Optics and Photonics

Lecture 02: Dielectric Waveguides and Optical Fibers

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Dielectric Waveguides and Optical Fibers

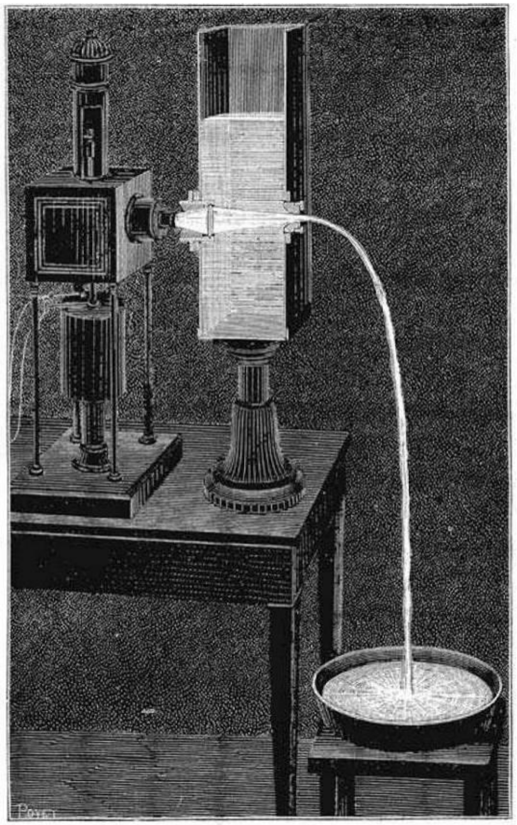
"The introduction of optical fiber systems will revolutionize the communications network. The low-transmission loss and the large bandwidth capability of the fiber systems allow signals to be transmitted for establishing communications contacts over large distances with few or no provisions of intermediate amplification."

[Charles K. Kao (one of the pioneers of glass fibers for optical communications) *Optical Fiber Systems: Technology, Design, and Applications* (McGraw-Hill Book Company, New York, USA, 1982), p. 1]

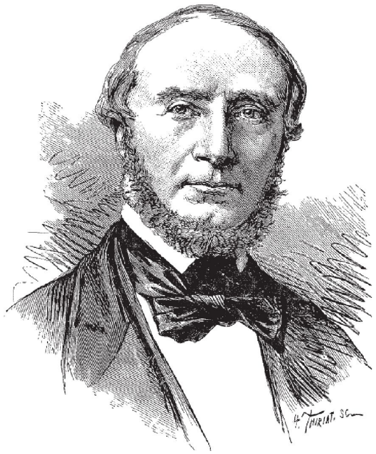
Courtesy of the Chinese University of Hong Kong

Charles Kao at the naming ceremony of Minor Planet (3463) "Kaokuen" by Nanjing's Purple Mountain Observatory in July 1996. Charles Kao and his colleagues carried out the early experiments on optical fibers at the Standard Telecommunications Laboratories Ltd (the research center of Standard Telephones and Cables) at Harlow in the United Kingdom, during the 1960s. He shared the Nobel Prize in 2009 in Physics with Willard Boyle and George Smith for "groundbreaking achievements concerning the transmission of light in fibers for optical communication." In a milestone paper with George Hockam published in the IEE Proceedings in 1966 they predicted that the intrinsic losses of glass optical fibers could be much lower than 20 dB/km, which would allow their use in long distance telecommunications. Today, optical fibers are used not only in telecommunications but also in various other technologies such as instrumentation and sensing. From 1987 to his retirement in 1996, professor Kao was the Vice Chancellor of the Chinese University of Hong Kong.(Courtesy of the Chinese University of Hong Kong.)

