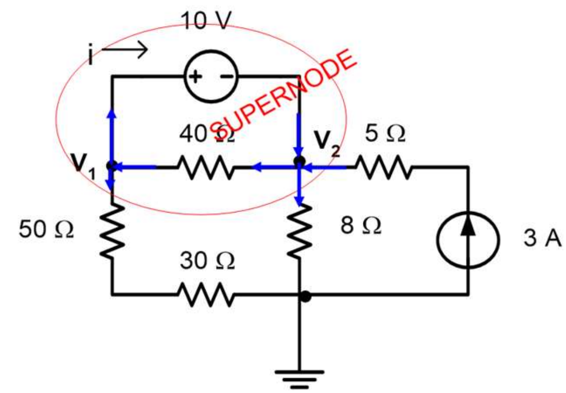


Supernode Equation: $V_s = V_1 - V_2$



Solution:

1) Mark essential nodes and assign unknown node voltages and indicate the reference node
2) Perform KCL at each marked non-reference nodes using Ohm's law to formulate the equations in terms of node voltages.



At Node V1

$$\frac{V_2 - V_1}{40} = i + \frac{V_1}{80}$$

At Node V2:

$$3 + i = \frac{V_2}{8} + \frac{V_2 - V_1}{40}$$

Super-Node Equation

$$V_1 - V_2 = 10$$

$$V_1 = 30.91 \qquad V_2 = 20.91$$

$$i = -0.636\,[A]$$

Finding current through the voltage source

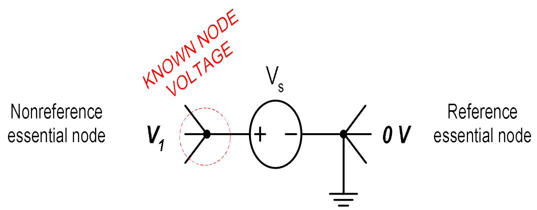
$$\frac{V_2 - V_1}{40} = i + \frac{V_1}{80}$$

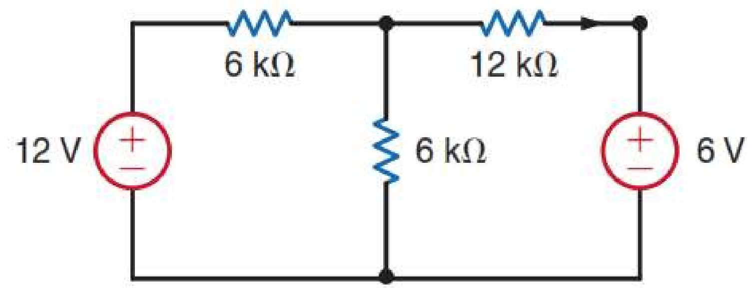
negative sign → Delivering energy/ power

$$P_{voltage\ source} = i * V = (-0.636) * 10 = -6.36\,[W]$$

❷ NODAL ANALYSIS TECHNIQUE → STEP ?

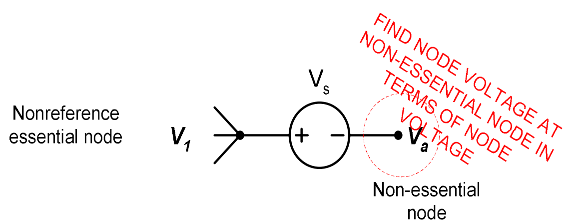
- **Case 2: Voltage source between a reference essential node and a non-reference essential node.**

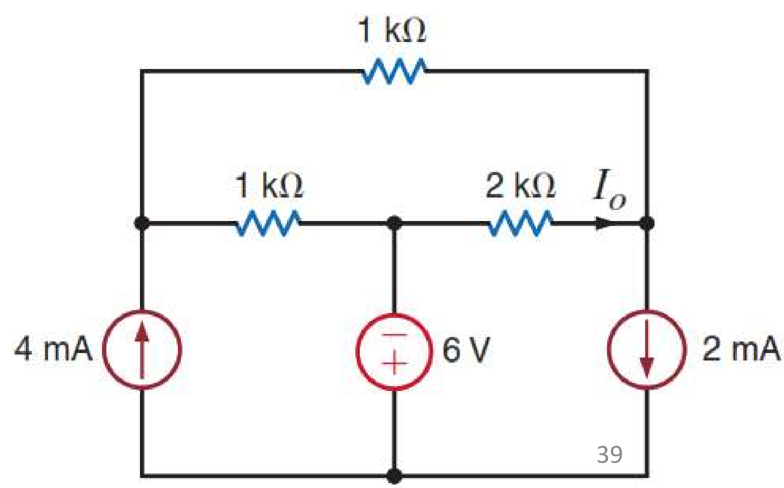




➡ *Known node voltage:* $V_1 = V_s$

- **Case 3: Voltage source between an essential node and a non-essential node.**





➡ *Node voltage at non-essential node:* $V_a = V_1 - V_s$

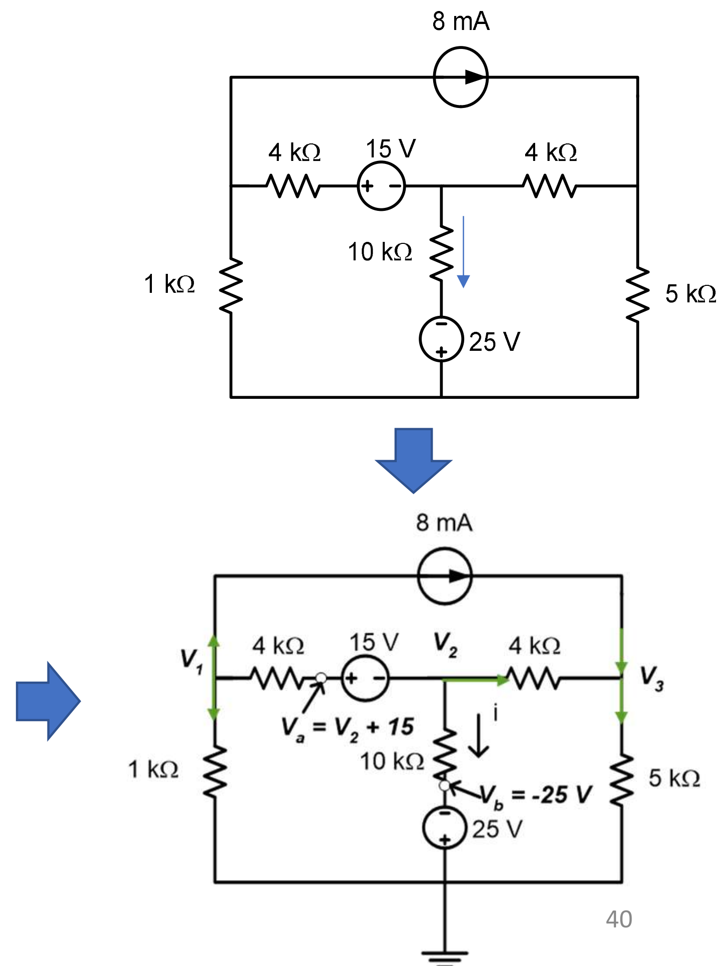
❷ NODAL ANALYSIS TECHNIQUE → STEP ?

Example 2:

- Voltage source between an essential node and a non-essential node
- Find the current through the 10 kΩ resistor. Show your work according to the nodal analysis procedure

Solution:

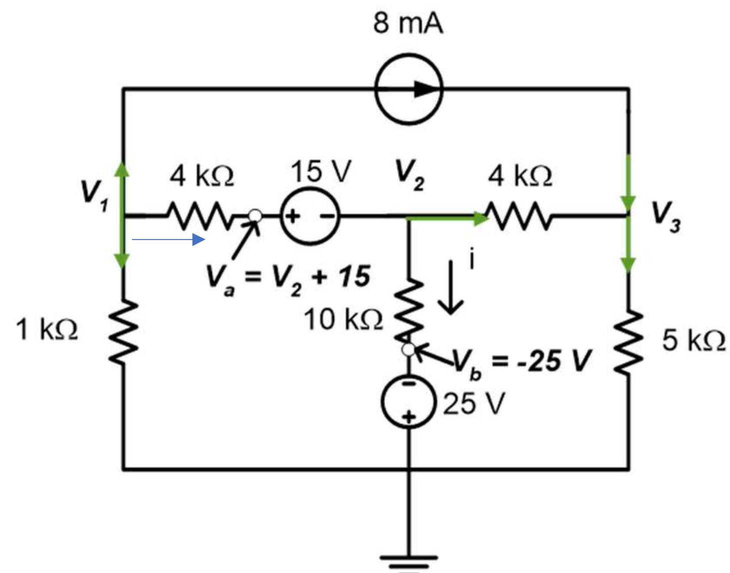
1) Mark essential nodes and assign unknown node voltages and indicate the reference node.
2) For voltage sources, indicate the node voltages at both ends with respect to the assigned unknown node voltages at the essential nodes
3) Perform KCL at each marked non-reference nodes using Ohm's law to formulate the equations in terms of node voltages



❷ NODAL ANALYSIS TECHNIQUE → STEP ?

Example 2:

- Voltage source between an essential node and a non-essential node
- Find the current through the 10 kΩ resistor. Show your work according to the nodal analysis procedure



At Node V1:

V_a

$$\frac{V_1}{1k} + \frac{V_1 - (V_2 + 15)}{4k} + 8\ mA = 0$$

$$\Rightarrow 5V_1 - V_2 = -17$$

At Node V2

$$\frac{V_2 + 25}{10k} + \frac{-V_1 + (V_2 + 15)}{4k} + \frac{V_2 - V_3}{4k} = 0\ mA$$

$$\Rightarrow -10V_1 + 18V_2 - 10V_3 = -250$$

At Node V3

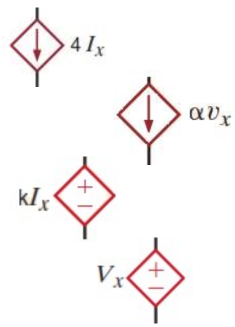
$$-\frac{V_3}{5k} + \frac{-V_3 + V_2}{4k} + 8\ mA = 0$$

$$\Rightarrow -5V_2 + 9V_3 = 160$$

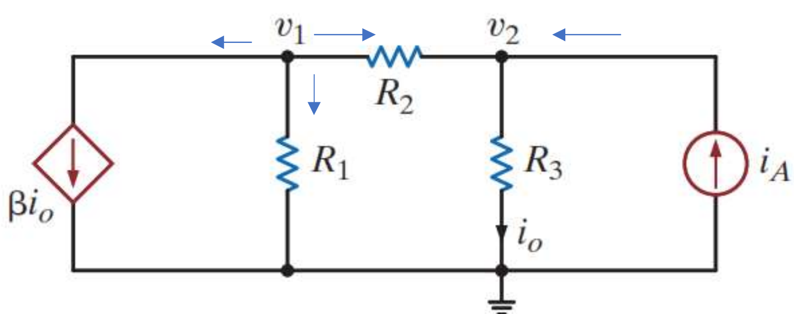
$$\Rightarrow V_1 = -5.43\,;\, V_2 = -10.17\,;\, V_3 = 12.13$$

❷ NODAL ANALYSIS TECHNIQUE → STEP ?

- **Case 4: Circuit with dependent sources**
  - ✓ Current - Controlled Current sources
  - ✓ Voltage - Controlled Current sources
  - ✓ Current - Controlled Voltage sources
  - ✓ Voltage - Controlled Voltage sources



Example 3: Let us determine the node voltages for this network,



$\beta = 2$ $R_1 = 12\ k\Omega$

$R_2 = 6\ k\Omega$ $R_3 = 3\ k\Omega$

$i_A = 2\ mA$

Solution for example 3:

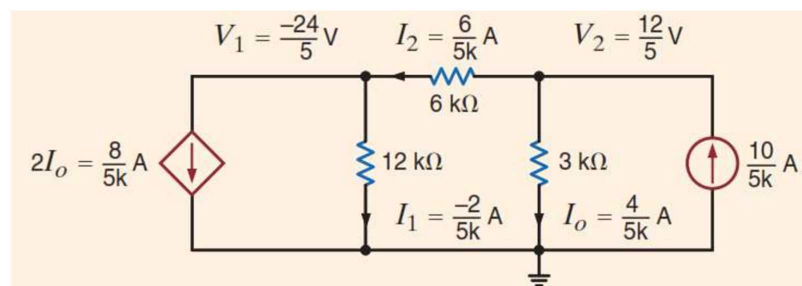
At node v_1

$$\beta i_o + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0$$

At node v_2

$$\frac{v_2 - v_1}{R_2} + i_o - i_A = 0$$

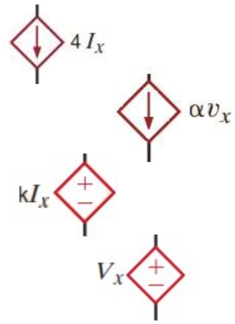
where $i_o = v_2/R_3$.



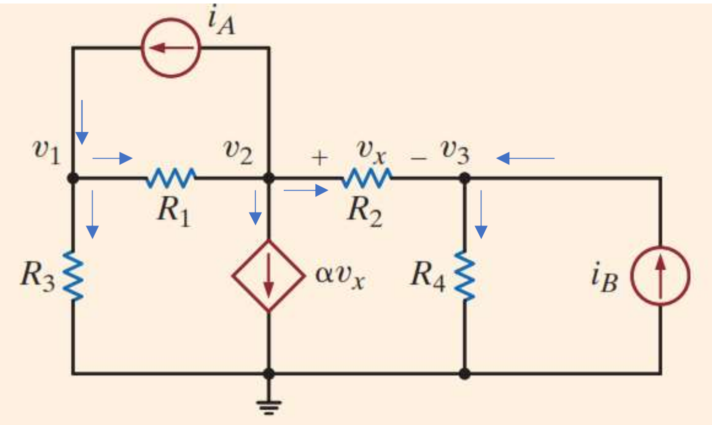
MODULE 2: DC CIRCUIT ANALYSIS – 6 HRS

❷ NODAL ANALYSIS TECHNIQUE → STEP ?

- **Case 4: Circuit with dependent sources**
  - ✓ Current - Controlled Current sources
  - ✓ Voltage - Controlled Current sources
  - ✓ Current - Controlled Voltage sources
  - ✓ Voltage - Controlled Voltage sources



Example 4: Let us determine the node voltages for this network,



$\alpha = 2$ R_1= 1 kΩ

R_2= 2 kΩ R_3= 2 kΩ

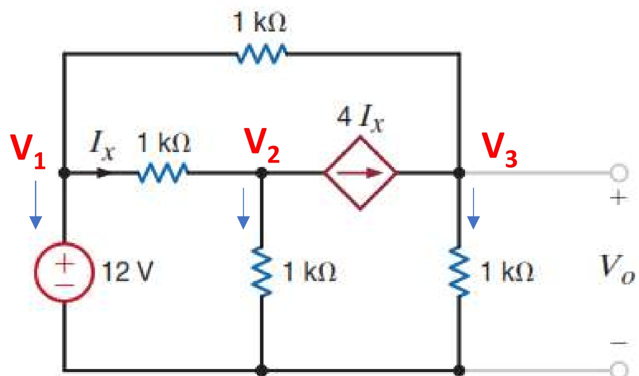
R_4= 4 kΩ

i_A= 2 mA

i_B= 4 mA

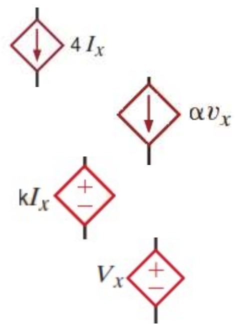
→ v_1= 12 V, v_2= 16 V, v_3= 16 V

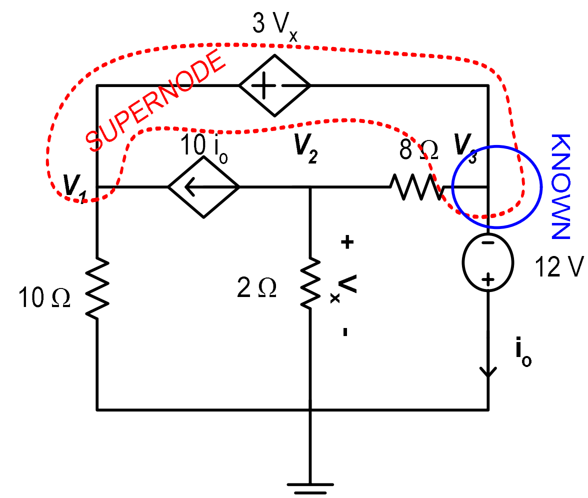
Example 5: Determine the voltage V_0,



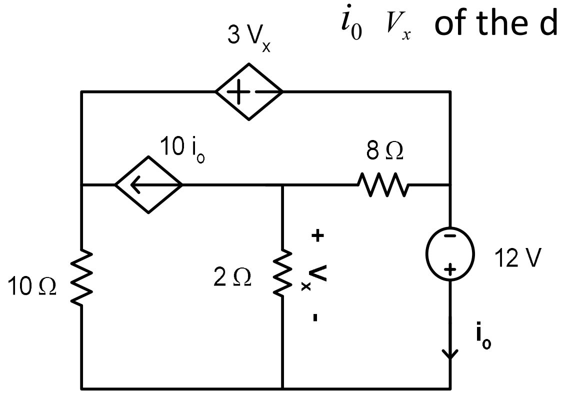
❷ NODAL ANALYSIS TECHNIQUE → STEP ?

- **Case 4: Circuit with dependent sources**
  - ✓ Current - Controlled Current sources
  - ✓ Voltage - Controlled Current sources
  - ✓ Current - Controlled Voltage sources
  - ✓ Voltage - Controlled Voltage sources





Example 6: Use the node-voltage method to find both dependent terms i_0 V_x of the dependent sources of the circuit



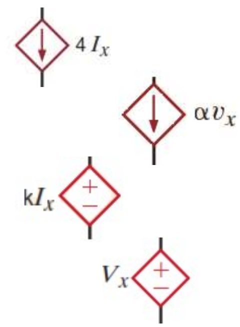
Solution:

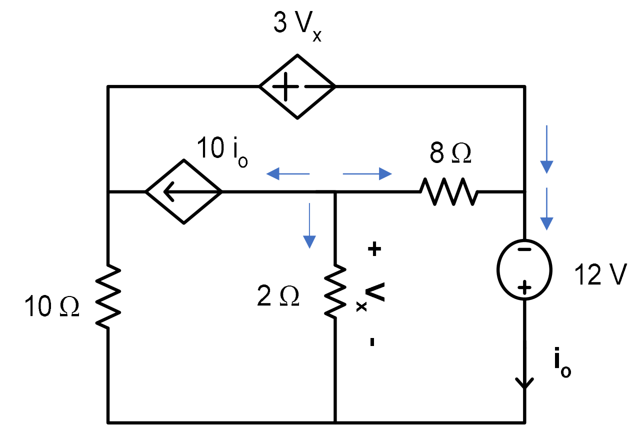
✓ Mark essential nodes and assign <u>unknown node voltages</u> and indicate the reference node

✓ Perform KCL at each marked <u>non-reference nodes</u> using Ohm's law to formulate the equations in terms of node voltages

❷ NODAL ANALYSIS TECHNIQUE → STEP ?

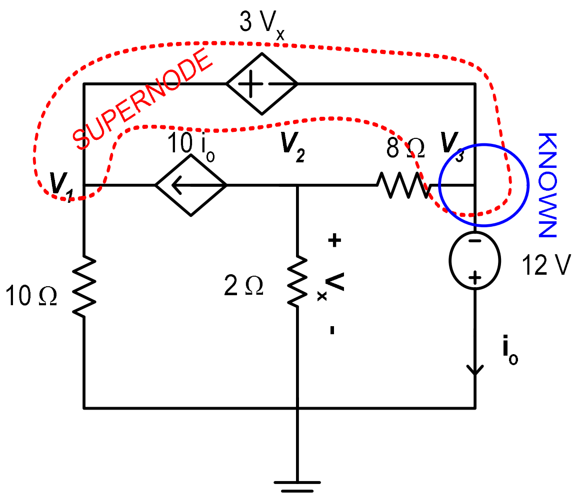
- **Case 4: Circuit with dependent sources**
  - ✓ Current - Controlled Current sources
  - ✓ Voltage - Controlled Current sources
  - ✓ Current - Controlled Voltage sources
  - ✓ Voltage - Controlled Voltage sources





Example 6:

Solution:



KCL at Node V_2

$$\frac{V_2}{2}+\frac{V_2+12}{8}=-10i_0$$

➡ $5V_2+80i_0=-12$

➡ $i_0=\frac{V_2+12}{8}-\frac{V_1}{10}+10i_0 \Rightarrow 720i_0-8V_1+10V_2=-120$

Super-node equation

$$3V_x=V_1+V_3=V_1+12$$

Constraint equation

$$V_x=V_2$$

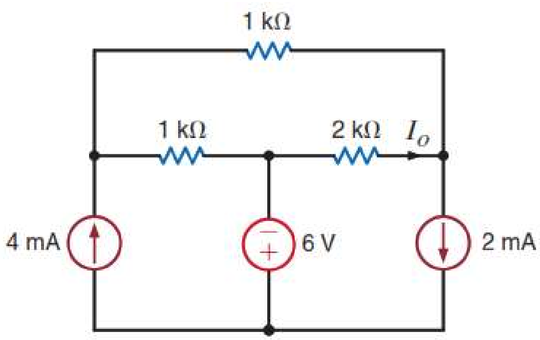
➡ $-V_1+3V_2=12$

➡ $V_1=-6.51\,;V_2=1.83\,;i_0=-0.264$

❷ NODAL ANALYSIS TECHNIQUE

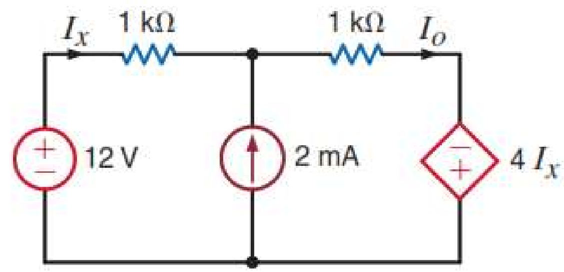
- **Problem N°1**

Find I_0 in the circuit using nodal analysis.



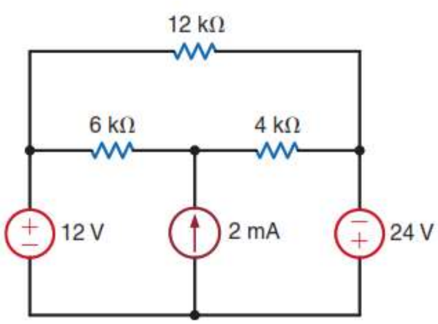
- **Problem N°2**

Find I_0 in the circuit using nodal analysis.