Jean-Daniel Colladon and the Light Guiding in a Water Jet



1841



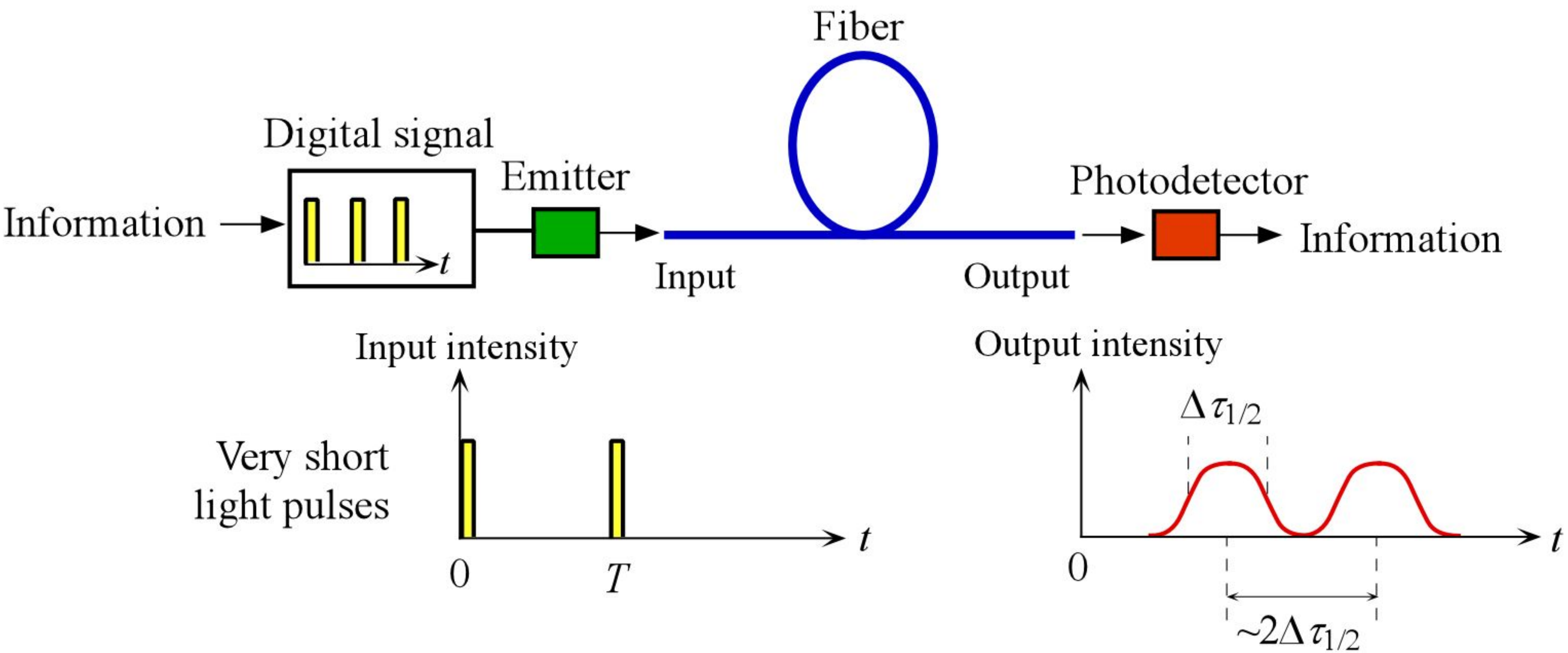
Light is guided along a water jet as demonstrated by Jean-Daniel Colladon. This illustration was published in *La Nature*, *Revue des Sciences*, in 1884 (p. 325). His first demonstration was around 1841. (*Comptes Rendes*, 15, 800-802, Oct. 24, 1842). A similar demonstration was done by John Tyndall for the Royal Institution in London in his 1854 lecture. Apparently, Michael Faraday had originally suggested the experiment to John Tyndall though Faraday himself probably learned about it either from another earlier demonstration or through Jean-Daniel Colladon's publication. Although John Tyndall is often credited with the original discovery of a water-jet guiding light, Tyndall, himself, does not make that claim but neither does he attribute it to someone else. (The fountain, courtesy of Conservatoire Numérique des Arts et Métiers, France; Colladon's portrait, courtesy of Musée d'histoire des sciences, Genève, Switzerland.)

Reference: Jeff Hecht, "Illuminating the Origin of Light Guiding," *Optics & Photonics News*, **10**, 26, 1999 and his wonderful book *The City of Light* (Oxford University Press, 2004) describe the evolution of the optical fiber from the water jet experiments of Colladon and Tyndall to modern fibers with historical facts and references.

Narinder Singh Kapany

Narinder Singh Kapany was born in Punjab in India, studied at the Agra University and then obtained his PhD from the Imperial College of Science and Technology, University of London in 1955. He held a number of key-positions in both academia and industry, including a Regents Professor at the University of California, Berkeley, the University of California, Santa Cruz (UCSC), the Director of the Center for Innovation and Entrepreneurial Development at UCSC. He made significant contributions to optical glass fibers starting in 1950s, and essentially coined the term *fiber optics* in the 1960s. His book *Fibre Optics: Principles and Applications*, published in 1967, was the first in optical fibers. (*Courtesy of Dr. Narinder S. Kapany*)