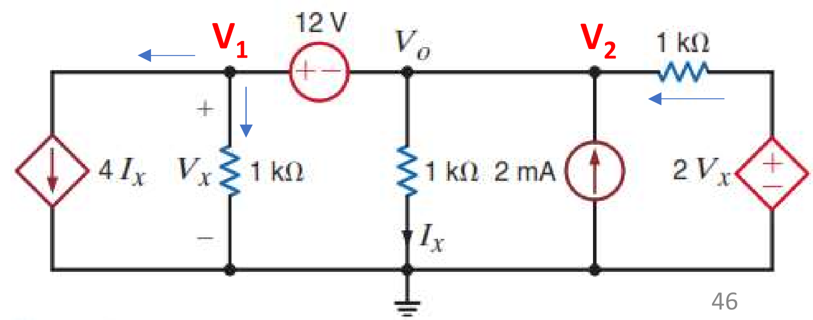
- **Problem N°3**

Calculate the power supplied by the 2-mA current source



- **Problem N°4**

Use nodal analysis to find V_0 in the circuit



❸ MESH/ LOOP ANALYSIS TECHNIQUE

- **Concept:**

→ Approach of Kirchhoff's Voltage Law (KVL) to find all circuit variables without having to sacrifice any of the elements.

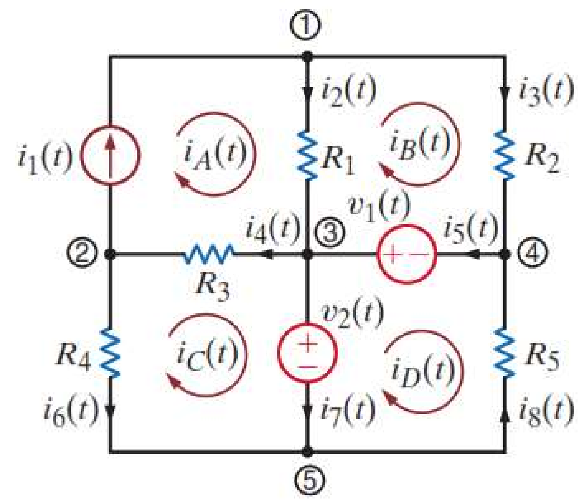
→ General procedure which is making use of Loop/Mesh currents in circuit analysis as key solutions.

- **Importance terms**

Mesh/Loop Current: Assigned unknown current flows around the perimeter of the me

Element Current: : Actual current through any element or branch in the circuit.

When Mesh Current = Element Current?



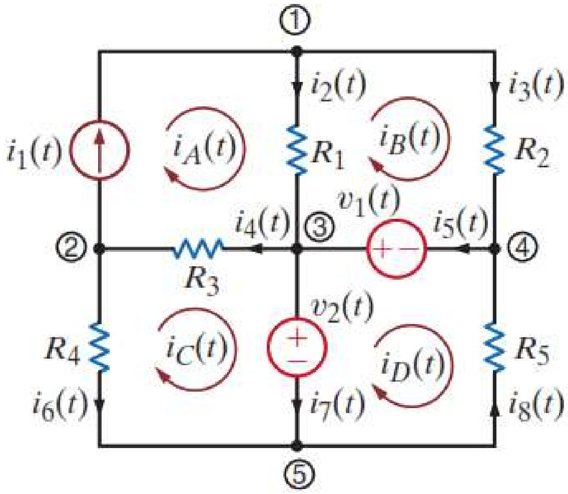
- **Why use Loop/ Mesh Current?**

→ Further reduce the number of equations to be solved simultaneously.

→ N° of independent equations = N° of the marked loop/ mesh.

→ Element voltages and currents can be obtained in few steps using the solved loop/ mesh current.

❸ MESH/ LOOP ANALYSIS TECHNIQUE



$i_1(t) = i_A(t)$ ← Loop A

$i_2(t) = i_A(t) - i_B(t)$ ← Branch R_1

$i_3(t) = i_B(t)$ ← Loop B

$i_4(t) = i_A(t) - i_C(t)$ ← Branch R_3

$i_5(t) = i_B(t) - i_D(t)$ ← Branch v_1

$i_6(t) = -i_C(t)$ ← Loop C

$i_7(t) = i_C(t) - i_D(t)$ ← Branch v_2

$i_8(t) = -i_D(t)$ ← Loop D

Six - Steps

1. Clearly label all circuit parameters and distinguish the unknown parameters from the known.

2. Identify all meshes/Loops of the circuit.

3. Assign mesh/Loop currents and label polarities.

4. Apply KVL at each mesh/Loop and express the voltages in terms of the mesh currents.

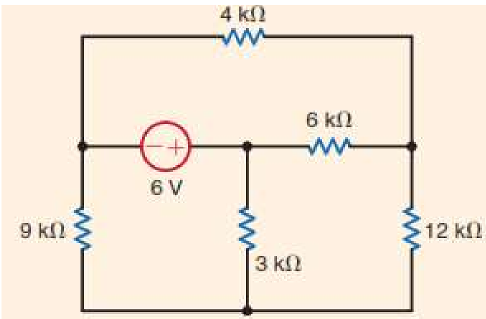
5. Solve the resulting simultaneous equations for the mesh/Loop currents.

6. Now that the mesh/Loop currents are known, the voltages may be obtained from Ohm's law.

MODULE 2: DC CIRCUIT ANALYSIS – 6 HRS

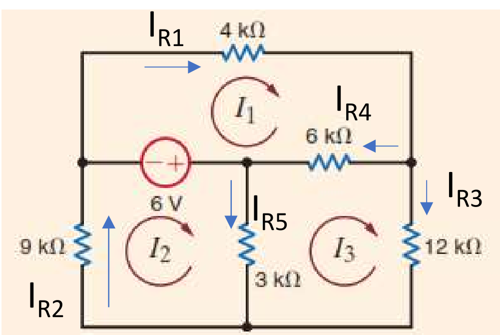
❸ MESH/ LOOP ANALYSIS TECHNIQUE

Example 1: calculate the current through resistors using loop analysis technuque



Solution:

✓ *Assign mesh currents and label polarities*



✓ *Apply KVL at each mesh/loop and express the voltages in terms of the mesh currents.*

Three-mesh equations (KVL) in this case:

KVL - Mesh 1 with mesh current I_1

I_{R1} I_{R4}

$4I_1 + 6(I_1 - I_3) + 6 = 0$

Mesh 2 with mesth current I_2

$9I_2 + 3(I_2 - I_3) - 6 = 0$

Mesh 3 with I_3

$-6(I_1 - I_3) + 12I_3 - 3(I_2 - I_3) = 0$

and then

$I_1 = -0.6757$ mA

$I_2 = 0.4685$ mA

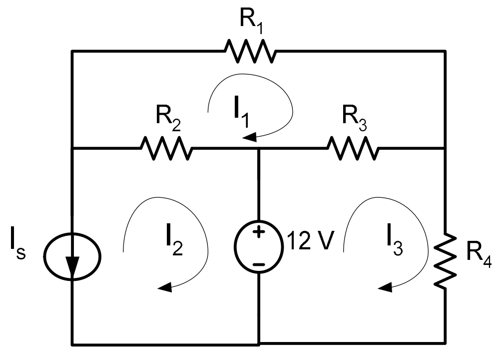
$I_3 = -0.1261$ mA

$$\begin{cases} I_{R1} = I_1 \\ I_{R2} = I_2 \\ I_{R3} = I_3 \\ I_{R4} = I_1 - I_3 \\ I_{R5} = I_2 - I_3 \end{cases}$$

❸ MESH/ LOOP ANALYSIS TECHNIQUE

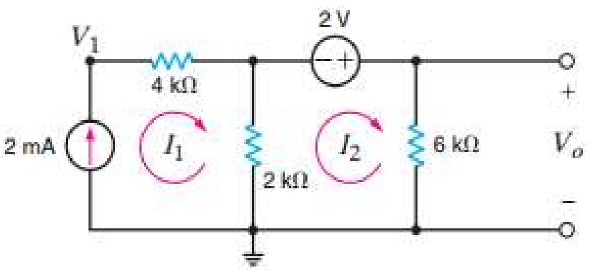
APPLYING MESH/ LOOP ANALYSIS ON CIRCUIT WITH CURRENT SOURCES

- **Case 1: Current source located at the outer most boundary**



- **Connecting mesh current immediately known ($I_2 = -I_s$)**
- **No need to apply KVL around that loop/mesh.**
- **Mesh Current = Element Current = Current Source Value**

Example 1: Let us find both V_0 and V_1 in the circuit in this circuit:



Solution:

Although it appears that there are 02-unknown mesh/loop currents, the current goes directly through the current source and, therefore, is constrained to be $I_1 = 2\,mA$

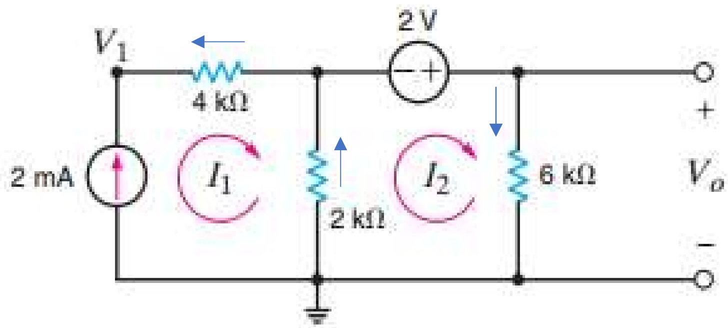
Hence, only the current is unknown → KVL for the right-loop I_2

❸ LOOP ANALYSIS TECHNIQUE

APPLYING LOOP ANALYSIS ON CIRCUIT WITH CURRENT SOURCES

- **Case 1: Current source located at the outer most boundary**

Example 1: Let us find both V_0 and V_1 in the circuit in this circuit:



Solution:

$I_1 = 2\ mA$

Only the current I_2 is unknown parameter.
KVL for the right- mesh I_2 is: $2*(I_2 - I_1) - 2 + 6*I_2 = 0$

$I_2 = 3/4\ mA$ $V_0 = I_2 * 6 = 9/2\,[V]$

To obtain V_1 (voltage cross the current source),
we apply KVL around any closed path.

$V_1 - 4*I_1 - 2 + 6*I_2 = 0$ ← Example: for loop contain mesh 1 & 2

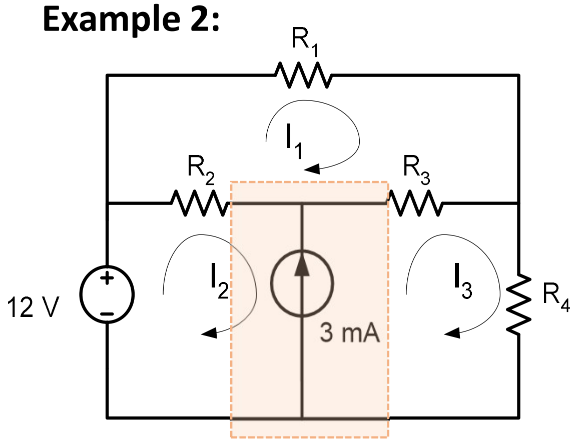
$V_1 = 21/2\,[V]$

❸ LOOP ANALYSIS TECHNIQUE

APPLYING MESH ANALYSIS ON CIRCUIT WITH CURRENT SOURCES

- **<u>Case 2</u>: Current source located at the boundary between 2 - meshes**

Example 2:



- **Enclose the current source and combine the two loops to form a SUPER-MESH.**
- **KVL is performed around the super-mesh; <u>do not consider</u> voltage across current source.**
- **Formulate simultaneous mesh equation – express the relationship between mesh currents that form the s/mesh and current source that it encloses**

Solution:

Super-Mesh equation $I_3 - I_2 = I_s$

KVL for mesh (I_1):

$$R_1 * I_1 + (I_1 - I_2) * R_2 + (I_1 - I_3) * R_3 = 0$$

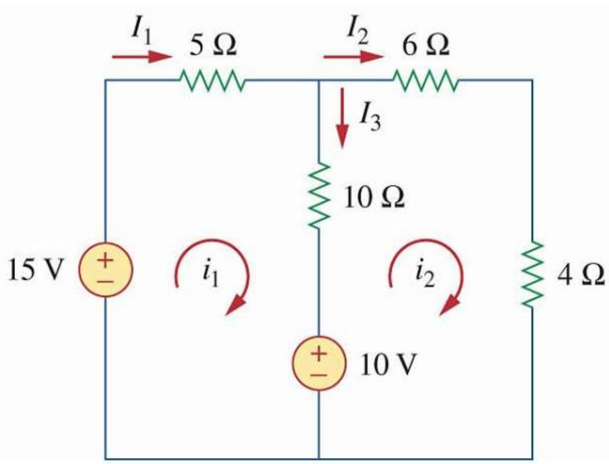
KVL for mesh (I_2 and I_3):

$$-12 + (I_2 - I_1) * R_2 + (I_3 - I_1) * R_3 + I_3 * R_4 = 0$$

❸ MESH/ LOOP ANALYSIS TECHNIQUE

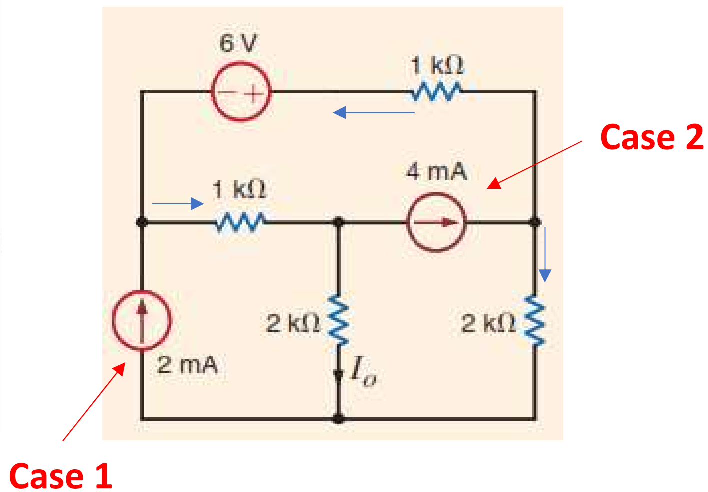
Example 3:

Find the branch currents I_1, I_2, and I_3, using mesh analysis



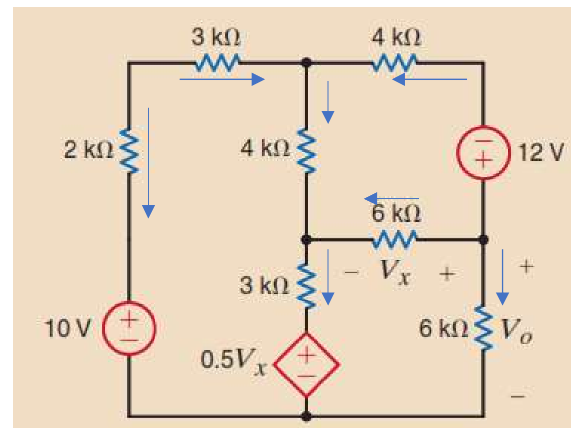
Example 4:

Let us find I_0 in the circuit
Calculate the power supplied by sources
Calculate the power dissipated by resistors



Example 5:

Let us find V_0 , V_x in the circuit

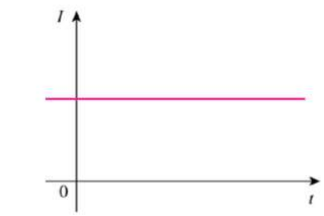


MODULE 1 & 2: TUTORIAL - 3 HRS

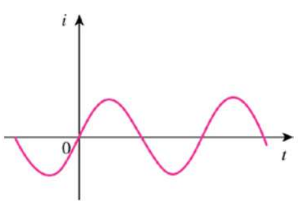
❹ SUMMARY

A. BASIC CONCEPTS

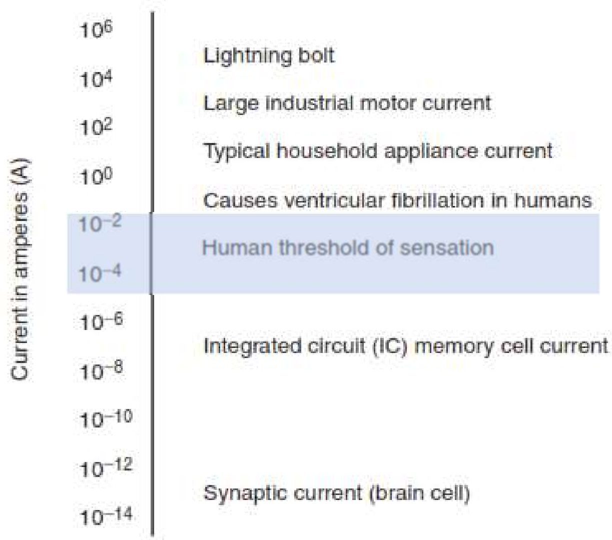
- What is an Electric Circuit ? is an interconnection of electrical components; simple & complex circuits
- Basic electrical quantities an Electric Circuit ? Current (A); Voltage (V); Power (W)
- Electric Circuit Type ? → Direct current (DC); Alternating current (AC)



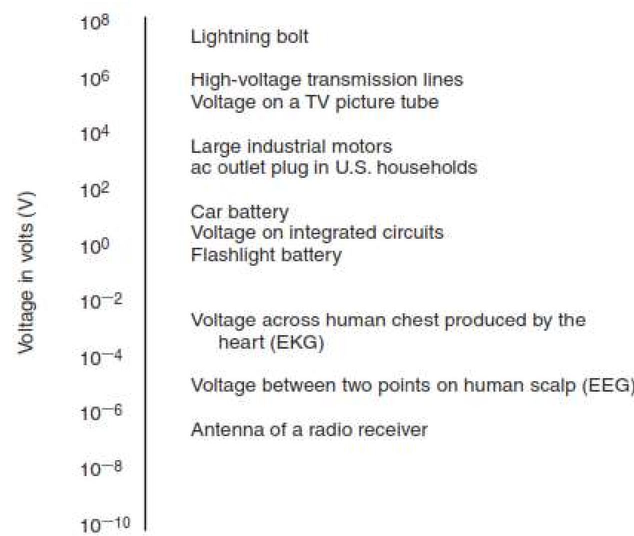
remains constant with time



varies sinusoidally with time



Current Magnitude



Voltage Magnitude

MODULE 1 & 2: TUTORIAL - 3 HRS

❹ SUMMARY

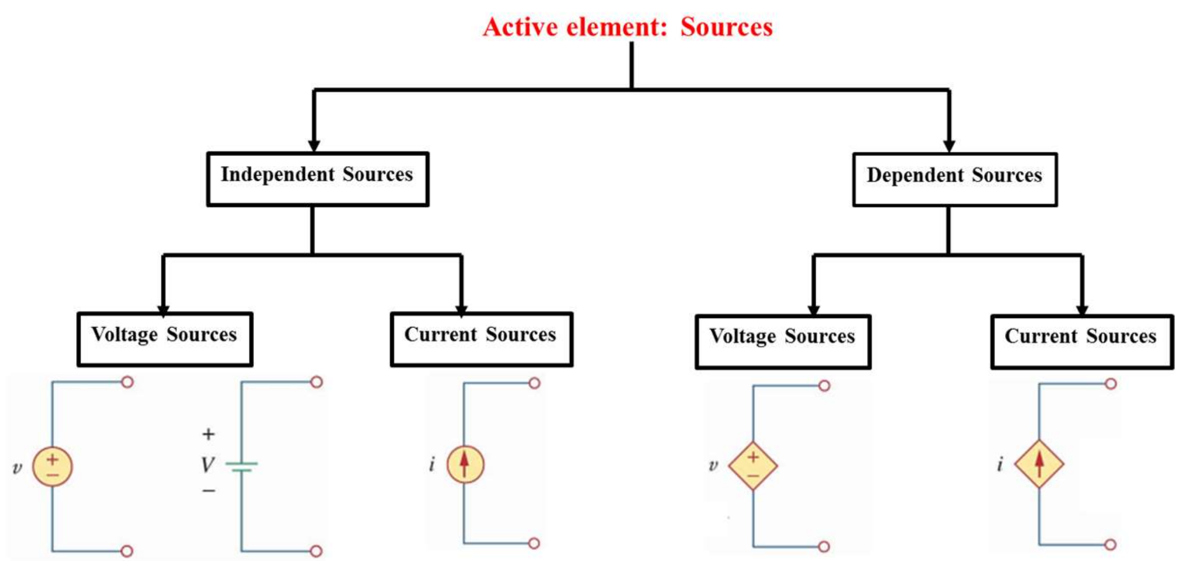
A. BASIC CONCEPTS

- Electric Circuit Components ?

<u>Passive</u> components or passive elements
Components or elements that <u>absorb or store power</u>
Passive: resistors, capacitors, and inductors

<u>Active</u> components or active elements
Components that are not passive! that is, components that <u>deliver power</u>
Active: current/ voltage sources

- Electric Circuit Sources ?



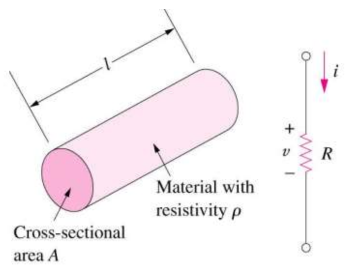
MODULE 1 & 2: TUTORIAL - 3 HRS

❹ SUMMARY

A. BASIC CONCEPTS

- Electric Circuit Components ? ➡ **Passive**: resistors, capacitors, and inductors

Resistor → Resistance (Ω)



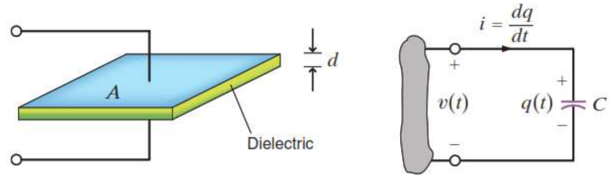
$$R = \rho \frac{L}{A}$$

$$v(t) = Ri(t)$$ **Ohm's Law**

Instantaneous power dissipated in a resistor

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R} = Ri^2(t)$$

Capacitor → Capacitance (F)



$$C = \frac{\varepsilon_0 A}{d}$$

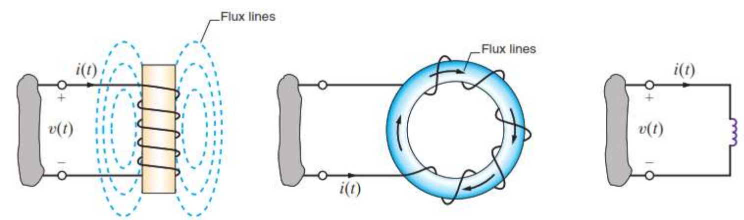
$$i_C(t) = \frac{dq(t)}{dt} = \frac{dq}{dv} * \frac{dv}{dt} = C * \frac{dv}{dt}$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} idt$$

Instantaneous power stored in the capacitor

$$p(t) = C\frac{dv(t)}{dt} * v(t)$$

Inductor → Inductance (H)



$$L = \frac{d\Phi}{di}$$

$$v_L(t) = \frac{d\Phi(t)}{dt} = \frac{d\Phi}{di} * \frac{di}{dt} = L * \frac{di}{dt}$$

Instantaneous power stored in the Inductor

$$p(t) = v(t) * i(t) = L * \frac{di}{dt} * i(t)$$

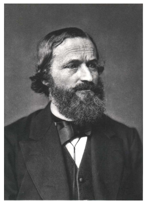
MODULE 1 & 2: TUTORIAL - 3 HRS

❹ **SUMMARY**

B. BASIC LAWS
- **Ohm's Law**
- **Kirchhoff's Current Law**
- **Kirchhoff's Voltage Law**

→ **To analyze single-loop and single node-pair circuits**
To analyze simple circuit → solve characteristic equation

Georg Simon Ohm



Gustav Kirchhoff

C. CIRCUIT ANALYSIS TECHNIQUES
- **Nodal Analysis** (node voltage; KCL)
- **Mesh Analysis** (mesh current; KVL)

→ **to calculate all currents and voltages in circuits that contain multiple nodes and loops**

A circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis.

Three techniques for solving linearly independent simultaneous equations:

- Gaussian elimination
- Matrix analysis
- MATLAB mathematical software package → *see Module 3 (Tutorial)*

❹ PROBLEMS

PROBLEM N°1

Consider the circuit shown in Figure 1

a) Write the nodal equations describing the circuit.

b) Determine the branch currents by nodal voltages.

c) Write loop equations describing the circuit.

d) Determine the branch currents by loop currents.

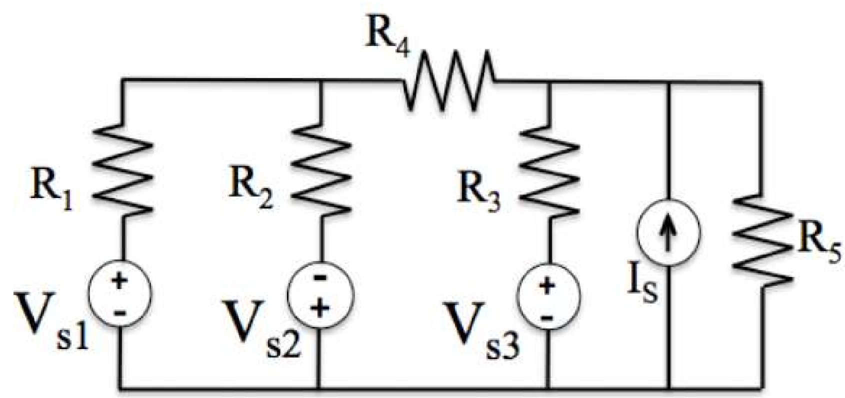


Figure 1

(final exam, 2015)

e) Give Vs1 = 100V, Vs2 = 80V, Vs3 = 24V, Is = 4A, R1 = 5 Ω, R2 = 15 Ω, R3 = 4 Ω, R4 = 8 Ω, R5 = 6 Ω. Determine the voltages across R3, R4 and R5.

SOLUTION N°1

❹ PROBLEMS

SOLUTION N°1

Note that:

$$V_a = V_{S1}\,;\, V_b = -V_{S2}\,;\, V_c = V_{S3}\,;$$

a) Write the nodal equations describing the circuit

b) Determine the branch currents by nodal voltages.

Mark essential nodes and assign unknown node voltages $V_1;V_1$; reference node.

Ohm's law:

Nodal Equations, apply KCL:

- At node 1:

$$I_{R1} + I_{R2} - I_{R4} = 0;$$

$$\frac{V_{S1}-V_1}{R_1} + \frac{-V_1-V_{S2}}{R_2} - \frac{V_1-V_2}{R_4} = 0$$

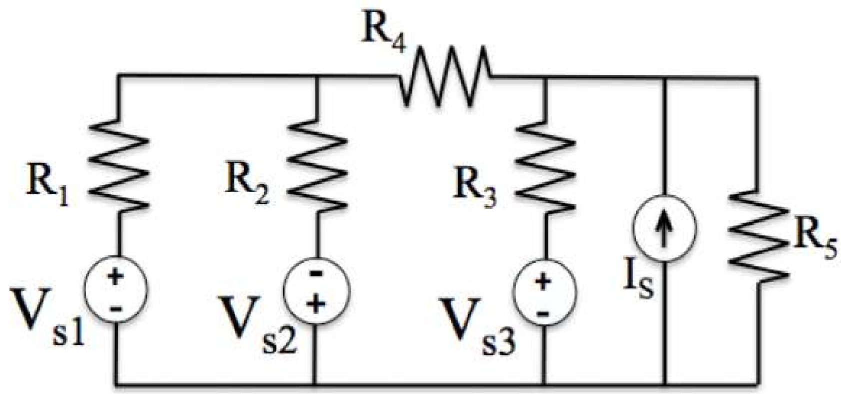
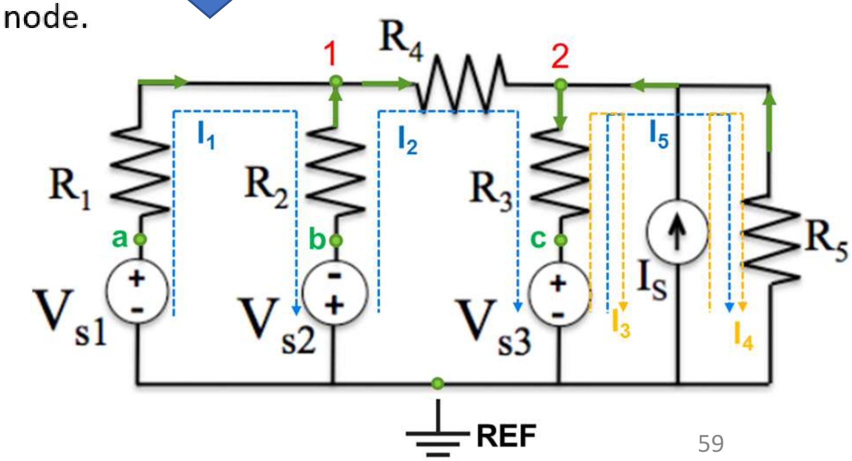


Figure 1



MODULE 1 & 2: TUTORIAL - 3 HRS

❹ PROBLEMS

SOLUTION N°1

Note that:

$$V_a = V_{S1}\ ;\ V_b = -V_{S2}\ ;\ V_c = V_{S3}\ ;$$

- At node 1:

$$\rightarrow \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)*V_1 - \frac{1}{R_4}*V_2 = V_{S1}*\frac{1}{R_1} + V_{S2}\frac{1_2}{R_2}\ ;$$

$$40V_1 - 15V_2 = 3040 \qquad (1)$$

- At node 2:

$$I_{R4} + I_{R5} + I_S - I_{R3} = 0;$$

$$\frac{V_1 - V_2}{R_4} - \frac{V_2}{R_5} + I_S - \frac{V_2 - V_{S3}}{R_2} = 0\ ;$$

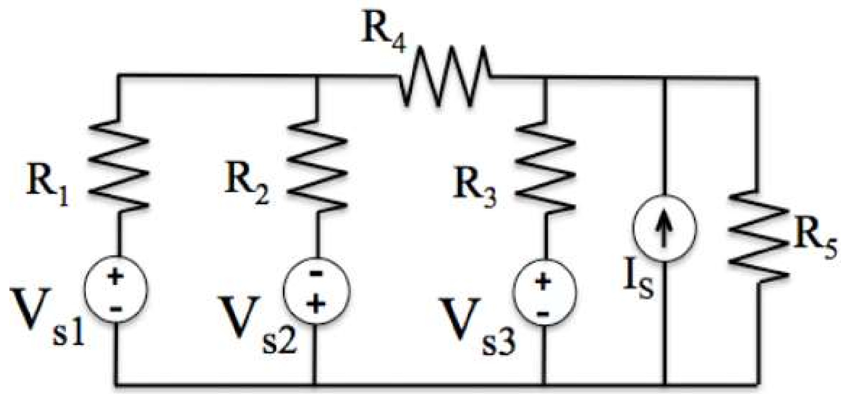
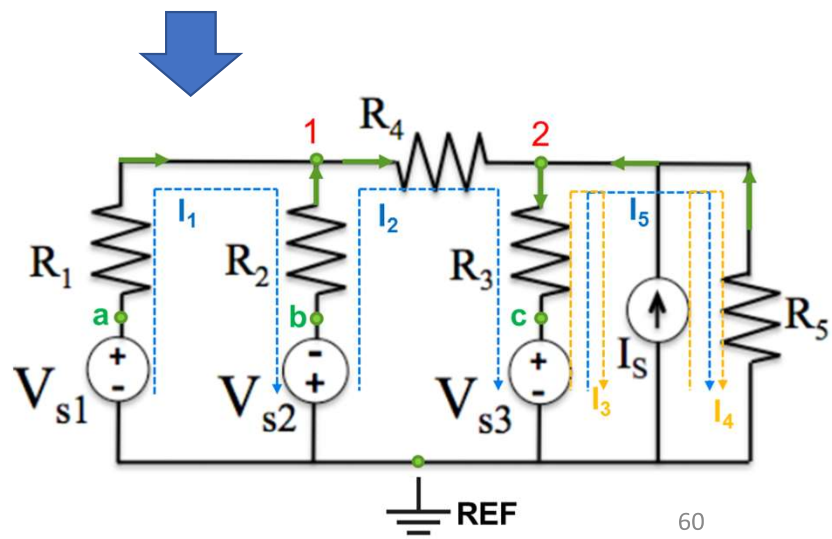


Figure 1



❹ **PROBLEMS**

SOLUTION N°1

Note that:

$$V_a = V_{S1}\ ; V_b = -V_{S2}\ ; V_c = V_{S3}\ ;$$

- At node 2:

$$\frac{1}{R_4} * V_1 - \left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_4}\right) * V_2 \quad = -I_S - V_{S3}\frac{1_2}{R_2};$$

$$15V_1 - 43V_2 = -672 \qquad\qquad (2)$$

From (1) and (2), we obtain: $V_1 = 46.78V\ ; V_2 = 29.26$

$$I_{R1} = 10.64A\ ;\ I_{R2} = -8.52A\ ;\ I_{R3} = 1.32A;\ I_{R4} = 2.19A;\ I_{R2} = -4.88A$$

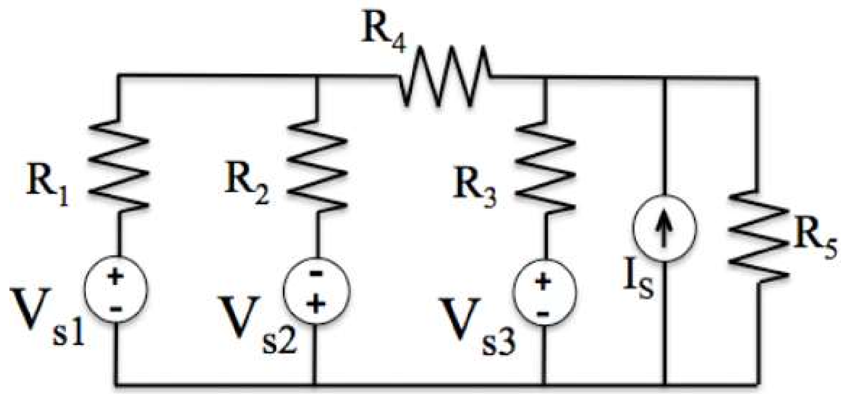
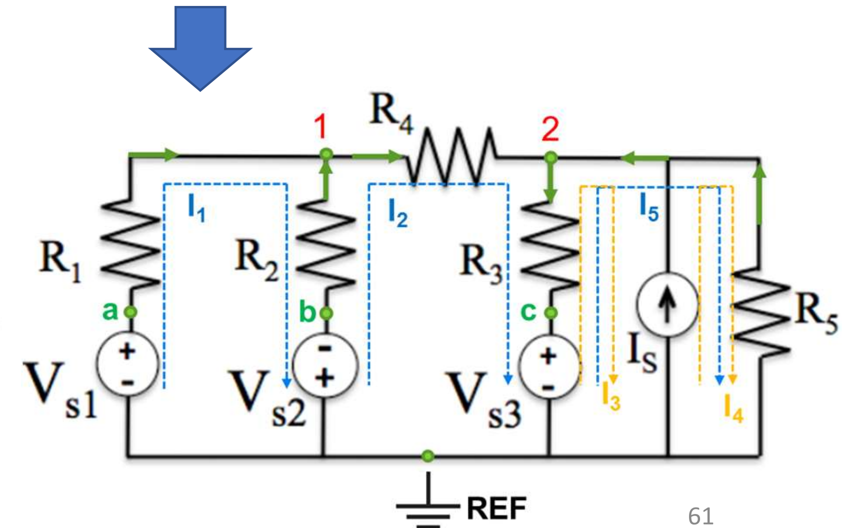


Figure 1



4 **PROBLEMS**

SOLUTION N°1

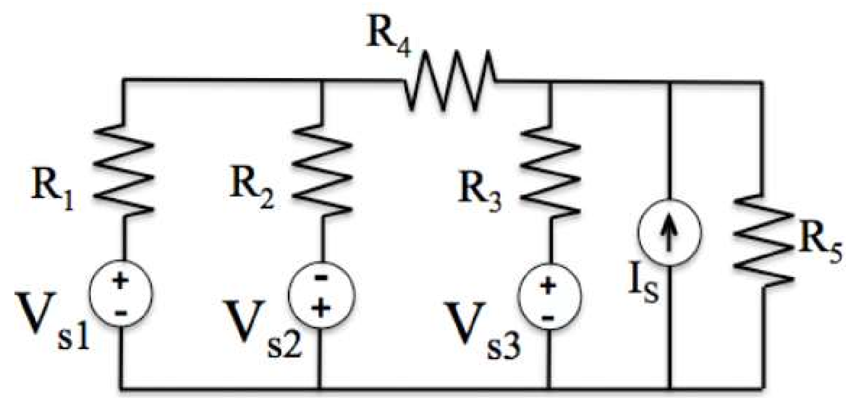


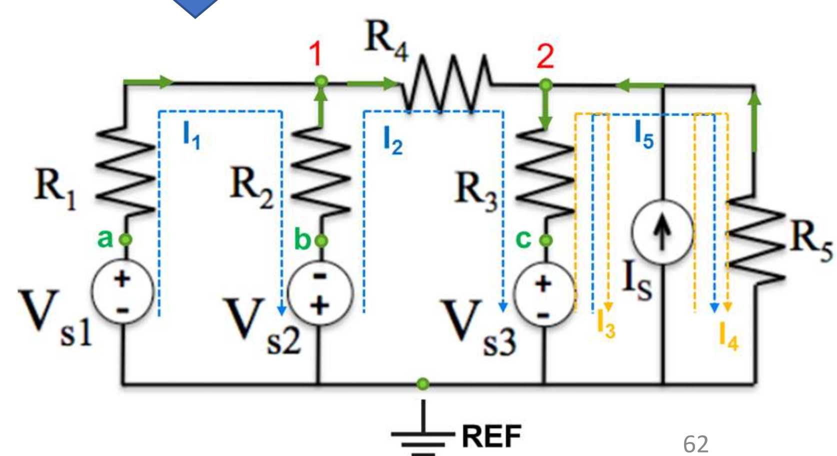
Figure 1

c) Write loop equations describing the circuit.

d) Determine the branch currents by loop currents.

First draw the meshes to be examined in the circuit.

They are labelled in figure with current I_1; I_2; I_3; I_4; I_5 with below.



Loop Equations, apply KVL:

Loop 1: $(R_1 + R_2)*I_1 - R_2*I_2 = V_{S2} + V_{S1}$

Loop 2: $(-R_2)*I_2 + (R_2 + R_3 + R_4)*I_2 - R_3*I_3 = -V_{S3} - V_{S2}$

Super-Mesh: $I_4 - I_3 = I_S$

4 PROBLEMS

SOLUTION N°1

Note that, the voltage across the current source V_x can determine:

Loop 3: $-V_{S3} - R_3 * I_2 + R_3 * I_3 - V_x = 0$,

Loop 4: $R_5 * I_4 + V_x = 0$

So: $-R_3 * I_2 + R_3 * I_3 + R_5 * I_4 = V_{S3}$

→ $I_{R1} = I_1 = 10.64A$; $I_{R2} = I_2 - I_1 = -8.5A$;

$I_{R4} = I_2 = 2.19A$; $I_{R3} = I_2 - I_5 = 1.3A$

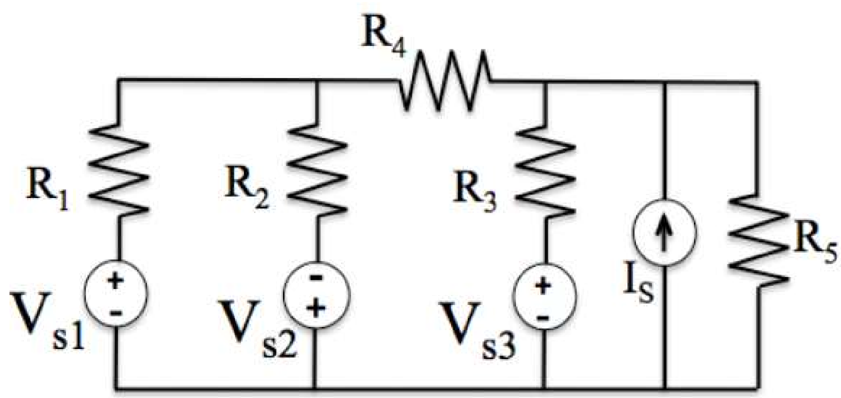
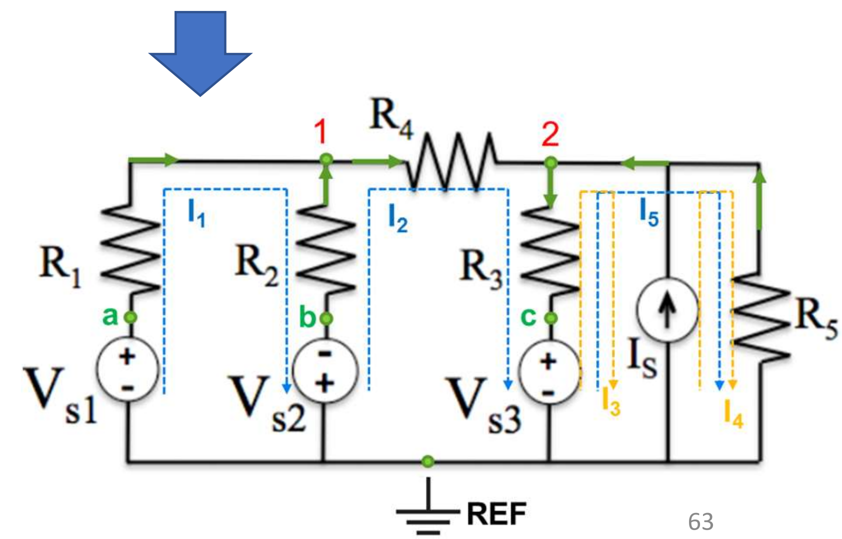


Figure 1



❹ PROBLEMS

PROBLEM N°2

Question 1 (6 pts) Consider the circuit shown in Figure 1.

a) Write the nodal equations describing the circuit
b) Determine the branch currents by nodal voltages
c) Write the loop equations of the circuit
d) Determine the branch currents by loop currents

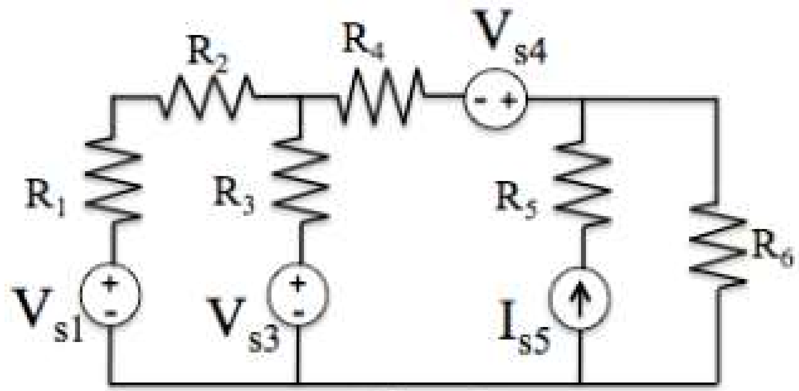
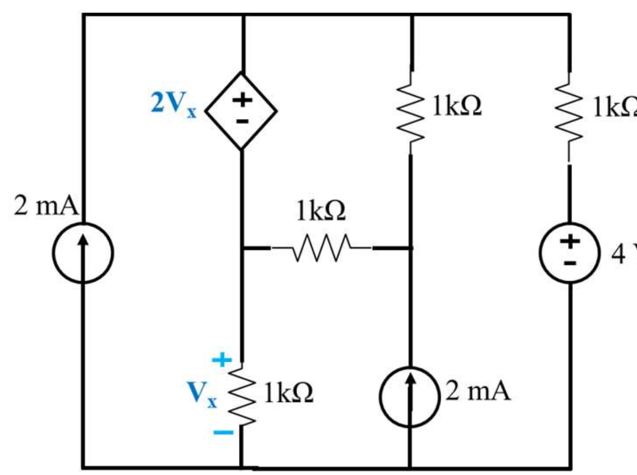


Figure 1

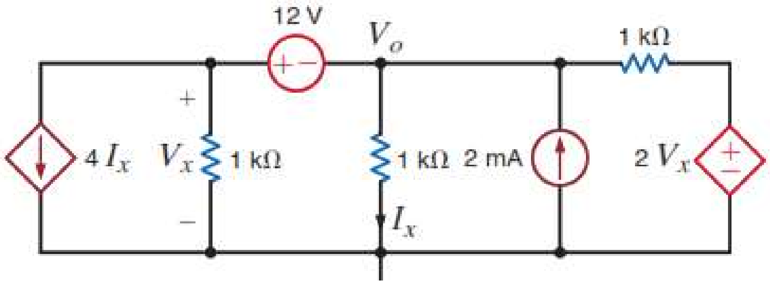
PROBLEM N°3

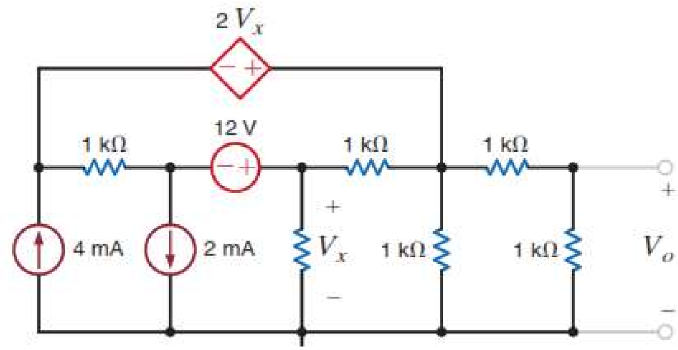
Let us find V_x in the circuit
Calculate the power supplied by sources
Calculate the power dissipated by resistors



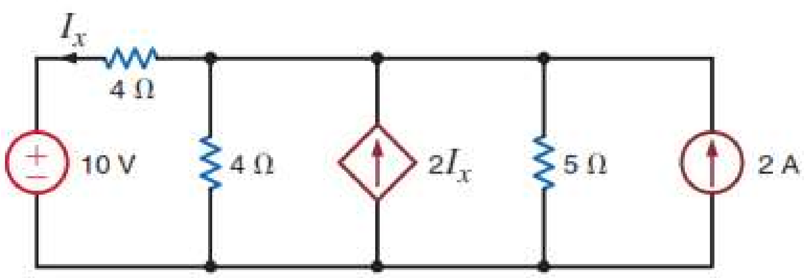
❹ PROBLEMS

PROBLEM N°4 Use nodal analysis to find V_0 in the circuit 4a & 4b

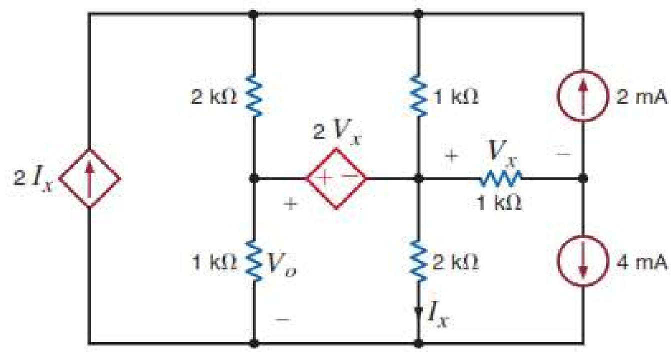




PROBLEM N°5 Find the power supplied by the 2-A current source in this circuit using loop analysis.