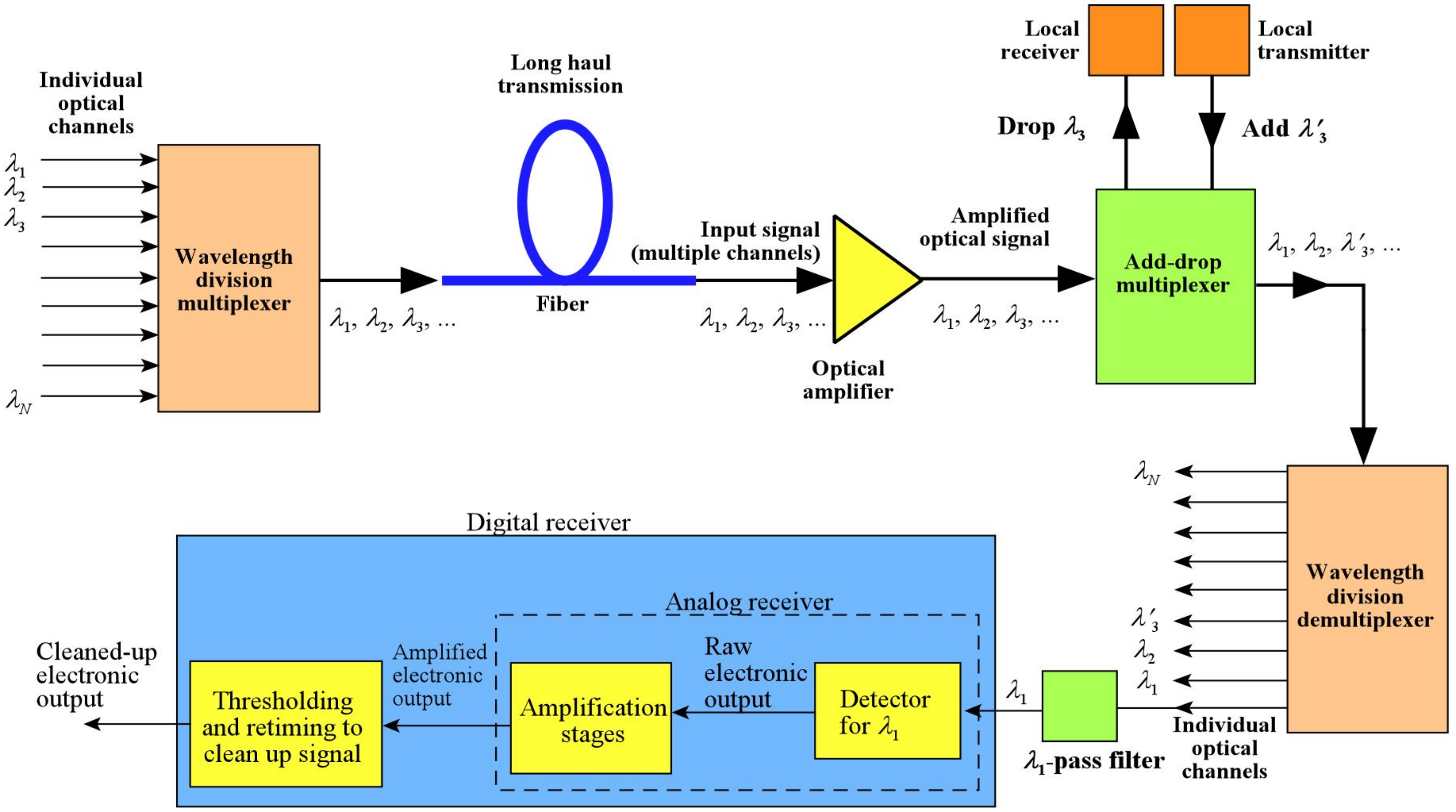
A Century and Half Later



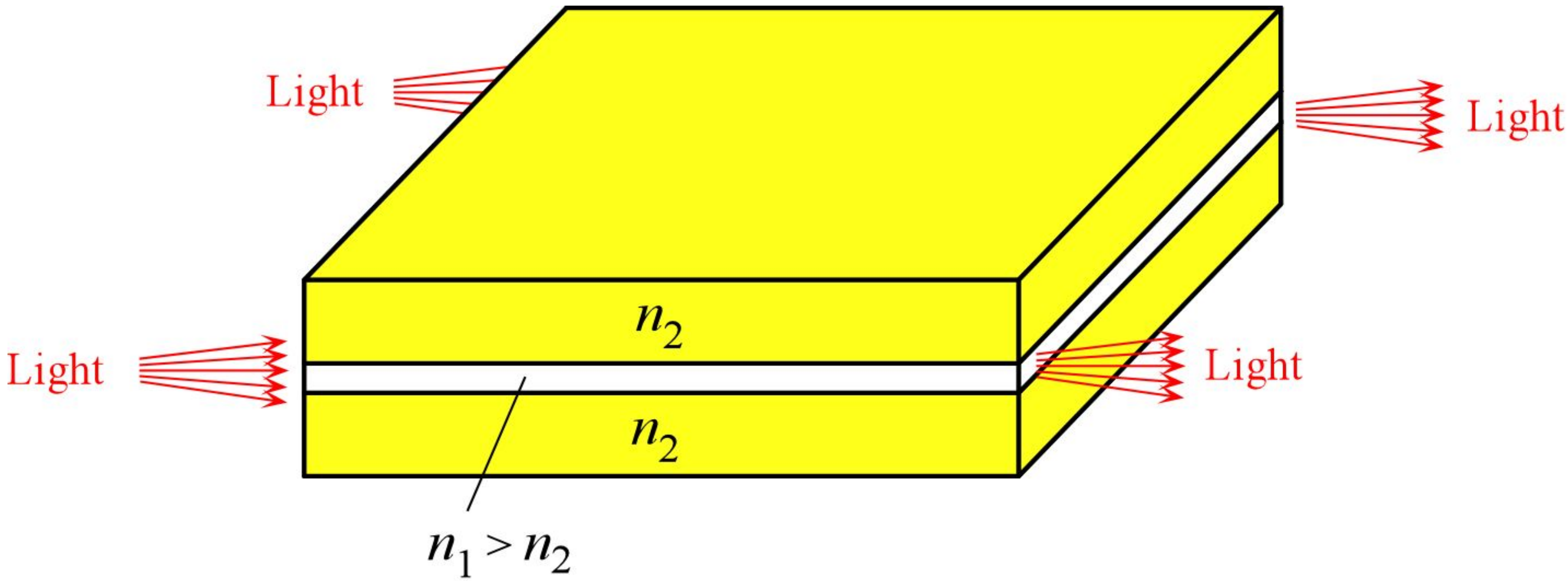
Light has replaced copper in communications. Photons have replaced electrons.

Will "Photonics Engineering" replace Electronics Engineering?