PROBLEM N°6 Find V_0 in the circuit in this circuit using loop analysis.



MODULE 3: NETWORK THEOREMS – 6 HRS

Objectives:

- to understand the concepts of linearity and equivalence
- to learn how to analyze electric circuits using the principle of superposition
- to learn how to calculate a Thévenin & Norton equivalent circuit for a linear circuit
- to learn how to use the maximum power transfer theorem

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

Large complex electrical circuits → **Simplify circuit analysis**

How to convert ? → **Circuit Theorems or Network Theorems**

- ✓ **Circuit linearity**
- ✓ **Superposition theorem**
- ✓ **Thévenin's theorem**
- ✓ **Norton's theorem**
- ✓ **Max power transfer**

A. Circuit Linearity

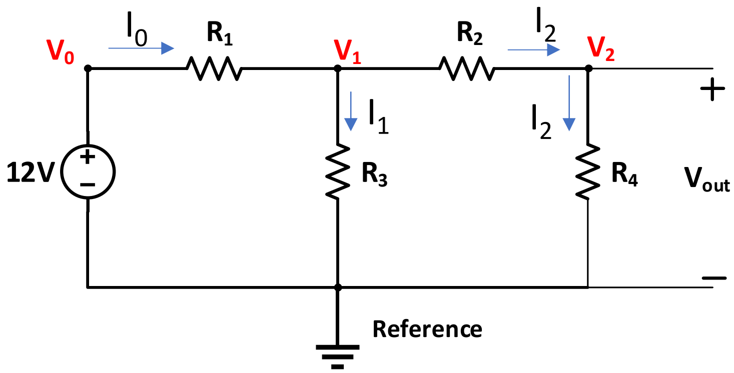
All the circuits we analyze: linear circuits, which are described by a set of linear algebraic equations.

MODULE 3: NETWORK THEOREMS – 6 HRS

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

A. Circuit Linearity

All the circuits we analyze: linear circuits, which are described by a set of linear algebraic equations.



Therefore, the assumption that $\mathbf{V_{out} = 1\ V}$ produced a source voltage of $\mathbf{V_{source} = 6\ V}$

However, the actual source voltage is:
$\mathbf{V_{source} = 12\ V}$,
the actual output voltage is: $\mathbf{V_{out} = 2\ V}$

linearity

Example 1: $R_1 = 2\ k\Omega$; $R_2 = 4k\Omega$; $R_3 = 3\ k\Omega$; $R_4 = 2\ k\Omega$
Input: $V_{source} = 12V$; determine V_{out} = ?

We will use linearity and simply assume that the output voltage is:
$\mathbf{V_{out} = 1\ V}$

This assumption will yield a value for the source voltage $\mathbf{V_{source}}$

$V_2 = V_{out} = 1\ V$, then $I_2 = V_2/R_4 = 0.5\ mA$

$V_1 = R_2 * I_2 + V_2 = 4 * I_2 + V_2$, then $V_1 = 3\ V$ Hence $I_1 = V_1/R_3 = 1\ mA$

Now, applying KCL at node V_1

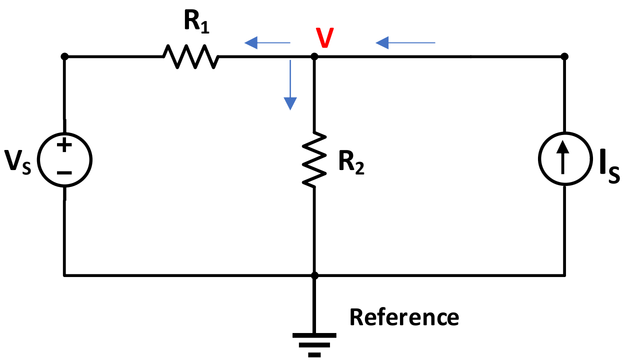
$I_0 = I_1 + I_2 = 1.5\ mA$, then $V_0 = R_1 * I_0 + V_1 = 2 * I_0 + V_1$, then $V_0 = 6\ V$

Therefore, $\mathbf{V_{source} = V_0 = 6\ V}$

MODULE 3: NETWORK THEOREMS – 6 HRS

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

A. Circuit Linearity



Example 2

Suppose we use variables instead of fixed values for all of the independent voltage and current sources. We can then use nodal analysis to find all node voltages in terms of the source values

Node voltage at node 1: **V**

$$-\frac{V-V_s}{R_1}-\frac{V}{R_2}+I_S=0$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V=\frac{1}{R_1}V_S+I_S;$$

We have two kinds of sources: V_S, I_S

V, R_1, R_2, ...linearity;

→ $V=a_1V_1+a_2V_2+\ldots+b_1I_1+b_2I_2\ldots$

Linearity theorem

For any electrical circuit containing resistors and independent sources, every node voltage and branch current is a linear function of all source values and has the form of $\sum a_iU_i$

Where: U_i, source values; a_i, suitably dimensioned constant.

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

A. Circuit Linearity

Problem 1 Find $\mathbf{I_0}$ in the circuit in **figure a & b** using linearity and the assumption that $\mathbf{I_0 = 1}$ **mA.**

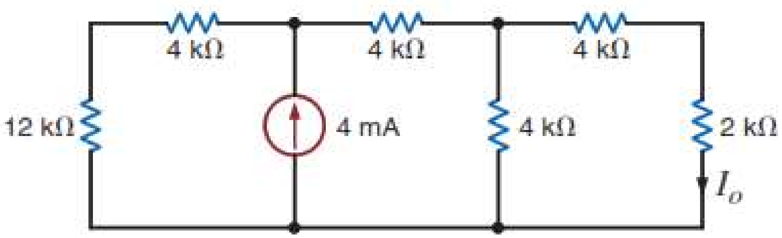


Figure a

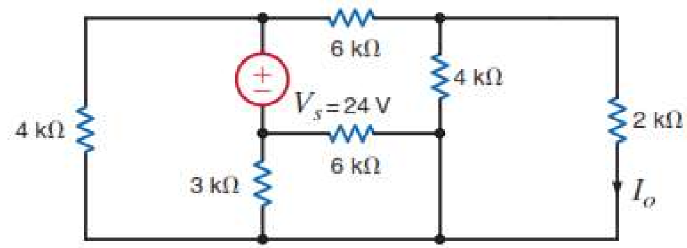
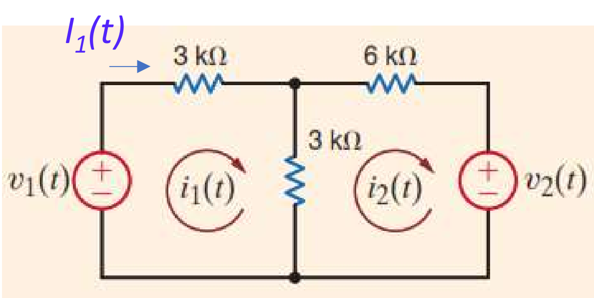


Figure b

B. Superposition Theorem

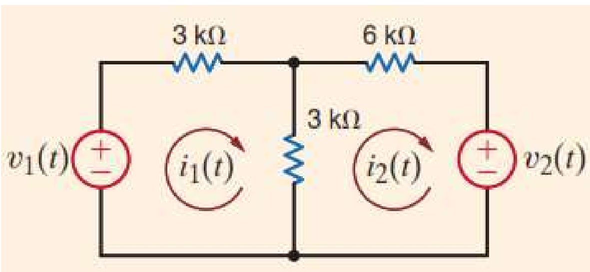


Example 1: we examine the simple circuit, two-sources contribute to the current in this circuit

The actual values of the sources are left unspecified so that we can examine the concept of superposition

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem



Example 1: we examine the simple circuit, two-sources contribute to the current in this circuit

The actual values of the sources are left unspecified so that we can examine the concept of superposition

The mesh equations for this circuit are

$$6ki_1(t) - 3ki_2(t) = v_1(t)$$
$$-3ki_1(t) + 9ki_2(t) = -v_2(t)$$

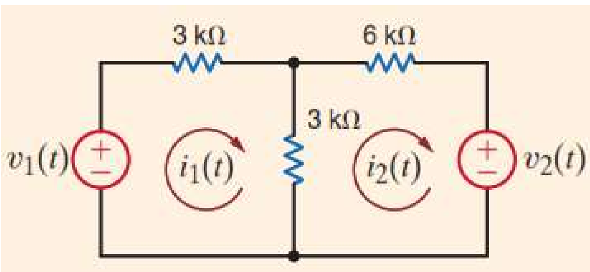
$$\Rightarrow \quad i_1(t) = \frac{v_1(t)}{5k} - \frac{v_2(t)}{15k}$$

The current $i_1(t)$ has a component due to $v_1(t)$ and a component due to $v_2(t)$

In view of the fact that $i_1(t)$ has two components, one due to each independent source.
it would be interesting to examine what each source acting alone would contribute to $i_1(t)$

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem



The current $i_1(t)$ has a component due to $v_1(t)$ and a component due to $v_2(t)$

In view of the fact that $i_1(t)$ has two components, one due to each independent source.
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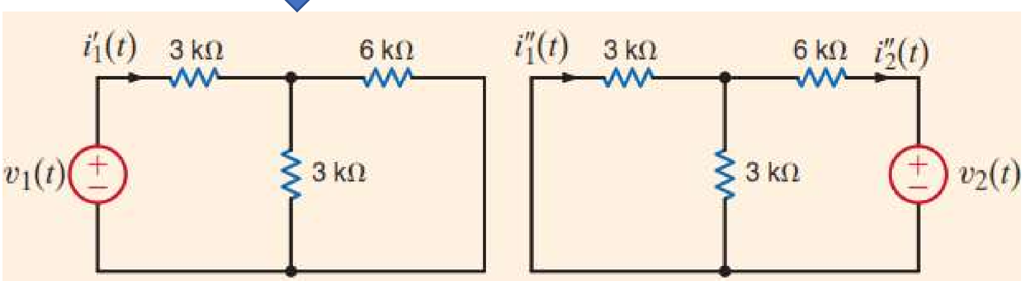


Acting alone by $v_1(t)$　　*Acting alone by $v_2(t)$*

Acting alone by $v_1(t)$ → ***$v_2(t) = 0$ V***

meaning that the source $v_2(t)$ is replaced with a short circuit

$$i_1'(t) = \frac{v_1(t)}{3k + \dfrac{(3k)(6k)}{3k + 6k}} = \frac{v_1(t)}{5k}$$

Acting alone by $v_2(t)$ → ***$v_1(t) = 0$ V***

$$i_2''(t) = -\frac{v_2(t)}{6k + \dfrac{(3k)(3k)}{3k + 3k}} = \frac{-2v_2(t)}{15k}$$

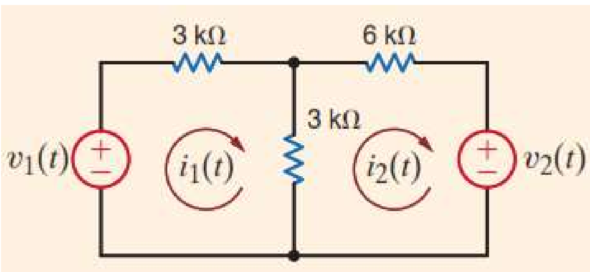
Then, using current division, we obtain

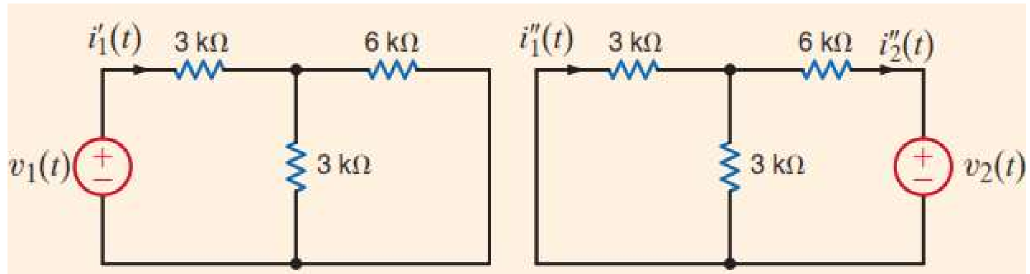
$$i_1''(t) = \frac{-2v_2(t)}{15k}\left(\frac{3k}{3k + 3k}\right) = \frac{-v_2(t)}{15k}$$

MODULE 3: NETWORK THEOREMS – 6 HRS

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem





Acting alone by $v_1(t)$ ***Acting alone by $v_2(t)$***

The current $i_1(t)$ has a component due to $v_1(t)$ and a component due to $v_2(t)$

$$i_1(t) = i_1'(t) + i_1''(t) = \frac{v_1(t)}{5\text{k}} - \frac{v_2(t)}{15\text{k}}$$

Concept of superposition

which provides us with this ability to reduce a complicated problem to several easier problems - each containing only a single independent source

Total current through or voltage across a resistor or branch

- Determine by adding effects due to each source acting independently
- Replace a voltage source with a short-circuit
- Replace a current source with an open-circuit
- Find results of branches using each source independently
- Algebraically combine results

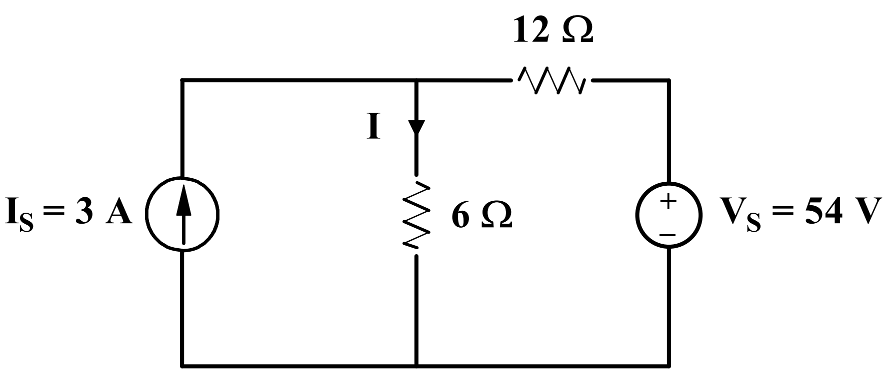
❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem

Concept of superposition

- Power
  - Not a linear quantity
  - Found by squaring voltage or current
- Superposion's theorem does not apply to power
  - To find power using superposition → Determine voltage or current → Calculate power

Example 2: Find the current **I** by using superposition



Solution

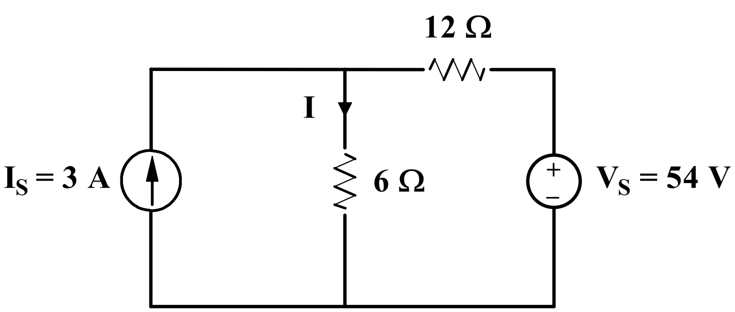
- ✓ First, deactivate the source $\mathbf{I_S}$ and find **I** in the 6 Ω resistor
- ✓ Second, deactivate the source $\mathbf{V_S}$ and find **I** in the 6 Ω resistor.
- ✓ Sum algebraically the two-currents for the total current.

MODULE 3: NETWORK THEOREMS – 6 HRS

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem

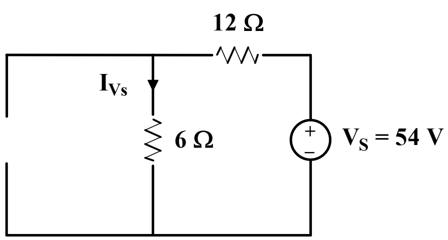
Example 2: Find the current **I** by using superposition



Solution

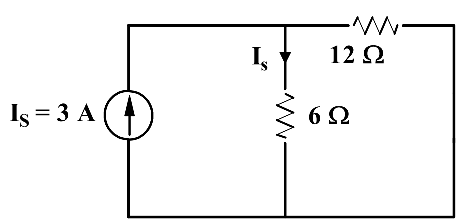
- ✓ First, deactivate the source I_S and find I in the 6 Ω resistor
- ✓ Second, deactivate the source V_S and find I in the 6 Ω resistor.
- ✓ Sum algebraically the two currents for the total current.

Step 1: Open-source I_S



$I_{Vs} = 3$ A

Step 2: Short-source V_S



$$I_S = \frac{3*12}{(3+12)} = 2\,A$$

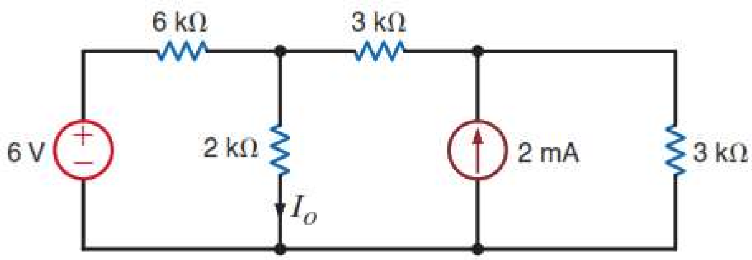
Total current I: $I = I_S + I_{VS} = 5$ A

MODULE 3: NETWORK THEOREMS – 6 HRS

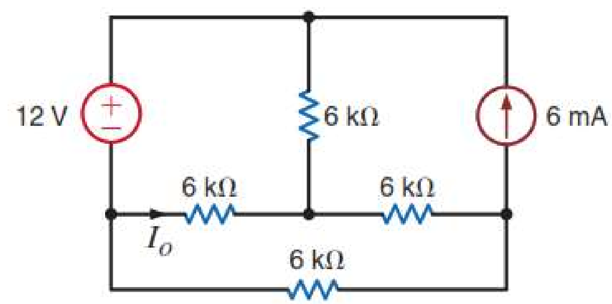
❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

B. Superposition Theorem

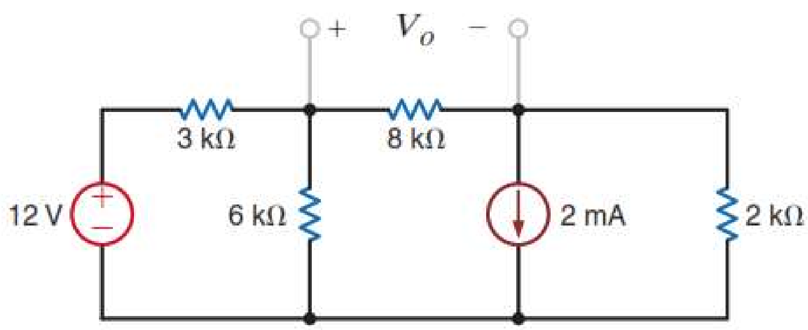
Example 2: Find the current $\mathbf{I_0}$ by using superposition



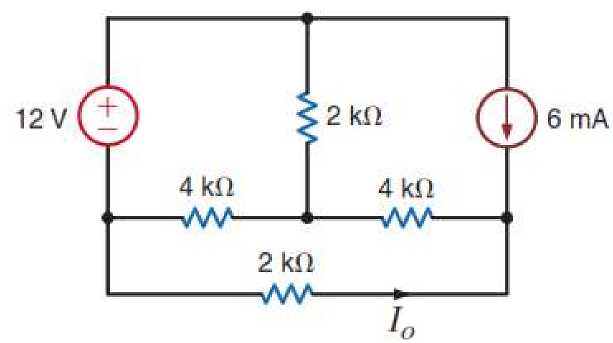
Example 3: Find the current $\mathbf{I_0}$ by using superposition



Example 4: Find the current $\mathbf{V_0}$ by using superposition



Example 5: Find the current $\mathbf{I_0}$ by using superposition



MODULE 3: NETWORK THEOREMS – 6 HRS

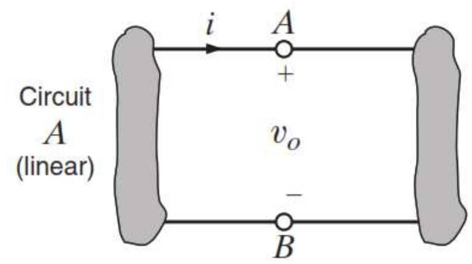
❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

C. Thévenin's and Norton's Theorems

Léon Charles Thévenin
1857 - 1926

Edward Lawry Norton
1898–1983

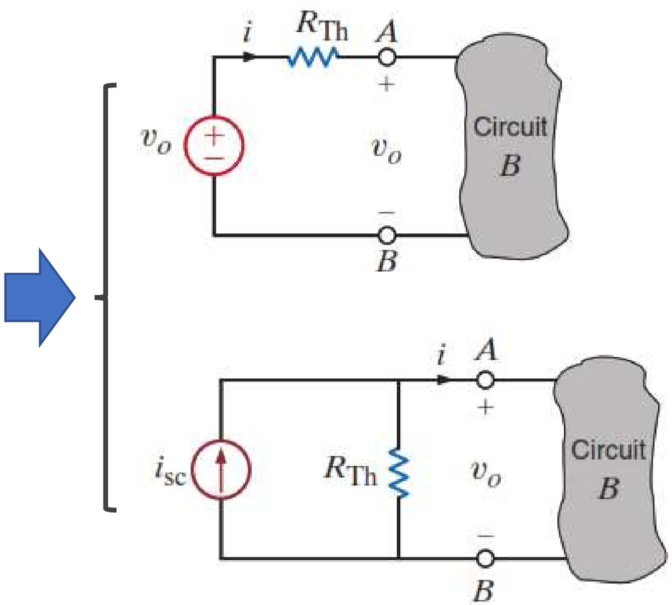
is the balance of the original network <u>exclusive of the load</u> and must be <u>linear</u>



is the load and may be linear or nonlinear

Thévenin's theorem tells us that we can replace the entire network, exclusive of the load, by an equivalent circuit that contains <u>only an independent voltage source in series with a resistor</u> in such a way that the current–voltage relationship at the load is unchanged.