WAVELENGTH DIVISION MULTIPLEXING: WDM

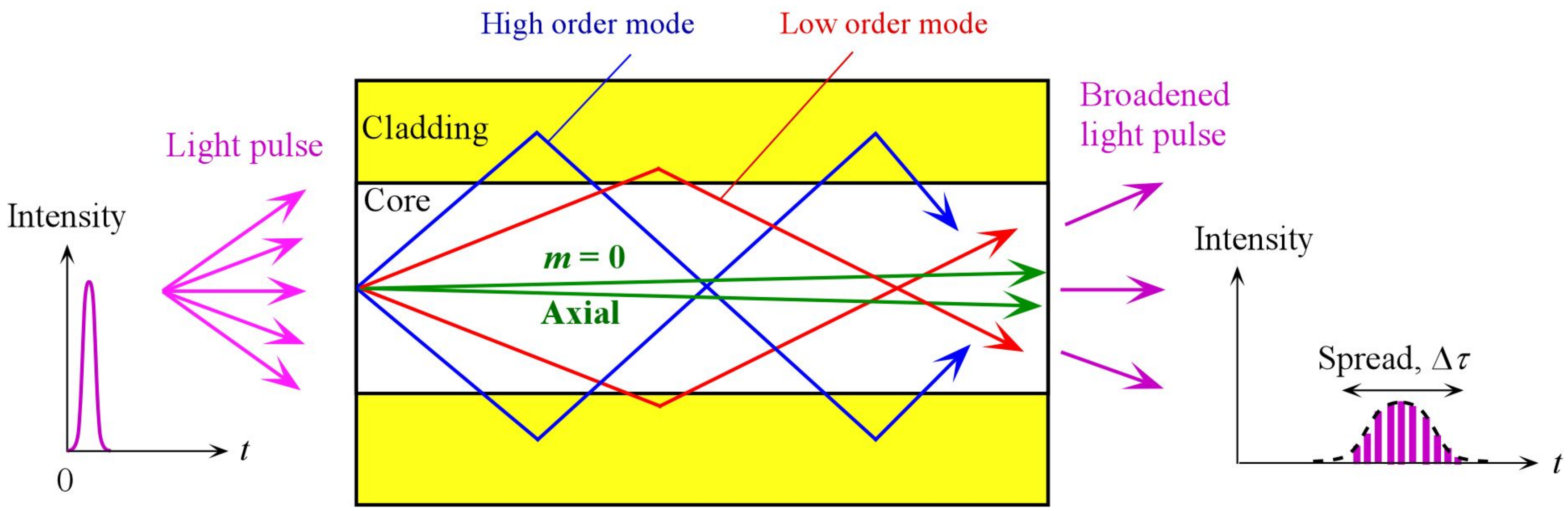


Planar Optical Waveguide



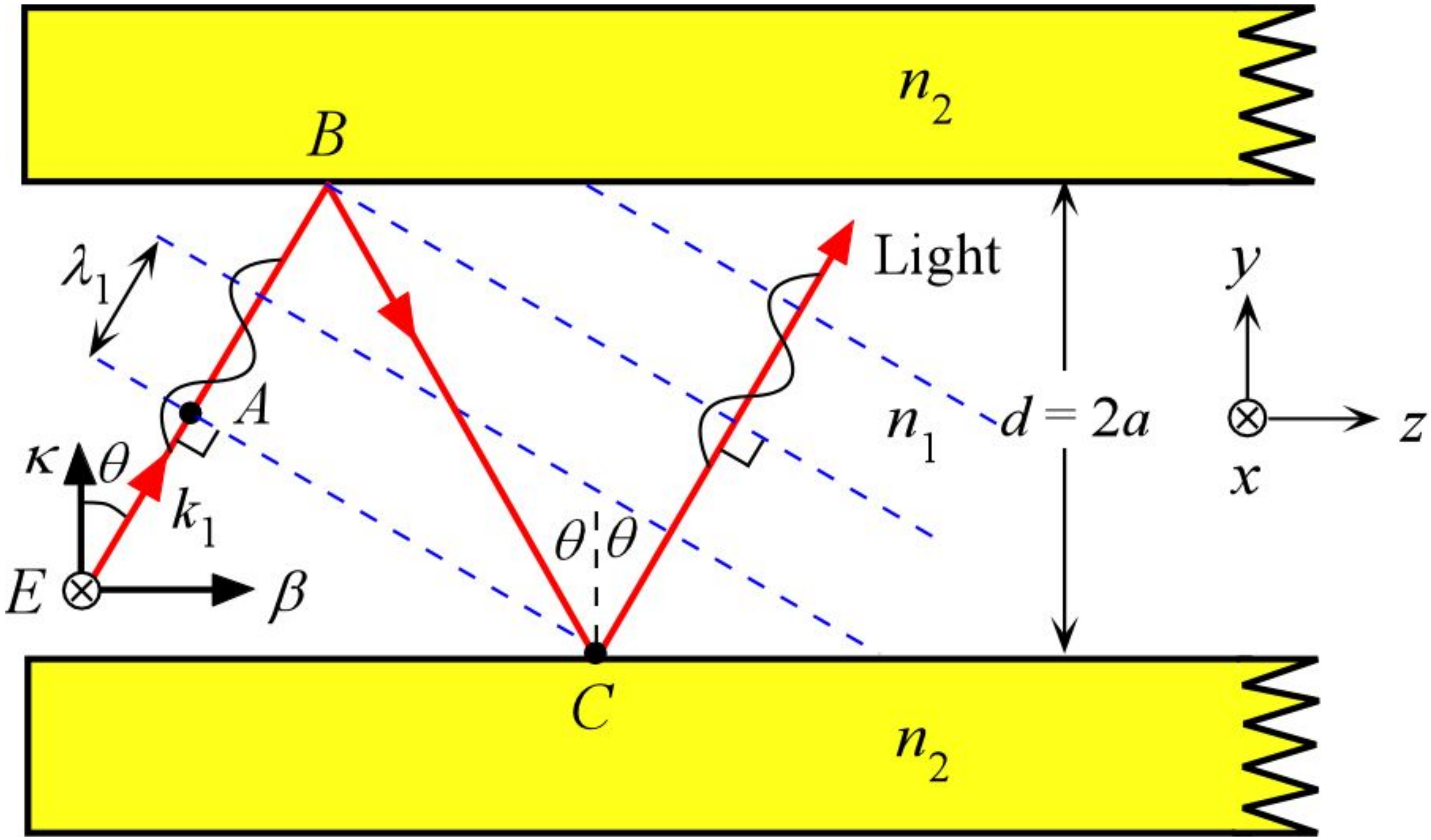
A planar dielectric waveguide has a central rectangular region of higher refractive index n_1 than the surrounding region which has a refractive index n_2. It is assumed that the waveguide is infinitely wide and the central region is of thickness $2a$. It is illuminated at one end by a nearly monochromatic light source.

Optical Waveguide



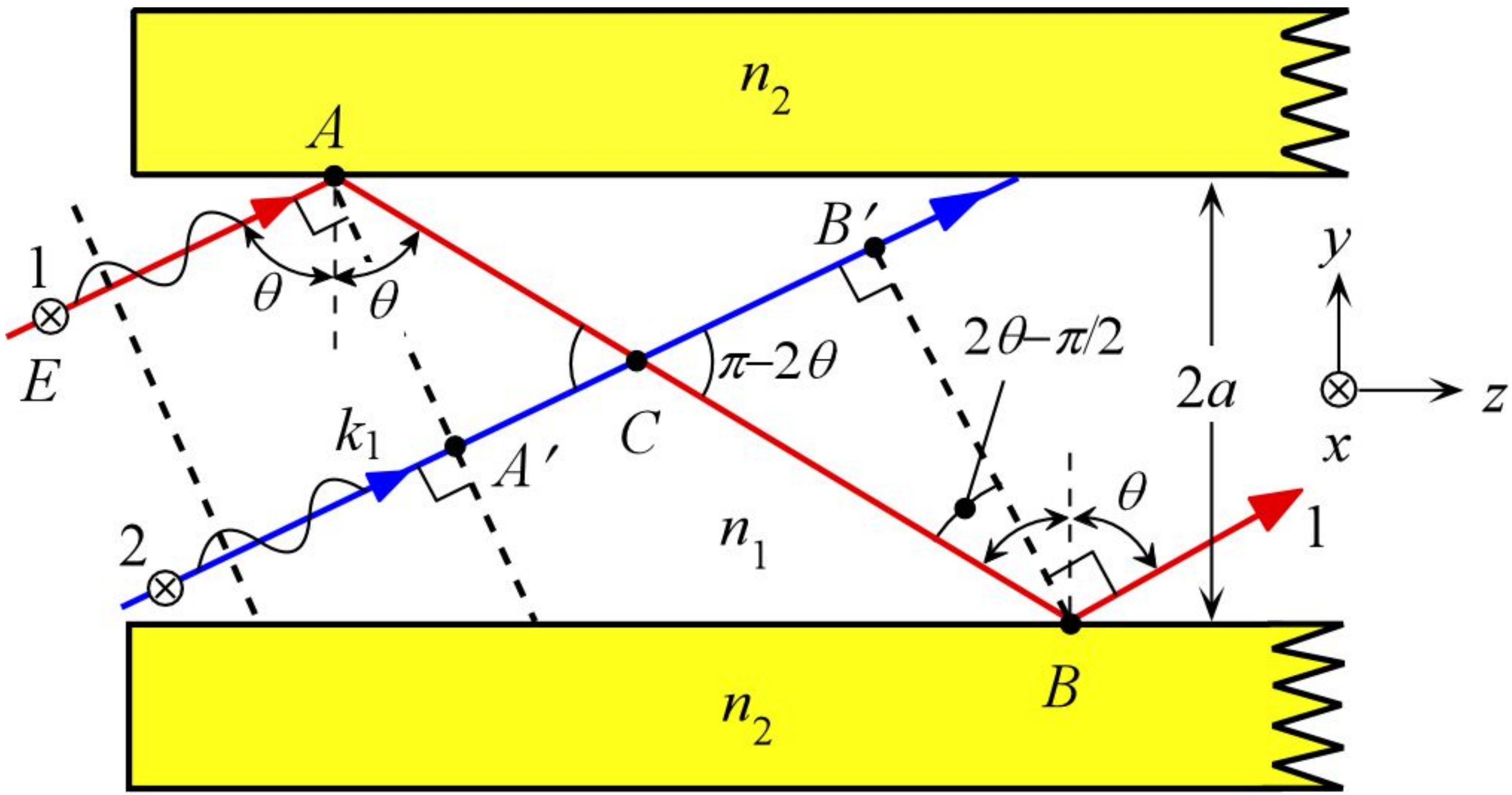
Light waves zigzag along the guide. Is that really what happens?

Waves Inside the Core



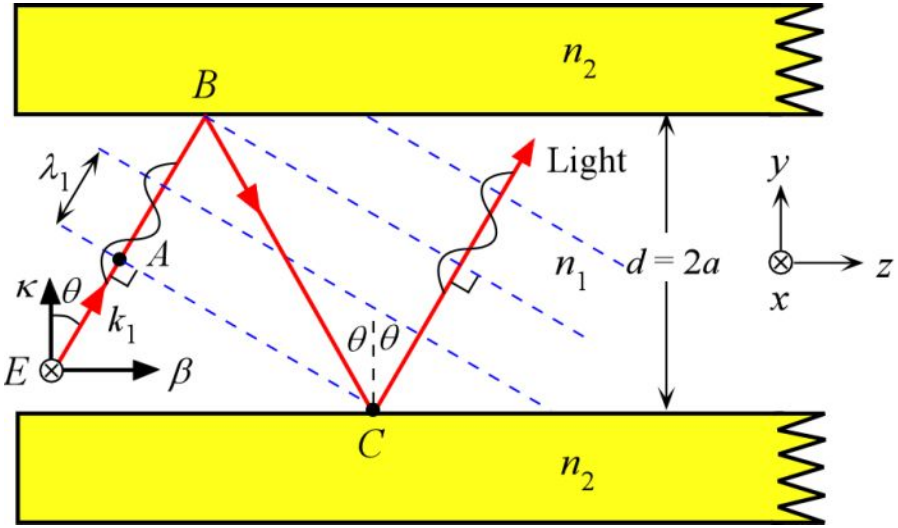
A light ray traveling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave. *E* is parallel to *x*. (λ_1 and k_1 are the wavelength and the propagation constant inside the core medium n_1 *i.e.* $\lambda_1 = \lambda/n_1$.)