Norton's theorem is identical to the preceding statement except that the equivalent circuit is an <u>independent current source in parallel with a resistor</u>.



Thévenin equivalent circuit

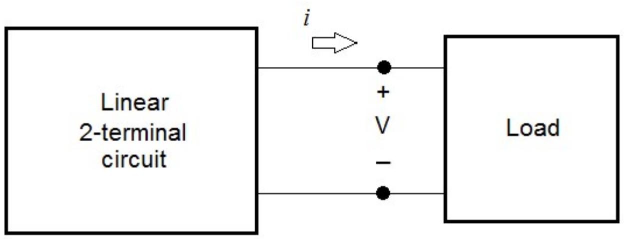
Norton equivalent circuit

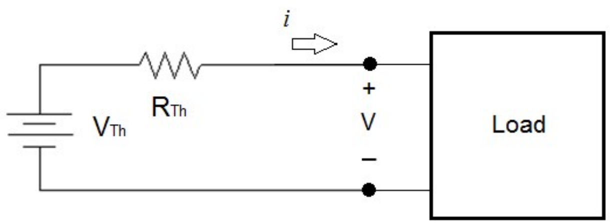
❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit

- A linear two-terminals circuit can be replaced with an equivalent circuit of an ideal voltage source, V_{Th}, in series with a resistor, R_{Th}.
- V_{Th} is equal to the open-circuit voltage at the terminals.
- R_{Th} is the equivalent or input resistance when the independent sources in the linear circuit are turned off





Linear circuit is a circuit where the voltage is directly proportional to the current (i.e., Ohm's Law is followed).

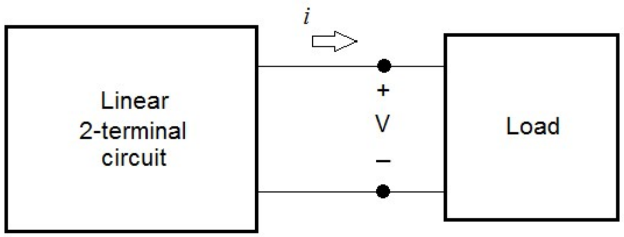
Two-terminals are the 2 nodes/2 wires that can make a connection between the circuit to the load.

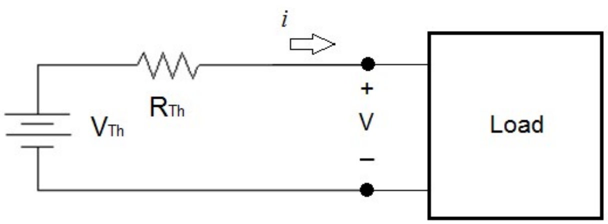
MODULE 3: NETWORK THEOREMS – 6 HRS

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit

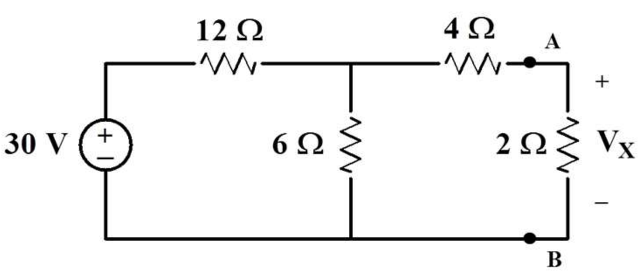




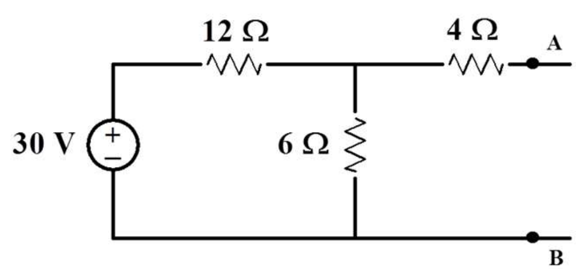
Steps to Determine V_{Th} and R_{Th}

1. Identify the load, which may be a resistor or a part of the circuit.
2. Replace the load with an open circuit.
3. Calculate V_{OC}. This is V_{Th}.
4. Turn off all independent voltage and currents sources in the linear 2-terminal circuit.
5. Calculate the equivalent resistance of the circuit. This is: R_{Th}.
   - The current through and voltage across the load in series with V_{Th} and R_{Th} is the load's actual current and voltage in the original circuit.

Example 1: Find the voltage $\mathbf{V_x}$



Solution for Example 1 First remove everything to the right of A-B



$$\Rightarrow V_{AB} = 30\frac{6\Omega}{6\Omega + 12\Omega} = 10\ V$$

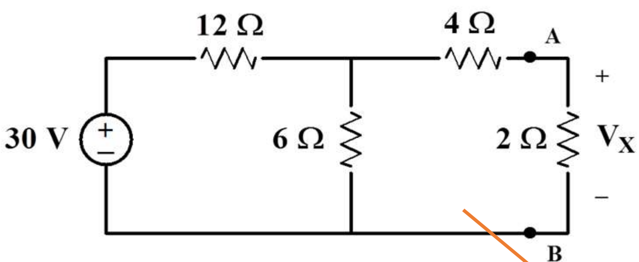
MODULE 3: NETWORK THEOREMS – 6 HRS

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

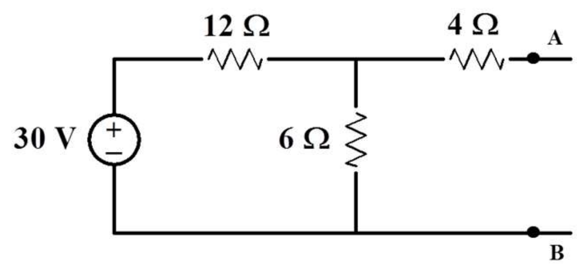
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit

Example 1: Find the voltage $\mathbf{V_x}$



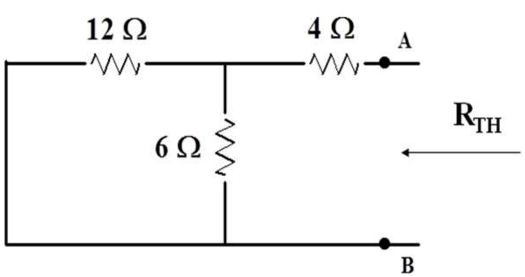
Solution for Example 1 First remove everything to the right of A-B



$$\Rightarrow V_{AB} = 30\frac{6\Omega}{6\Omega + 12\Omega} = 10\,V$$

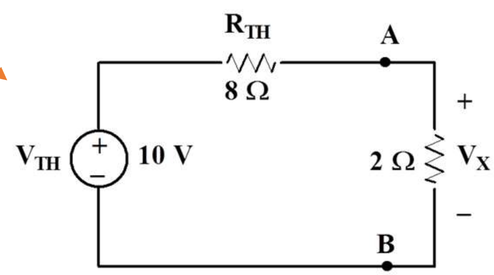
Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus there can be no voltage across the resistor

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals



$$R_{TH} = (12\Omega // 6\Omega) + 4\Omega = 8\Omega$$

After having found the Thevenin circuit, we connect this to the load in order to find V_X.



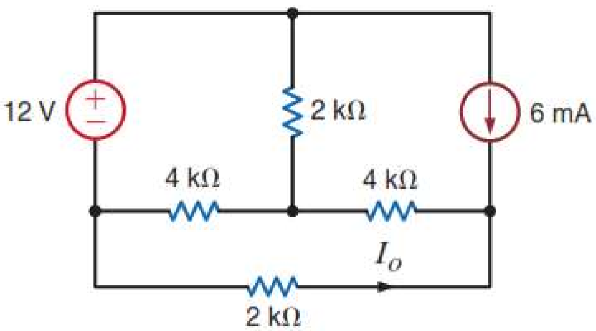
$$\Rightarrow V_X = \frac{10V.2\Omega}{2\Omega + 8\Omega} = 2\,V$$

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

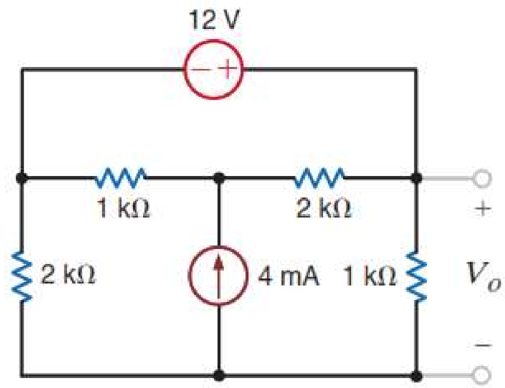
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit

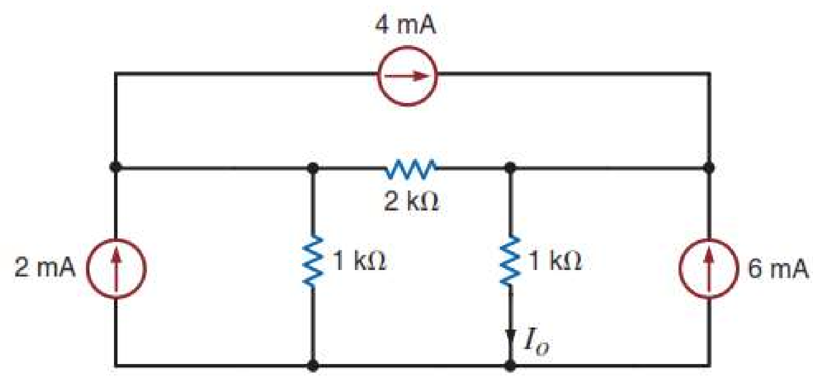
Example 2 Use Thévenin's theorem to find I_0 in the circuit



Example 3 Use Thévenin's theorem to find V_0 in the circuit



Example 4 Use Thévenin's theorem to find I_0 in the circuit

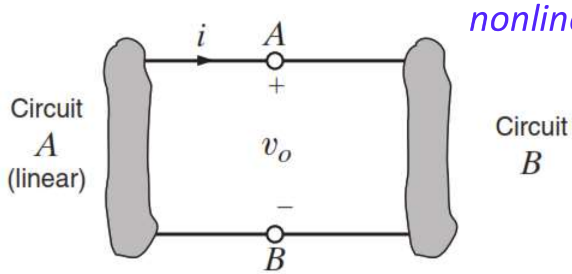


1 INTRODUCTION – ELECTRICAL NETWORK’S THEOREM

C. Thévenin’s and Norton’s Theorems **Thévenin equivalent circuit working with a dependent sources**

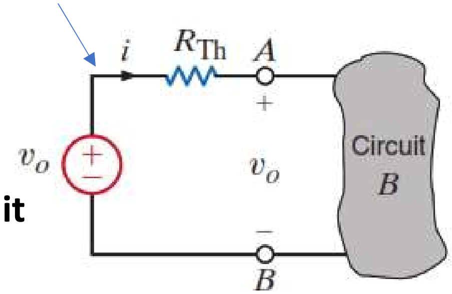
As we have stated earlier, the Thévenin or Norton equivalent of a network containing <u>only independent sources</u>

is the balance of the original network <u>exclusive of the load</u> and must be <u>linear</u>



is the load and may be linear or nonlinear

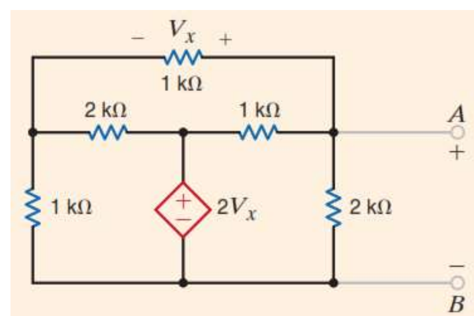
Circuit A delivers a current i to circuit B and produces a voltage across the input terminals of circuit B



Thévenin equivalent circuit For linear circuit A

Thévenin equivalent circuit working with a dependent sources ?

Example 1

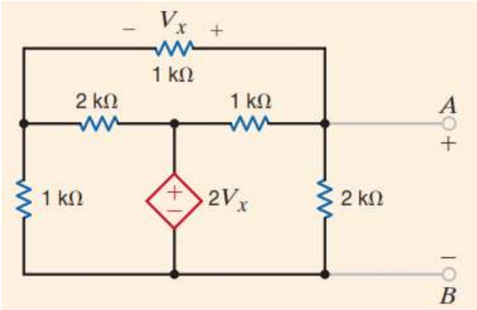


INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with a dependent sources

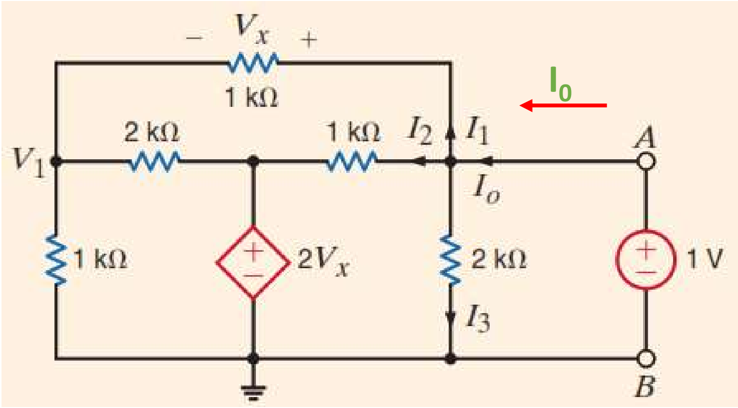
Thévenin equivalent circuit working with a dependent sources ?



Example 1

Our approach to this problem will be to apply a **1-V source** at the terminals A-B of the circuit and then compute the current I_0 and $R_{Th} = 1/I_0$

Thevenin equivalent resistance: ratio Voc/I



Example 1

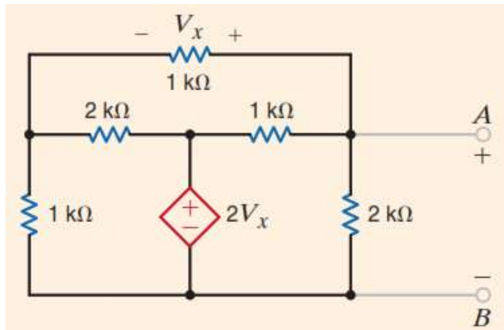
Circuit B delivers a current I_0 to circuit A and produces a voltage across the input terminals of circuit A

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

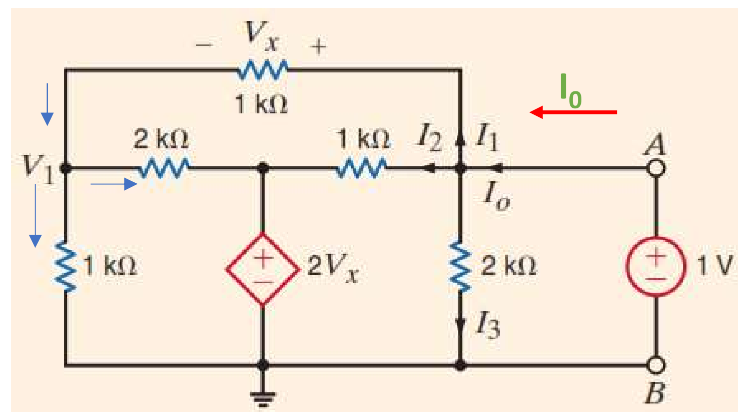
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with a dependent sources

Case 1: Thévenin equivalent circuit working with a only dependent sources ?



Example 1



KVL around the outer loop specifies that

$V_1 + V_x = 1$ V

The KCL equation at the node labeled V_1 is

$V1/1k + (V_1 – 2V_x)/2k - (V_1-1)/1k = 0$

→ $V_x = 1$ V

→ $I_1 = V_x/1 = 1$ mA $\quad$ $I_3 = 1/2 = 0.5$ mA

$I_2 = (1 - 2V_x)/1 = -1$ mA $\quad$ $I_0 = I_1 + I_2 + I_3 = 0.5$ mA → $R_{Th} = 1/I_0 = 2$ kΩ

1 INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

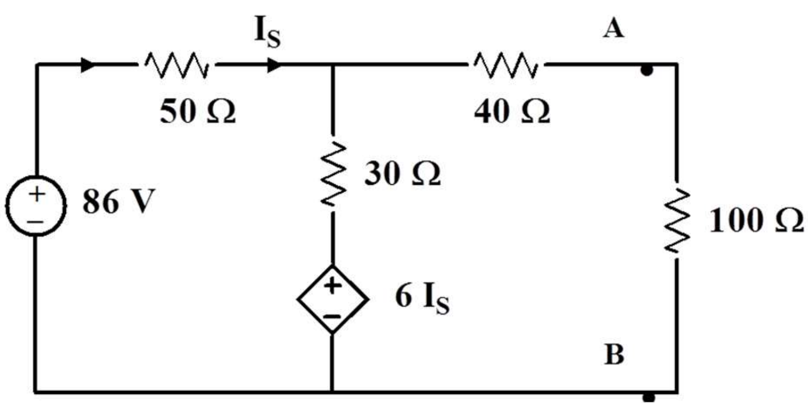
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with a dependent sources

Case 2: Thévenin equivalent circuit working with both dependent and independent sources ?

Example 2

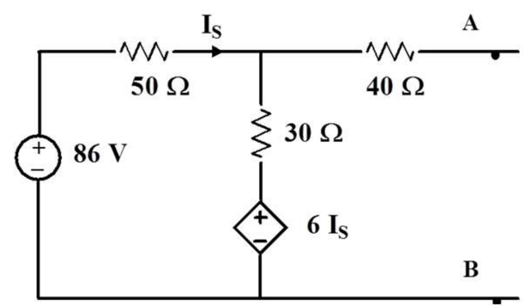
Find the voltage across the 100 Ω load resistor by first finding the Thevenin circuit to the left of terminals A-B



Solution for Example 2

➢ **Step 1: Find the open-voltage V_{TH}**

Remove the 100 Ω load resistor and find $V_{AB} = V_{TH}$ to the left of terminals A-B



$$-86 + 80I_S + 6I_S = 0 \rightarrow I_S = 1\,A$$
$$V_{AB} = 6I_S + 30I_S = \rightarrow 36V$$

$V_{TH} = V_{AB} = 36$ V

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

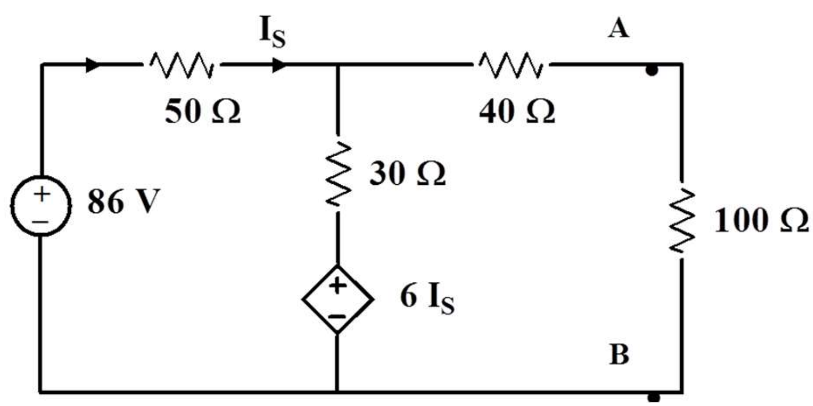
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with a dependent sources

Thévenin equivalent circuit working with both dependent and independent sources ?

Example 2

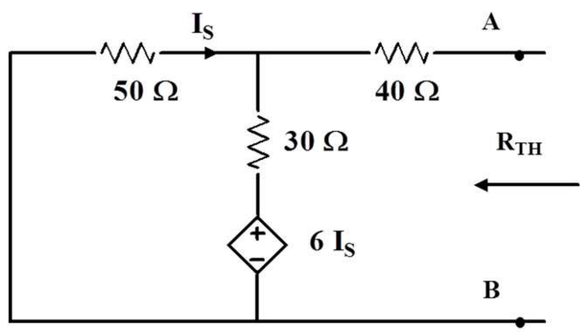
Find the voltage across the 100 Ω load resistor by first finding the Thevenin circuit to the left of terminals A-B



Solution for Example 2

- **Step 2: Find the Thevenin's equivalent resistance R_{TH}**

  We deactivate all independent sources but retain all dependent sources (controlled variable):



Technique to fine R_{TH} ⇐ Refer to the **Case 1** – previous slides

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

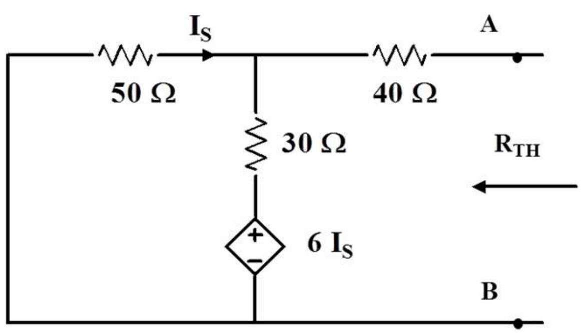
C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with both dependent and independent sources ?