Waves Inside the Core



Two arbitrary waves 1 and 2 that are initially in phase must remain in phase after reflections. Otherwise the two will interfere destructively and cancel each other.

Waveguide Condition and Modes



$$k_1 = kn_1 = 2\pi n_1/\lambda,$$

$$\Delta\varphi(AC) = k_1(AB + BC) - 2\varphi = m(2\pi)$$

$$BC = d/\cos\theta \text{ and } AB = BC\cos(2\theta)$$

$$AB + BC = BC\cos(2\theta) + BC = BC[(2\cos^2\theta - 1) + 1] = 2d\cos\theta$$

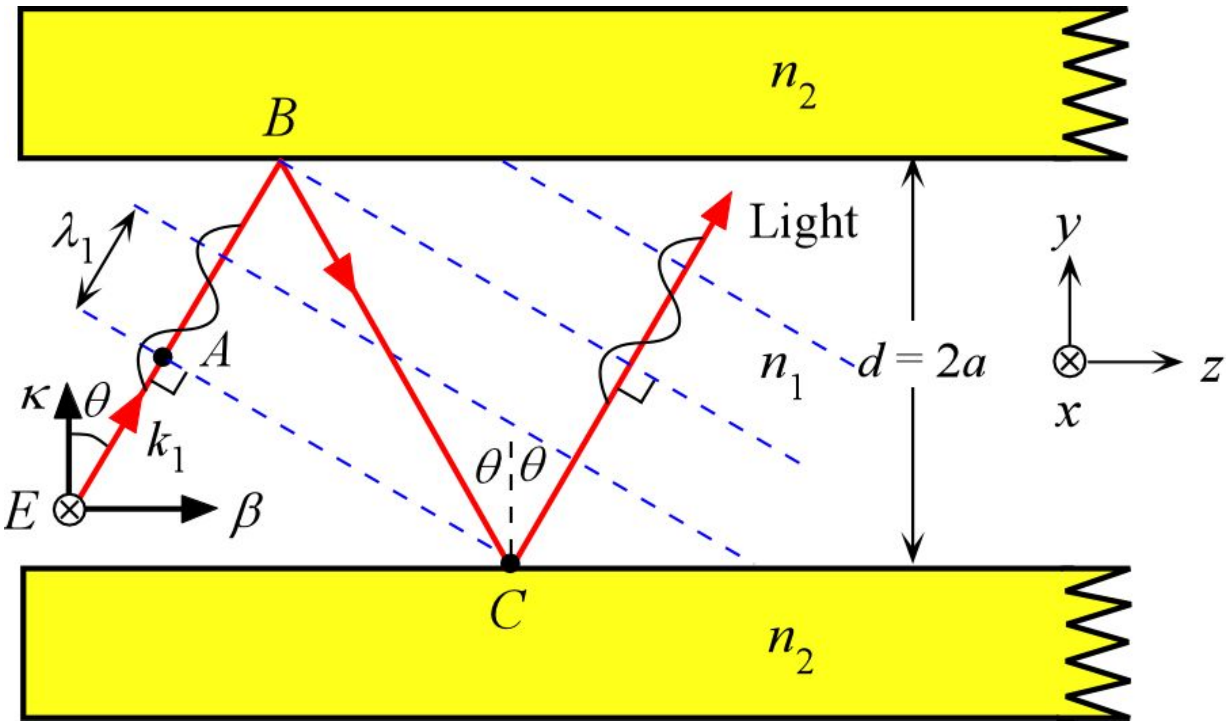
$$\mathbf{k_1[2d\cos\theta] - 2\varphi = m(2\pi)}$$

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m* = 0, 1, 2, 3 *etc
Integer

"Mode number"

Waveguide condition



$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

Propagation constant along the guide

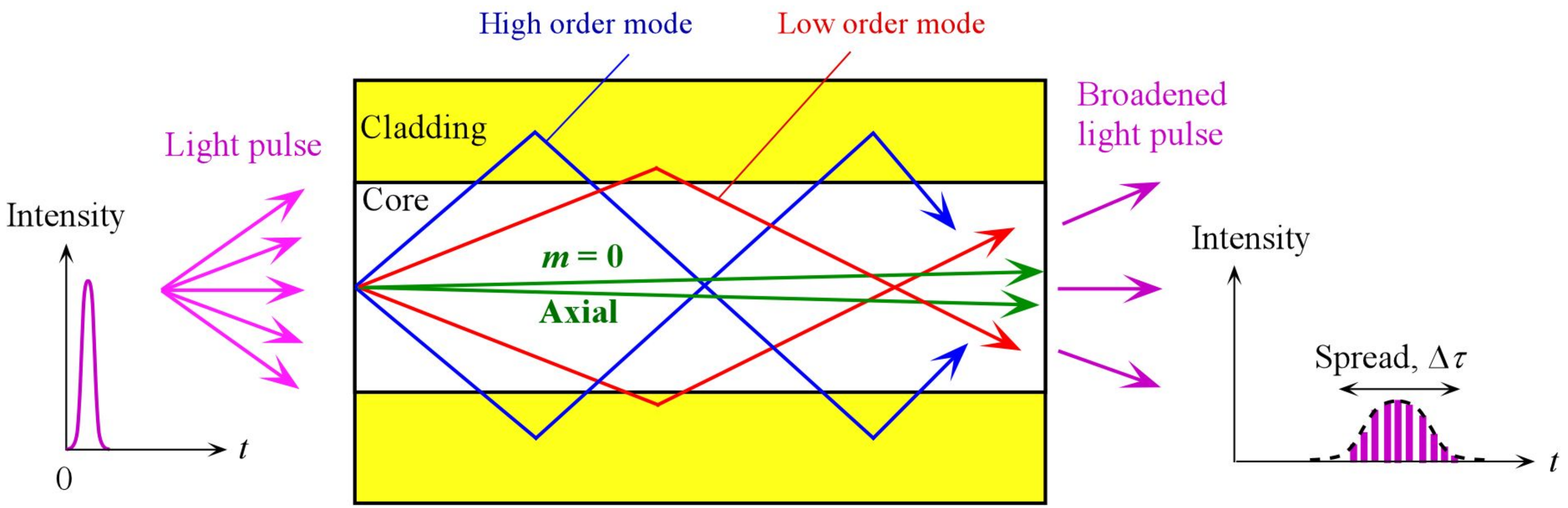
$$\kappa_m = k_1 \cos\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\cos\theta_m$$

Transverse Propagation constant

Waveguide Condition and Waveguide Modes

To get a propagating wave along a guide you must have constructive interference. All these rays interfere with each other. Only certain angles are allowed . Each allowed angle represents a **mode** of propagation.

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$



Waveguide Condition

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m **= integer,** n_1 **= core refractive index,** θ_m **is the incidence angle, 2*a* is the core thickness.**

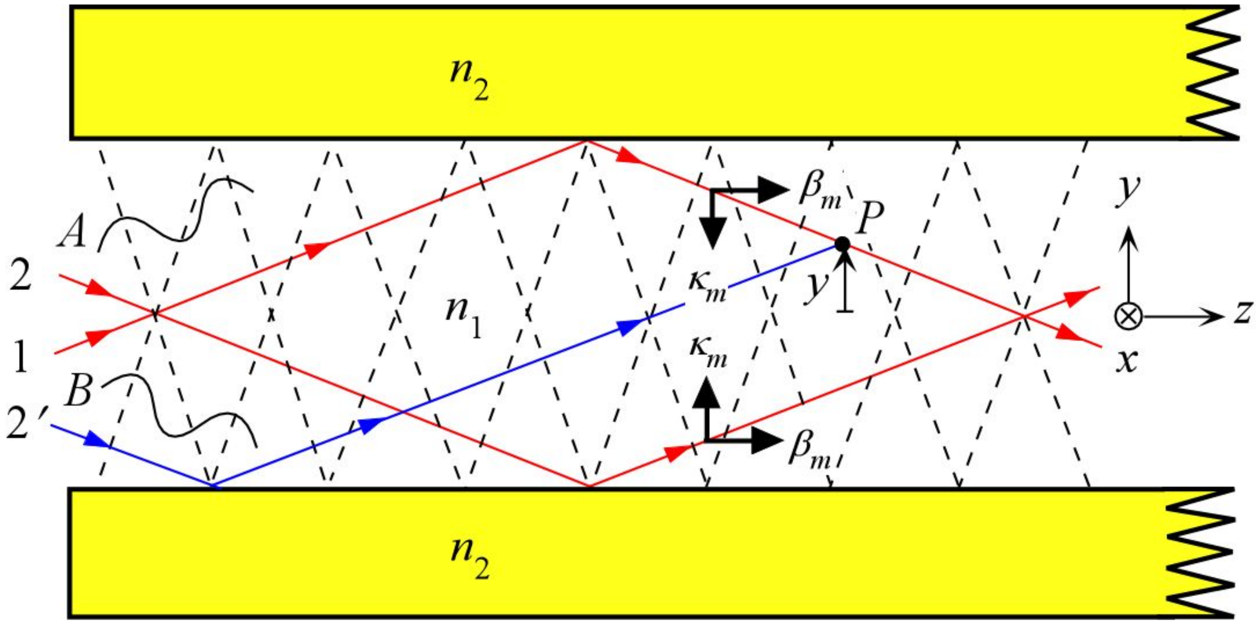
Minimum θ_m **and maximum *m* must still satisfy TIR.**

There are only a finite number of modes.