Solution for Example 2

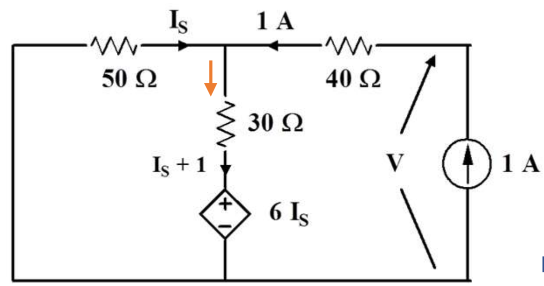
- **Step 2: Find the Thevenin's equivalent resistance R_{TH}**

We deactivate all independent sources but retain all dependent sources (controlled variable):



Thévenin equivalent circuit working with a dependent sources

We must apply either a voltage or current source at the load and calculate the ratio of this voltage to current to find R_{TH}.



Around the loop at the left we write the KVL equation

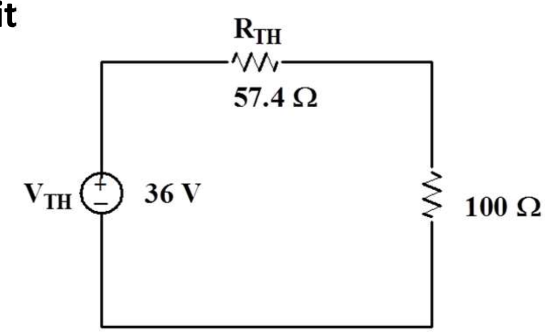
$$50I_S + 30(I_S + 1) + 6I_S = 0; --> I_S = \frac{-15}{43}A$$

➡ Find Voltage of the current source 1-A ?

$$50\left(\frac{-15}{43}\right) - 1(40) + V = 0 \quad \text{or} \quad V = 57.4 \text{ volts}$$

$$R_{TH} = \frac{V}{I} = \frac{V}{1} = 57.4\ \Omega$$

- **Step 3: Find the Thevenin's equivalent circuit**



➡ $$V_{100} = \frac{36x100}{57.4 + 100} = 22.9\ V$$

MODULE 3: NETWORK THEOREMS – 6 HRS

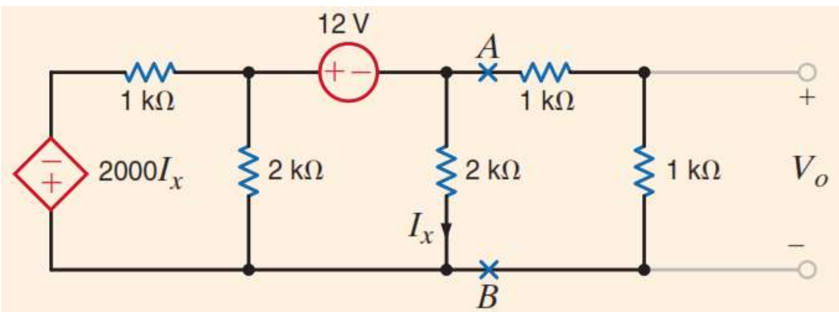
1 INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

C. Thévenin's and Norton's Theorems

Thévenin equivalent circuit working with a dependent sources

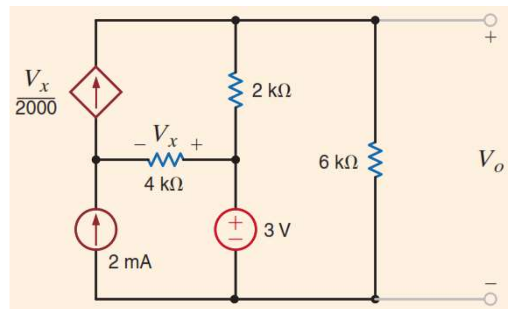
Example 3

Let us use Thévenin's theorem to find V_0



Example 4

Let us use Thévenin's theorem to find V_0



$$V_{oc} = 2kI_1 + 3 = 11\text{ V}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = 2\text{ k}\Omega$$

$$V_o = 11\left(\frac{6k}{2k + 6k}\right)$$

$$= \frac{33}{4}\text{ V}$$

Hidden:

$$V_{oc} = -6\ V$$

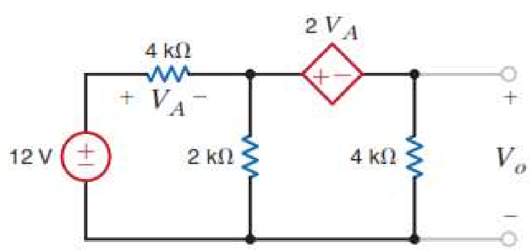
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{1}{3}\text{ k}\Omega$$

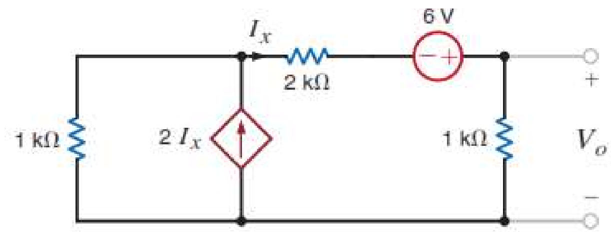
$$V_o = (-6)\left(\frac{1k}{1k + 1k + \frac{1}{3}k}\right) = \frac{-18}{7}\text{ V}$$

Example 5

Example 6

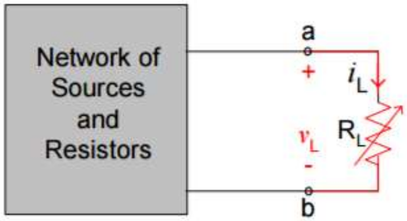
Let us use Thévenin's theorem to find V_0

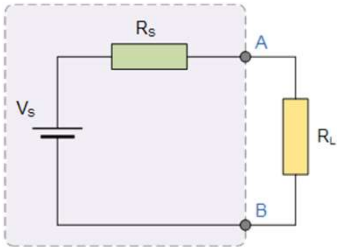




INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

D. Maximum Power Transfer





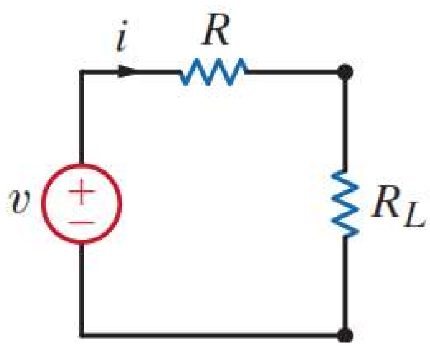
In circuit analysis we are sometimes interested in determining the maximum power that can be delivered to a load. By employing Thévenin's theorem, we can determine the maximum power that a circuit can supply and the manner in which to adjust the load to effect maximum power transfer.

Problem statements:

- ✓ What is the maximum power that can be delivered to a load?
- ✓ What is the value of load resistance R_L that maximizes the power?
- ✓ What is the efficiency of power transfer?

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

D. Maximum Power Transfer



Simple circuit

The power that is delivered to the load is given by the expression

$$P_{\text{load}} = i^2 R_L = \left(\frac{v}{R + R_L}\right)^2 R_L$$

✓ What is the value of load resistance R_L that maximizes the power?

→ We differentiate this expression with respect to R_L and equate the derivative to zero

$$\frac{dP_{\text{load}}}{dR_L} = \frac{(R + R_L)^2 v^2 - 2v^2 R_L (R + R_L)}{(R + R_L)^4} = 0$$

→ $R_L = R$
(for the Thevenin equivalent circuit: $R = R_{Th}$)

Results of maximum power transfer:

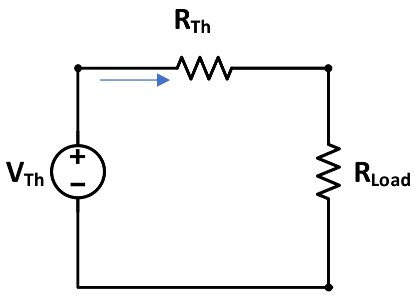
- The maximum power transfer takes place when the load resistance match to the Thevenin's resistance

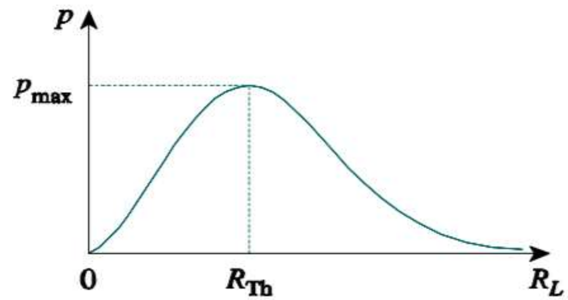
Using Thevenin equivalent circuit

The maximum power transferred to the load $P_{\max} = P_L, (R_L = R_{TH}) = \dfrac{V^2{}_{TH}}{4R_{TH}}$

INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

D. Maximum Power Transfer





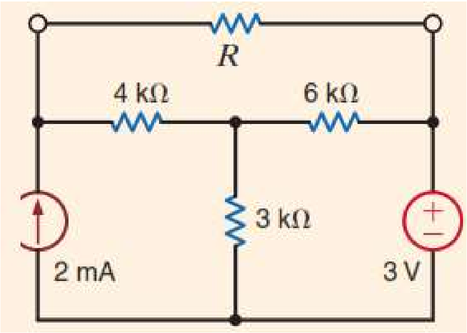
The maximum power transferred to the load

$$\begin{cases} P_{max} = P_L = \dfrac{V^2_{TH}}{4R_{TH}} \\ R_L = R_{TH} \end{cases}$$

Example 1: Find the voltage $\mathbf{V_x}$

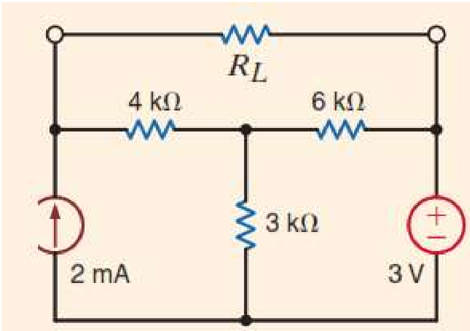
Let us find

- the value of R for maximum power transfer in the network
- the maximum power that can be transferred to this load R



Consider R = Load

Using Thevenin equivalent circuit

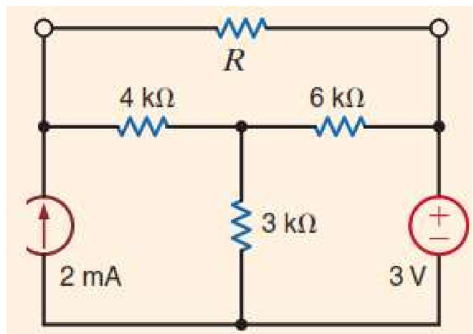


❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

D. Maximum Power Transfer

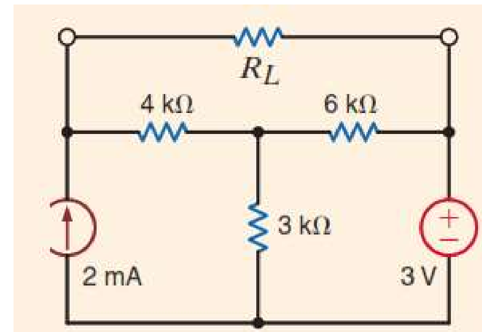
Example 1: Find the voltage $\mathbf{V_x}$

Let us find
- the value of R for maximum power transfer in the network
- the maximum power that can be transferred to this load R

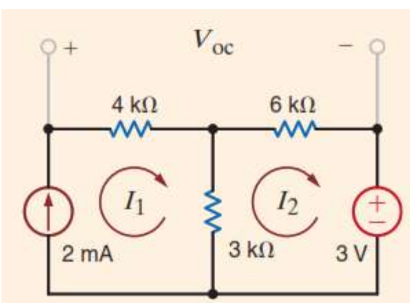


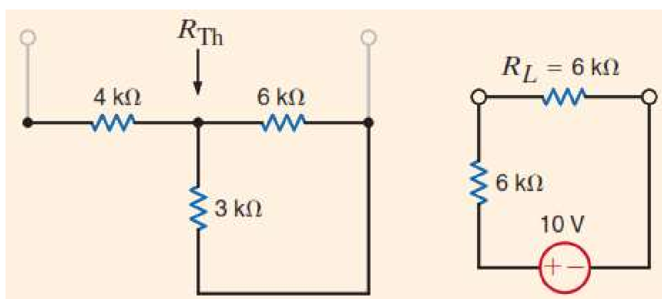
Consider R = Load

Using Thevenin equivalent circuit



- **Step 1: Find the Thevenin's equivalent voltage V_{OC} and resistance R_{TH}**





- **Step 2: Find the R for max power transfer by using Thevenin's equivalent circuit**

$$R_L = R_{Th} = 6\,\text{k}\Omega$$

$$P_L = \left(\frac{10}{12\text{k}}\right)^2 (6\text{k}) = \frac{25}{6}\,\text{mW}$$

MODULE 3: NETWORK THEOREMS – 6 HRS

1 INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

PROBLEM N°1

5.34 Use Thévenin's theorem to find V_o in the circuit using Fig. P5.34.

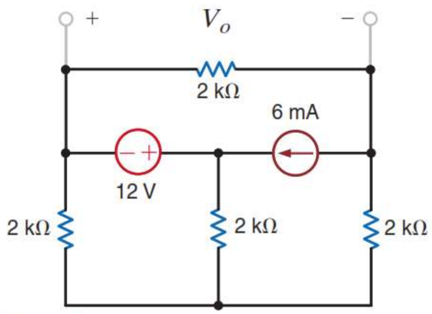


Figure P5.34

PROBLEM N°2

5.38 Find V_o in the circuit in Fig. P5.38 using Thévenin's theorem.

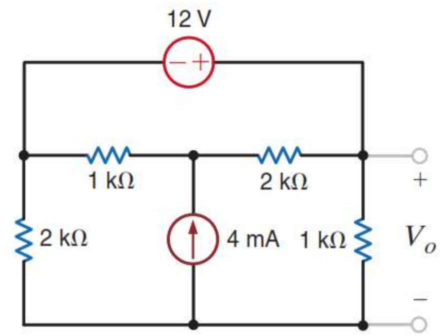


Figure P5.38

PROBLEM N°3

5.59 Find V_o in the network in Fig. P5.59 using Thévenin's theorem.

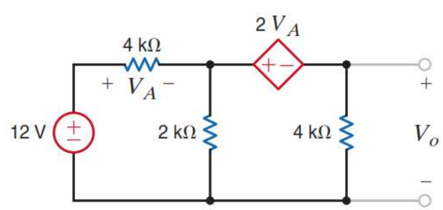


Figure P5.59

PROBLEM N°4

5.109 Calculate the maximum power that can be transferred to R_L in Fig. P5.109.

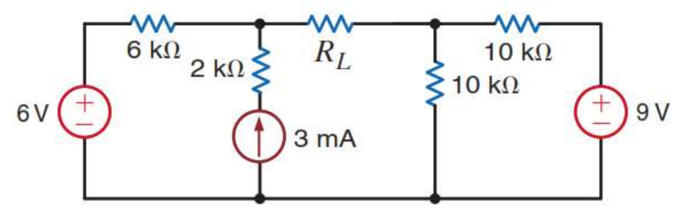


Figure P5.109

❶ INTRODUCTION – ELECTRICAL NETWORK'S THEOREM

D. Maximum Power Transfer

Question 3 (7 pts): In figure 3, give $V_{s1} = 10V$, $R_1 = 5\ \Omega$, $R_2 = 3\ \Omega$, $I_{s3} = 3A$, $R_3 = 6\ \Omega$, $R_4 = 2\ \Omega$, $V_{s4} = 10V$, $R_5 = 4\ \Omega$, $V_{s5} = 15V$, $V_s = 12V$, $I_s = 4A$. Find the value of R that will achieve maximum power transfer and determine that value of the maximum power.

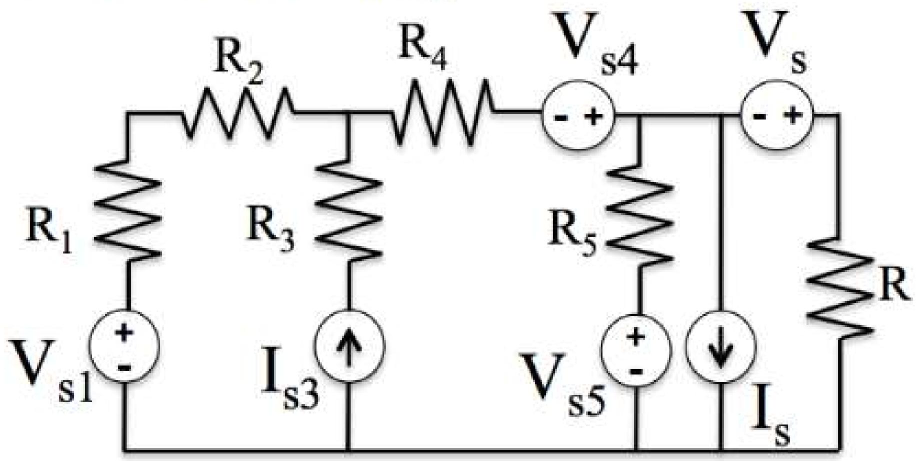


Figure 3

MODULE 3: TUTORIAL – 2 HRS

CONTENT

Network theorems

- **Linearity**
- **Principle of Superposition applying for a linear network containing multiple independent sources**
- **Using Thévenin's theorem: replace some portion of a network at a pair of terminals with a voltage source in series with a resistor**
- **Maximum power transfer can be achieved by selecting the load to be equal to Thevenin equivalent resistance found by looking into the network from the load terminals**

Matlab DC circuit analysis

- **The MATLAB program for solving the nodal voltages, mesh currents;**
- **The MATLAB functions diff and find → maximum power transfer**
- **The MATLAB Simulink for DC Circuit Modeling**

MODULE 3: TUTORIAL – 2 HRS

SUMMARY 1

Principle of Superposition

Textbook, page 196

Applying Superposition

Step 1. In a network containing multiple independent sources, each source can be applied independently with the remaining sources turned off.

Step 2. To turn off a voltage source, replace it with a short circuit, and to turn off a current source, replace it with an open circuit.

Step 3. When the individual sources are applied to the circuit, all the circuit laws and techniques we have learned, or will soon learn, can be applied to obtain a solution.

Step 4. The results obtained by applying each source independently are then added together algebraically to obtain a solution.

Applying Thévenin's Theorem

Textbook, page 211

Step 1. Remove the load and find the voltage across the open-circuit terminals, V_{oc}. All the circuit analysis techniques presented here can be used to compute this voltage.

Step 2. Determine the Thévenin equivalent resistance of the network at the open terminals with the load removed. Three different types of circuits may be encountered in determining the resistance, R_{Th}.

- **(a)** If the circuit contains only independent sources, they are made zero by replacing the voltage sources with short circuits and the current sources with open circuits. R_{Th} is then found by computing the resistance of the purely resistive network at the open terminals.
- **(b)** If the circuit contains only dependent sources, an independent voltage or current source is applied at the open terminals and the corresponding current or voltage at these terminals is measured. The voltage/current ratio at the terminals is the Thévenin equivalent resistance. Since there is no energy source, the open-circuit voltage is zero in this case.
- **(c)** If the circuit contains both independent and dependent sources, the open-circuit terminals are shorted and the short-circuit current between these terminals is determined. The ratio of the open-circuit voltage to the short-circuit current is the resistance R_{Th}.

Step 3. If the load is now connected to the Thévenin equivalent circuit, consisting of V_{oc} in series with R_{Th}, the desired solution can be obtained.

Maximum Power Transfer

- Maximum power transfer can be achieved by selecting the load R_L to be equal to R_{Th} found by looking into the network from the load terminals.

$$P_{\max} = P_L, (R_L = R_{TH}) = \frac{V^2{}_{TH}}{4R_{TH}}$$

PROBLEM N°1

Question 3 (7 pts): In figure 3, give $V_{s1} = 10V$, $R_1 = 5\ \Omega$, $R_2 = 3\ \Omega$, $I_{s3} = 3A$, $R_3 = 6\ \Omega$, $R_4 = 2\ \Omega$, $V_{s4} = 10V$, $R_5 = 4\ \Omega$, $V_{s5} = 15V$, $V_s = 12V$, $I_s = 4A$. Find the value of R that will achieve maximum power transfer and determine that value of the maximum power.

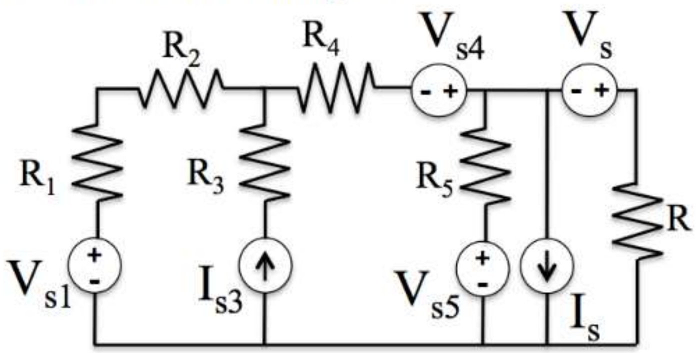
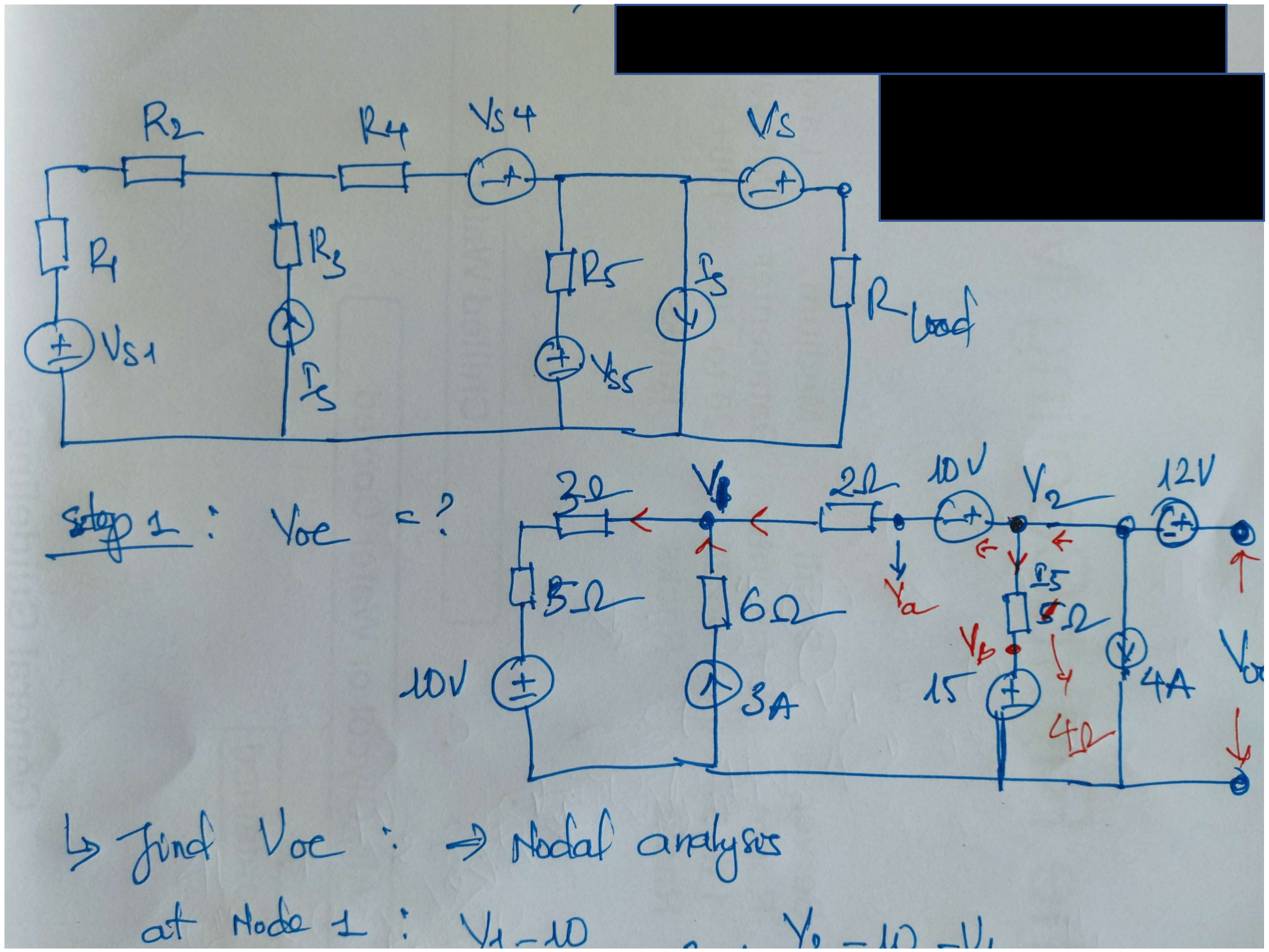
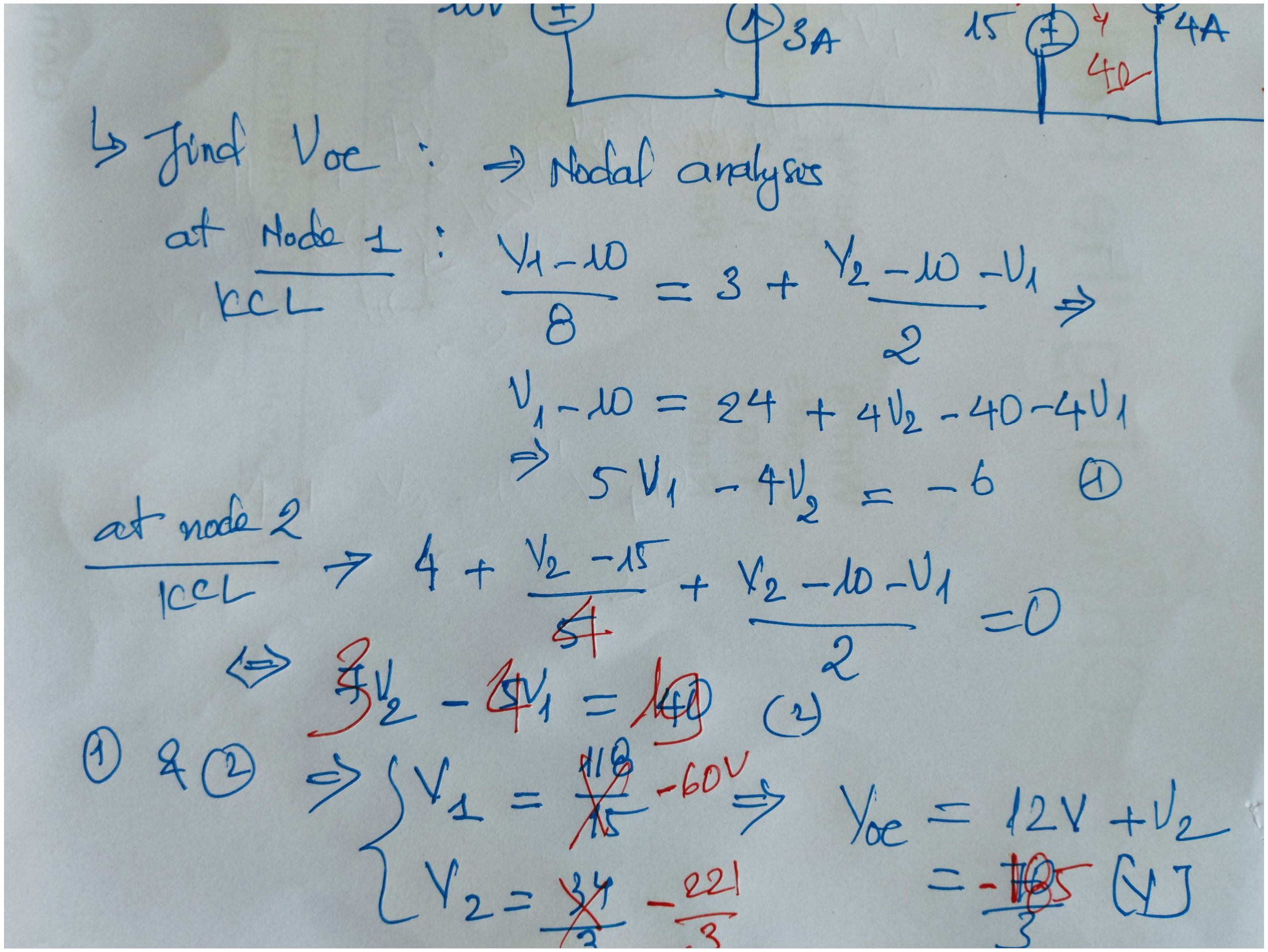


Figure 3

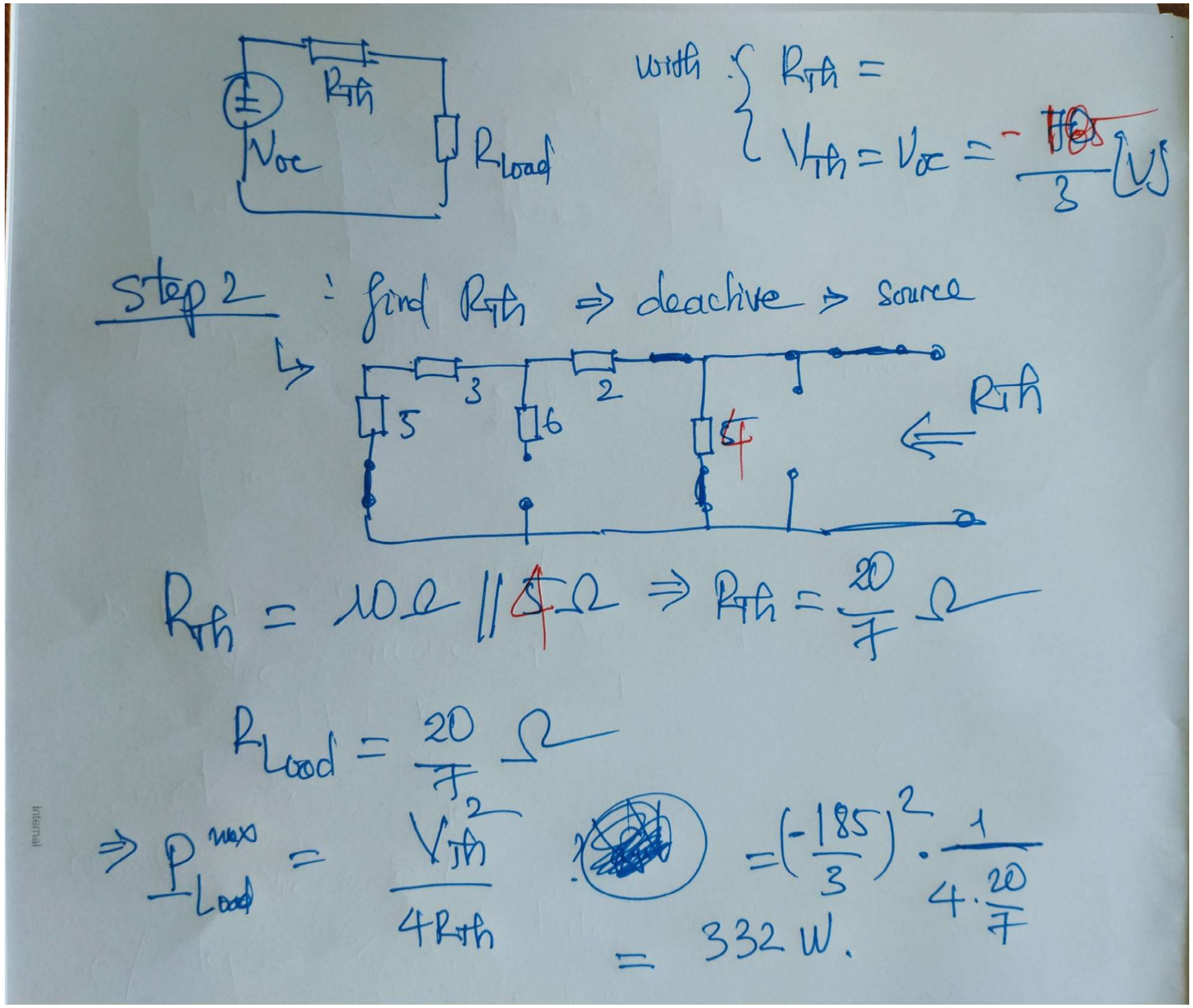
PROBLEM N°2


[redacted]
R2
R4
Vs4
Vs
R1
R3
R5
Is
Rload
Vs1
Is
Vs5
Step 1 : Voc = ?
3Ω
V1
2Ω
10V
V2
12V
5Ω
6Ω
Va
15Ω
4Ω
10V
3A
15
4A
Voc
↳ Find Voc : → Nodal analyses
at Node 1 :

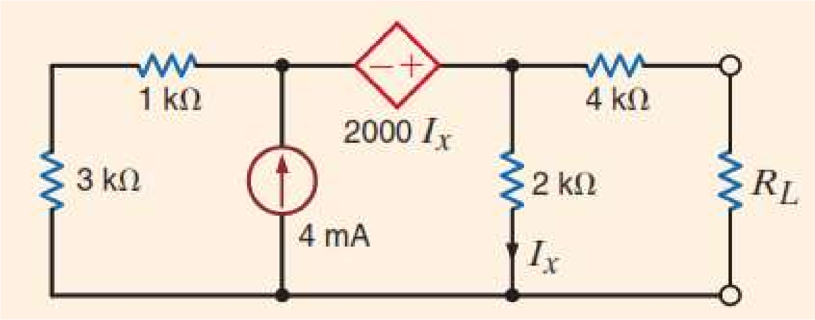
PROBLEM N°2

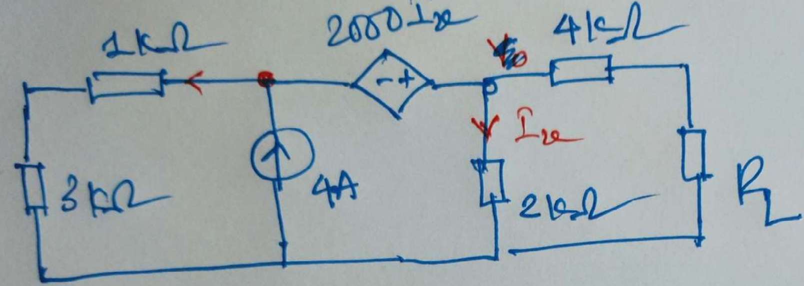

3A
15
4A
4Ω
↳ Find Voc : ⇒ Nodal analyses
at Node 1 : KCL
(V1 − 10)/8 = 3 + (V2 − 10 − V1)/2 ⇒
V1 − 10 = 24 + 4V2 − 40 − 4V1
⇒ 5V1 − 4V2 = −6 (1)
at node 2 KCL → 4 + (V2 − 15)/4 + (V2 − 10 − V1)/2 = 0
⇔ 3V2 − 4V1 = 40 (2)
(1) & (2) ⇒ V1 = −60V
V2 = −221/3
⇒ Voc = 12V + V2
= −185/3 [V]

PROBLEM N°2


RTh
Voc
RLoad
with { RTh =
VTh = Voc = -185/3 [V]
Step 2 : find RTh ⇒ deactive ⇒ source
3
2
5
6
4
RTh
RTh = 10Ω || 4Ω ⇒ RTh = 20/7 Ω
RLoad = 20/7 Ω
⇒ P max Load = VTh² / 4RTh = (-185/3)² · 1/(4 · 20/7)
= 332 W.

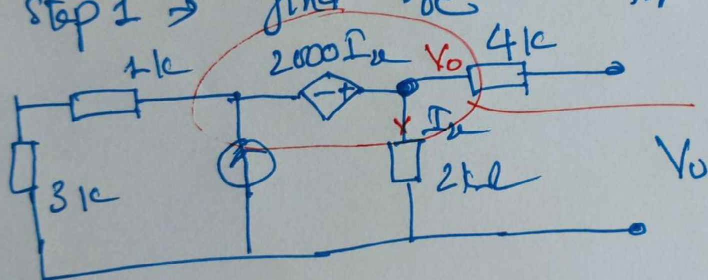
PROBLEM N°2 Find for maximum power transfer and the maximum power transferred to this load in the circuit in this figure





$R_L = ?$ for max power transfer

1/ step 1 → find $V_{oc} = V_{Th}$



↳ KCL for super-node:

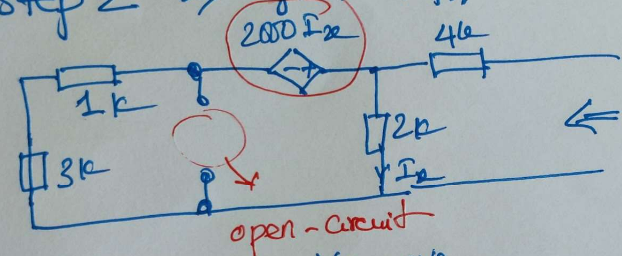
$$\frac{V_{oc} - 2000 I_x}{1k + 3k} + \frac{V_{oc}}{2k} = 4\,mA \quad (1)$$

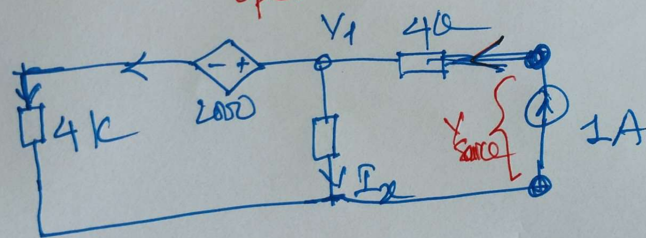
and $I_x = \dfrac{V_{oc}}{2k}$ (2)

and ⇒ $I_x = \frac{V_{oc}}{2k}$ (2)

(1) & (2) ⇒ $V_{oc} = 8\,V$

(2)/ step 2 → find R_{Th} :





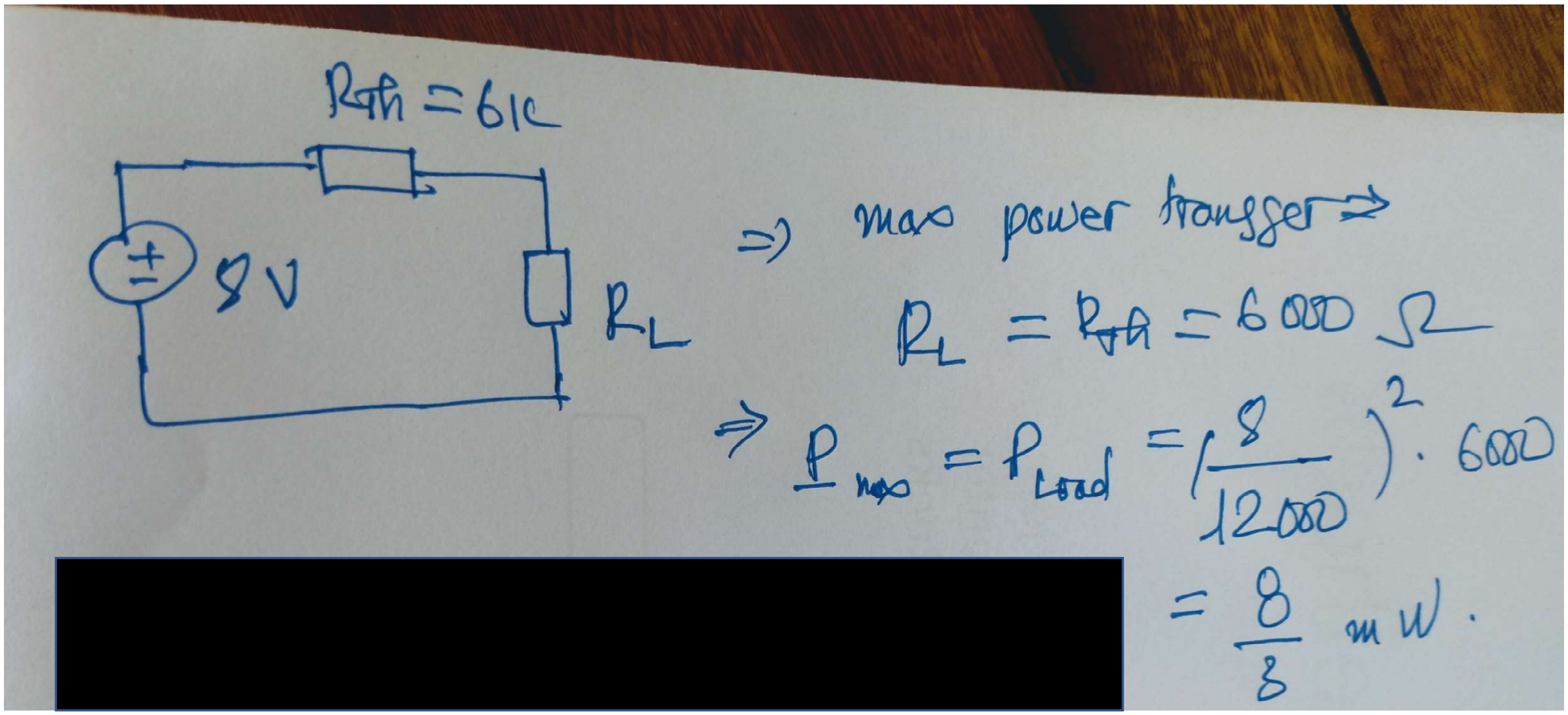
⇒ $R_{Th} = \frac{V_{source}}{1A}$

$$\begin{cases} I_x + \frac{V_1 - 2000 I_x}{4000} = 1A \\ I_x = \frac{V_1}{2000} \end{cases}$$

$$\Rightarrow \frac{V_1}{2000} + \frac{V_1 - 2000 \cdot \frac{V_1}{2000}}{4000} = 1$$

$\Rightarrow V_1 = 2000 \Rightarrow V_{source} = 6000$

$\Rightarrow R_{Th} = 6k\Omega$


Rth = 6k
8V
RL
⇒ max power transfer ⇒
RL = Rth = 6000 Ω
⇒ Pmax = PLoad = (8/12000)² · 6000
[redacted]
= 8/3 mW.

Matlab DC circuit analysis

When MATLAB is invoked, the command window will display the prompt >>

A matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

In MATLAB → may be entered as follows:

```
A = [1 2 3;  2 3 4; 3 4 5];
```

A matrix A can also be entered across three input lines as

```
A = [ 1 2 3
      2 3 4
      3  4 5];
```

If $Z * I = V$ and Z is non-singular, the left division, $Z\backslash V$ is equivalent to MATLAB expression

$$I = inv(Z) * V$$

where **inv** is the MATLAB function for obtaining the inverse of a matrix. The right division denoted by V/Z is equivalent to the MATLAB expression

$$I = V * inv(Z)$$

Matlab DC circuit analysis

APPLYING : NODAL ANALYSIS

If we assume that the admittance between nodes i and j: **(Y = 1/R)** Tổng dẫn !!! >< Tổng trở

Y_{ij} ,

The nodal equations:

$$Y_{11} V_1 + Y_{12} V_2 + \dots + Y_{1m} V_m = \sum I_1$$

$$Y_{21} V_1 + Y_{22} V_2 + \dots + Y_{2m} V_m = \sum I_2$$

$$Y_{m1} V_1 + Y_{m2} V_2 + \dots + Y_{mm} V_m = \sum I_m$$

where

$m = n - 1$

V_1, V_2 and V_m are voltages from nodes 1, 2 and so on ..., n with respect to the reference node.

$\sum I_x$ is the algebraic sum of current sources at node x.

can be expressed in matrix form as

$$[Y][V] = [I] \Rightarrow [V] = [Y]^{-1}[I]$$

$[Y]^{-1}$ is an inverse of $[Y]$.

In MATLAB, we can compute [V] by using the command

$$V = inv(Y) * I$$

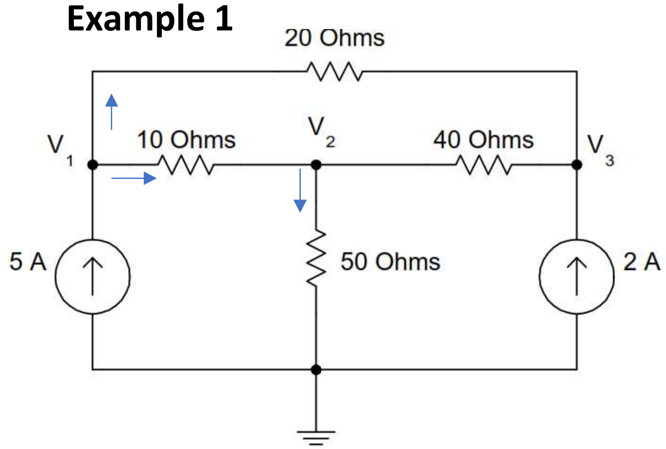
MODULE 3: TUTORIAL – 2 HRS

Matlab DC circuit analysis

APPLYING : NODAL ANALYSIS

In MATLAB, we can compute [V] by using the command $V = inv(Y) * I$

Example 1



Circuit with Nodal Voltages

Solution

- Using KCL and assuming that the currents leaving a node are positive, we have

For node 1,

$$\frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{20} - 5 = 0$$

i.e.,

$$0.15V_1 - 0.1V_2 - 0.05V_3 = 5$$

At node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{50} + \frac{V_2 - V_3}{40} = 0$$

i.e.,

$$-0.1V_1 + 0.145V_2 - 0.025V_3 = 0$$

At node 3,

$$\frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{40} - 2 = 0$$

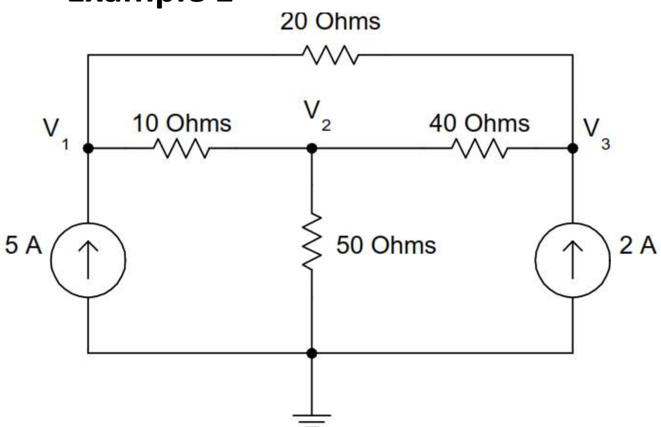
i.e.,

$$-0.05V_1 - 0.025V_2 + 0.075V_3 = 2$$

Matlab DC circuit analysis

APPLYING : NODAL ANALYSIS

Example 1



Circuit with Nodal Voltages

➢ In matrix form, we have

For node 1, $0.15V_1 - 0.1V_2 - 0.05V_3 = 5$

At node 2 $-0.1V_1 + 0.145V_2 - 0.025V_3 = 0$

At node 3 $-0.05V_1 - 0.025V_2 + 0.075V_3 = 2$

$$\begin{bmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.145 & -0.025 \\ -0.05 & -0.025 & 0.075 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

The MATLAB program for solving the nodal voltages is ?

MATLAB Script

```
diary ex4_1.dat
% program computes the nodal voltages

% given the admittance matrix Y and current vector I
% Y is the admittance matrix and I is the current vector
% initialize matrix y and vector I using YV=I form

   Y = [ 0.15    -0.1     -0.05;
      -0.1      0.145   -0.025;
      -0.05    -0.025    0.075];

   I = [5;
       0;
       2];
   % solve for the voltage
   fprintf('Nodal voltages V1, V2 and V3 are \n')
   v = inv(Y)*I
   diary
```

The final results obtained from MATLAB are

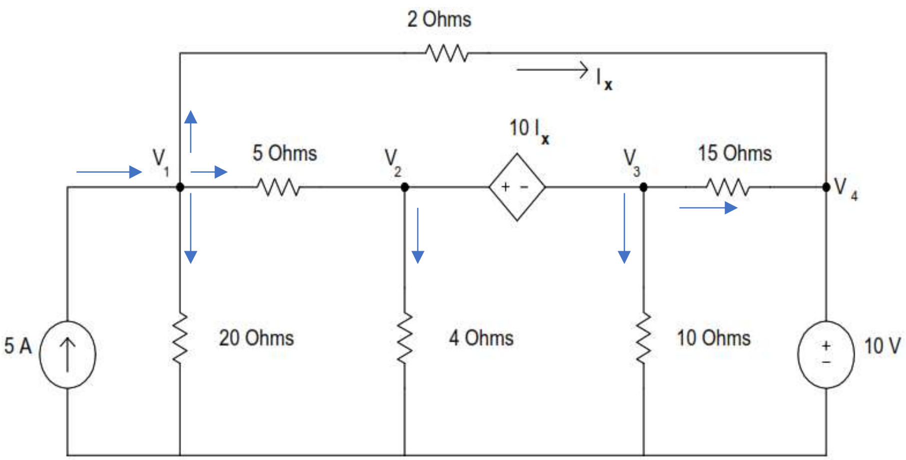
```
Nodal voltages V1, V2 and V3,

v =
    404.2857
    350.0000
    412.8571
```

MODULE 3: TUTORIAL – 2 HRS

Matlab DC circuit analysis

APPLYING : NODAL ANALYSIS

Example 2



Circuit with Dependent and Independent Sources

Solution

- Using KCL and the convention that currents leaving a node is positive, we have

At node 1

$$\frac{V_1}{20}+\frac{V_1-V_2}{5}+\frac{V_1-V_4}{2}-5=0$$

Simplifying, we get

$$0.75V_1-0.2V_2-0.5V_4=5$$

From supernodes 2 and 3, we have

$$\frac{V_3}{10}+\frac{V_2-V_1}{5}+\frac{V_2}{4}+\frac{V_3-V_4}{15}=0$$

Simplifying, we get

$$-0.2V_1+0.45V_2+0.1667V_3-0.06667V_4=0$$

At node 2,

$$V_2-V_3=10I_X$$

But

$$I_X=\frac{(V_1-V_4)}{2}$$

Thus

$$V_2-V_3=\frac{10(V_1-V_4)}{2}$$

At node 4, we have

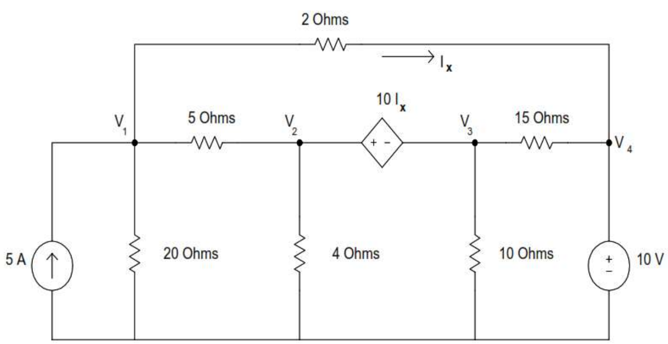
$$V_4=10$$

MODULE 3: TUTORIAL – 2 HRS

Matlab DC circuit analysis

APPLYING : NODAL ANALYSIS

Example 2



Circuit with Dependent and Independent Sources

- In matrix form, we have

$$\begin{bmatrix} 0.75 & -0.2 & 0 & -0.5 \\ -5 & 1 & -1 & 5 \\ -0.2 & 0.45 & 0.1667 & -0.06667 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

MATLAB Script

```
diary ex4_2.dat
% this program computes the nodal voltages
% given the admittance matrix Y and current vector I
% Y is the admittance matrix
% I is the current vector
% initialize the matrix y and vector I using YV=I

   Y = [0.75   -0.2   0   -0.5;
        -5     1  -1     5;
        -0.2   0.45  0.166666667   -0.0666666667;
         0      0   0      1];

 % current vector is entered as a transpose of row vector
 I = [5   0   0   10]';

% solve for nodal voltage
fprintf('Nodal voltages V1,V2,V3,V4 are \n')
V = inv(Y)*I
diary
```

The final results obtained from MATLAB are

```
V =
     18.1107
     17.9153
    -22.6384
     10.0000
```

Matlab DC circuit analysis

APPLYING : LOOP ANALYSIS

$$Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + \quad \dots \quad Z_{1n}I_n = \sum V_1$$

$$Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 + \quad \dots \quad Z_{2n}I_n = \sum V_2$$

$$Z_{n1}I_1 + Z_{n2}I_2 + Z_{n3}I_3 + \quad \dots \quad Z_{nn}I_n = \sum V_n$$

where

$I_1, I_2, \dots I_n$ are the unknown currents for meshes 1 through n.

$Z_{11}, Z_{22}, \dots, Z_{nn}$ are the impedance for each mesh through which individual current flows.

Z_{ij}, $j \# i$ denote mutual impedance.

$\sum V_x$ is the algebraic sum of the voltage sources in mesh x.

can be expressed in matrix form as $[Z][I] = [V]$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & \dots & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33} & \dots & Z_{3n} \\ .. & .. & . & \dots & .. \\ Z_{n1} & Z_{n2} & Z_{n3} & \dots & Z_{nn} \end{bmatrix} \quad I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ . \\ I_n \end{bmatrix} \quad V = \begin{bmatrix} \sum V_1 \\ \sum V_2 \\ \sum V_3 \\ .. \\ \sum V_n \end{bmatrix}$$

$$[I] = [Z]^{-1}[V]$$

In MATLAB, we can compute [*I*] by using the command

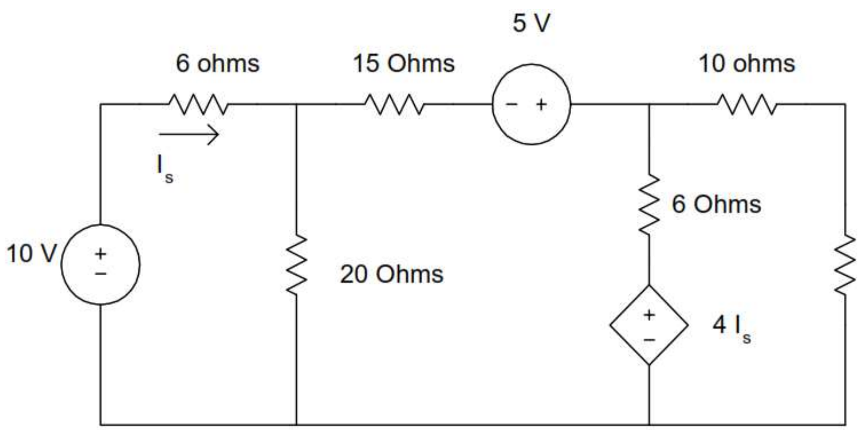
$$I = inv(Z) * V$$

Matlab DC circuit analysis

APPLYING : LOOP ANALYSIS

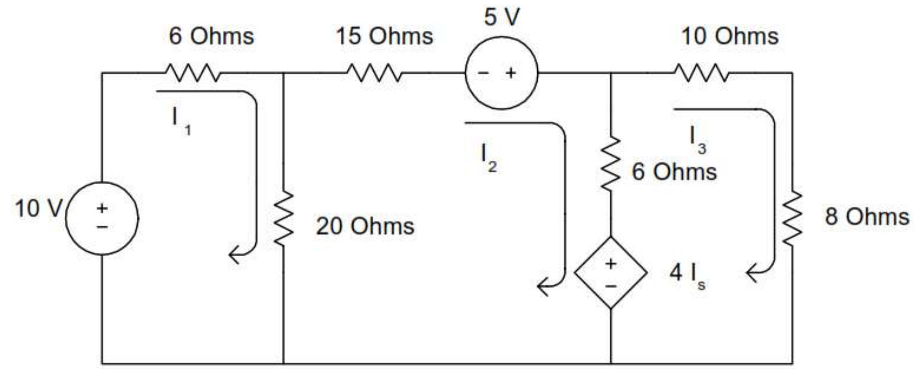
Example 1

Find the power dissipated by the 8 Ohm resistor and the current supplied by the 10-volt source



Solution

Using loop analysis and denoting the loop currents as I_1, I_2, I_3



By inspection,

$$I_S = I_1$$

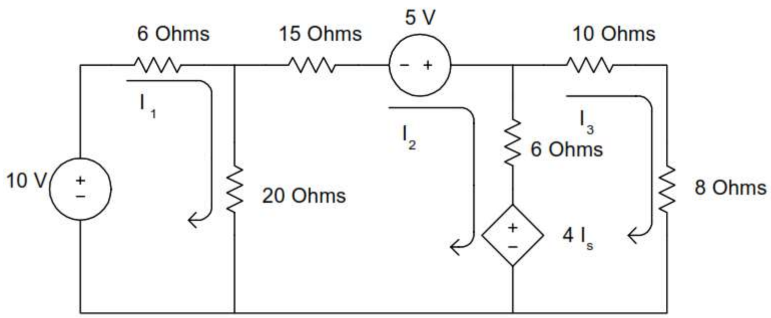
For loop 1,

$$-10 + 6I_1 + 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 10$$

Matlab DC circuit analysis

APPLYING : LOOP ANALYSIS



For loop 2,

$$15I_2 - 5 + 6(I_2 - I_3) + 4I_S + 20(I_2 - I_1) = 0$$

$$I_S = I_1 \Rightarrow -16I_1 + 41I_2 - 6_3 I = 5$$

For loop 3,

$$10I_3 + 8I_3 - 4I_S + 6(I_3 - I_2) = 0$$

$$I_S = I_1 \Rightarrow -4I_1 - 6I_2 + 24I_3 = 0$$

MATLAB Script

$$\begin{bmatrix} 26 & -20 & 0 \\ -16 & 41 & -6 \\ -4 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

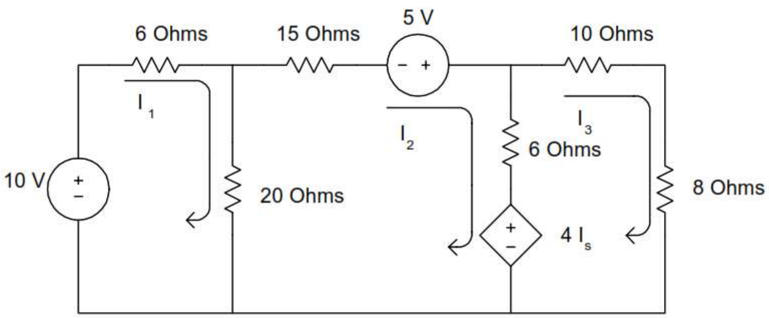
The power dissipated by the 8 Ohm resistor is

$$P = RI_3^2 = 8I_3^2$$

The current supplied by the source is $I_S = I_1$

Matlab DC circuit analysis

APPLYING : LOOP ANALYSIS



$$\begin{bmatrix} 26 & -20 & 0 \\ -16 & 41 & -6 \\ -4 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

The power dissipated by the 8 Ohm resistor is

$$P = RI_3^2 = 8I_3^2$$

The current supplied by the source is $I_S = I_1$

MATLAB Script

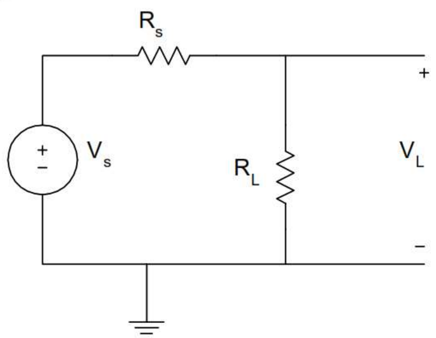
```
diary ex4_4.dat
% This program determines the power dissipated by
% 8 ohm resistor and current supplied by the
% 10V source
%
% the program computes the loop currents, given
% the impedance matrix Z and voltage vector V
%
% Z is the impedance matrix
% V is the voltage vector
% initialize the matrix Z and vector V of equation
% ZI=V

Z = [26   -20    0;
    -16    40   -6;
     -4    -6   24];
V = [10   5   0]';

% solve for loop currents
I = inv(Z)*V;
% the power dissipation in 8 ohm resistor is P
P = 8*I(3)^2;
% print out the results
fprintf('Power dissipated in 8 ohm resistor is %8.2f Watts\n',P)
fprintf('Current in 10V source is %8.2f Amps\n',I(1))
diary
```

```
Power dissipated in 8 ohm resistor is     0.42  Watts
Current in 10V source is     0.72 Amps
```

MODULE 3: TUTORIAL – 2 HRS

Matlab DC circuit analysis

MAXIMUM POWER TRANSFER



The voltage across the Load R_L is given as

$$V_L = \frac{V_s R_L}{R_s + R_L}$$

The power dissipated by the load R_L is given

$$P_L = \frac{V_L^2}{R_L} = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$

$$\frac{dP_L}{dR_L} = \frac{(R_s + R_L)^2 V_S - V_s^2 R_L (2)(R_s + R_L)}{(R_s + R_L)^4}$$

$$\frac{dP_L}{dR_L} = 0$$

Simplifying the above we get

$$(R_s + R_L) - 2R_L = 0$$

i.e.,

$$R_L = R_S$$

Let us discuss : MATLAB Diff and Find Functions

MODULE 3: TUTORIAL – 2 HRS

Matlab DC circuit analysis

MAXIMUM POWER TRANSFER

Let us discuss : MATLAB Diff and Find Functions

Numerical differentiation can be obtained using the backward difference expression

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

or by the forward difference expression

$$f'(x_n) = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$$

The derivative of *f(x)* can be obtained by using the MATLAB "diff" function as

$$f'(x) \cong diff(f)./diff(x).$$

If f is a row or column vector

$$f = [f(1) \quad f(2) \quad \ldots \quad f(n)]$$

then **diff(f)** returns a vector of difference between adjacent elements

$$diff(f) = [f(2) - f(1) \quad f(3) - f(2) \quad \ldots \quad f(n) - f(n-1)]$$

The output vector $diff(f)$ will be one element less than the input vector f.

The **find** function determines the indices of the nonzero elements of a vector or matrix. The statement

B = find(f)

will return the indices of the vector f that are nonzero.

For example, to obtain the points where a change in sign occurs, the statement

Pt_change = find(product < 0)

will show the indices of the locations in *product* that are negative.

MODULE 3: TUTORIAL – 2 HRS

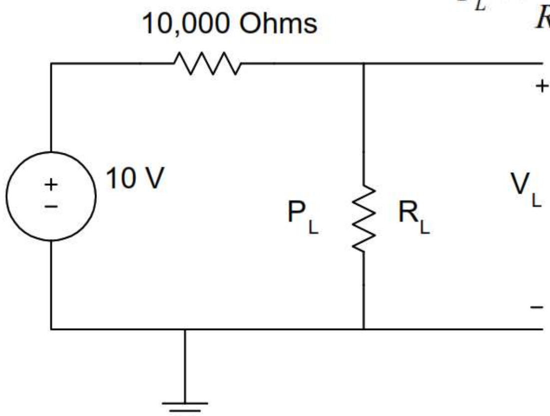
Matlab DC circuit analysis

MAXIMUM POWER TRANSFER

Let us discuss : MATLAB Diff and Find Functions

Assuming: R_L varies from 0 to 50KΩ, plot the power dissipated by the load. Verify that the maximum power dissipation by the load occurs when R_L is 10 KΩ.

$$P_L = \frac{V_L^2}{R_L} = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$



MATLAB Script

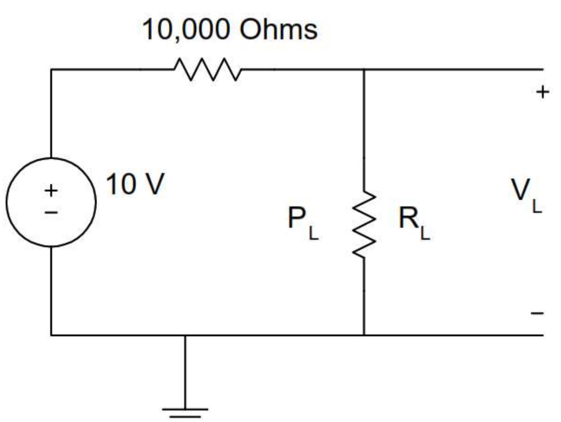
```
% maximum power transfer
% vs is the supply voltage
% rs is the supply resistance
% rl is the load resistance
% vl is the voltage across the load
% pl is the power dissipated by the load
vs = 10;    rs = 10e3;
rl = 0:1e3:50e3;
k = length(rl); % components in vector rl
% Power dissipation calculation
for i=1:k
 pl(i) = ((vs/(rs+rl(i)))^2)*rl(i);
end
% Derivative of power is calculated using backward difference
dp = diff(pl)./diff(rl);
rld = rl(2:length(rl)); % length of rld is 1 less than that of rl
% Determination of critical points of derivative of power
prod = dp(1:length(dp) - 1).*dp(2:length(dp));
crit_pt = rld(find(prod < 0));
max_power = max(pl); % maximum power is calculated
% print out results
```

Matlab DC circuit analysis

MAXIMUM POWER TRANSFER

Let us discuss : MATLAB Diff and Find Functions

Assuming: R_L varies from 0 to 50KΩ, plot the power dissipated by the load. Verify that the maximum power dissipation by the load occurs when R_L is 10 KΩ.

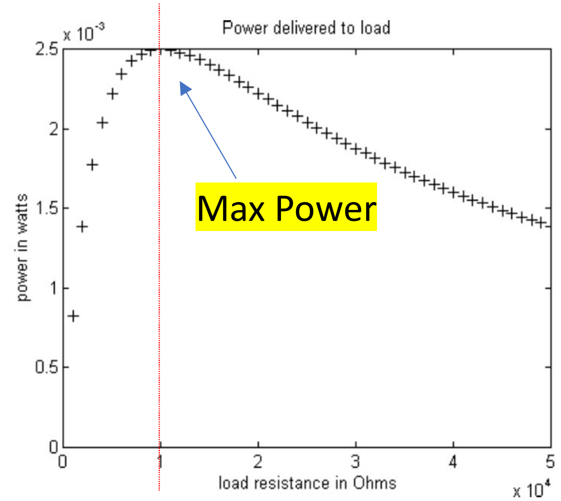


MATLAB Script

```
fprintf('Maximum power occurs at %8.2f Ohms\n',crit_pt)
fprintf('Maximum power dissipation is %8.4f Watts\n', max_power)
% Plot power versus load
plot(rl,pl,'+')
title('Power delivered to load')
xlabel('load resistance in Ohms')
ylabel('power in watts')
```

The results obtained from MATLAB are

Maximum power occurs at 10000.00 Ohms
Maximum power dissipation is 0.0025 Watts



Matlab Simulink DC circuit analysis

MID-TERM EXAM – 1.5 HRS

	Percentage	Type
Attendance/Attitude	10%	
Practical	20%	Group report & presentation
Mid-term	20%	Written exam: theory & application
Final exam	50%	Written exam: theory & application

- Question 1: theory
- Question 2 → Question 4: application
- Open-book !!

MODULE 4: OPERATIONAL AMPLIFIER– 6 HRS

MODULE 4: OPERATIONAL AMPLIFIER– 6 HRS

MODULE 4: OPERATIONAL AMPLIFIER– 6 HRS

MODULE 5: FIRST AND SECOND ORDER TRANSIENT CIRCUITS – 9 HRS

MODULE 5: FIRST AND SECOND ORDER TRANSIENT CIRCUITS – 9 HRS

FINAL EXAM – 2 HRS

	Percentage	Type
Attendance/Attitude	10%	
Practical	20%	Group report & presentation
Mid-term	20%	Written exam: theory & application
Final exam	50%	Written exam: theory & application

- Question 1: theory
- Question 2 → Question 4: application
- Open-book !!