Propagation along the guide for a mode *m* is

$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

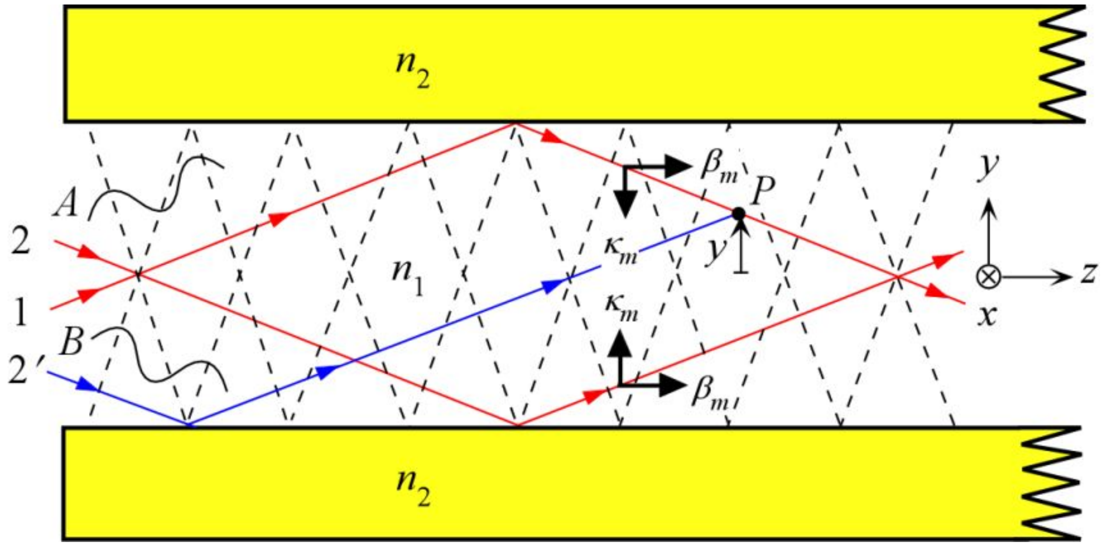
Waveguide Condition and Modes



To get a propagating wave along a guide you must have constructive interference. All these rays interfere with each other. Only certain angles are allowed . Each allowed angle represents a **mode** of propagation.

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Modes in a Planar Waveguide



We can identify upward (*A*) and downward (*B*) traveling waves in the guide which interfere to set up a standing wave along *y* and a wave that is propagating along *z*. Rays 2 and 2′ belong to the same wave front but 2′ becomes reflected before 2. The interference of 1 and 2′ determines the field at a height *y* from the guide center. The field *E(y, z, t)* at *P* can be written as

$$E(y,z,t) = E_m(y)\cos(\omega t - \beta_m z)$$

Traveling wave along *z*

Field pattern along *y*

Modes in a Planar Waveguide: Summary

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m = integer, n_1 = core refractive index, θ_m is the incidence angle, $2a$ is the core thickness.

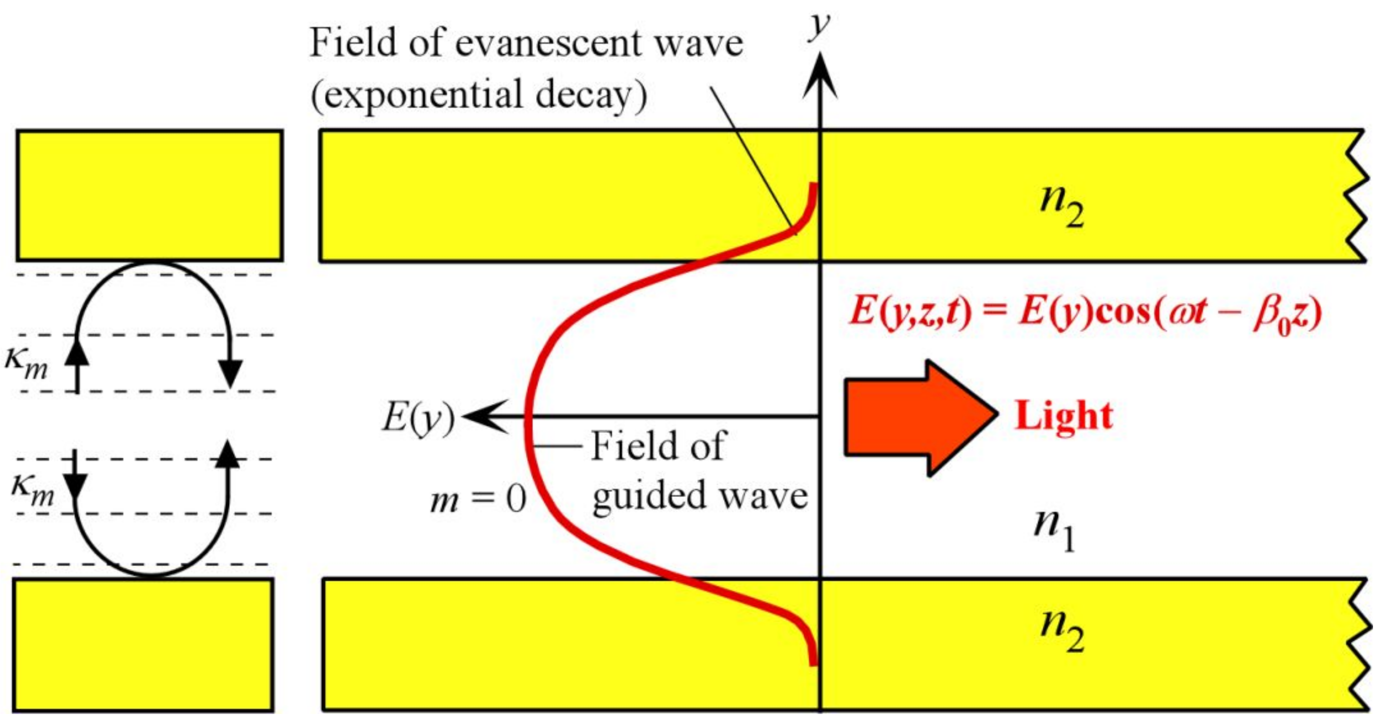
$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

$$E(y,z,t) = E_m(y)\cos(\omega t - \beta_m z)$$

Traveling wave along z

Field pattern along y

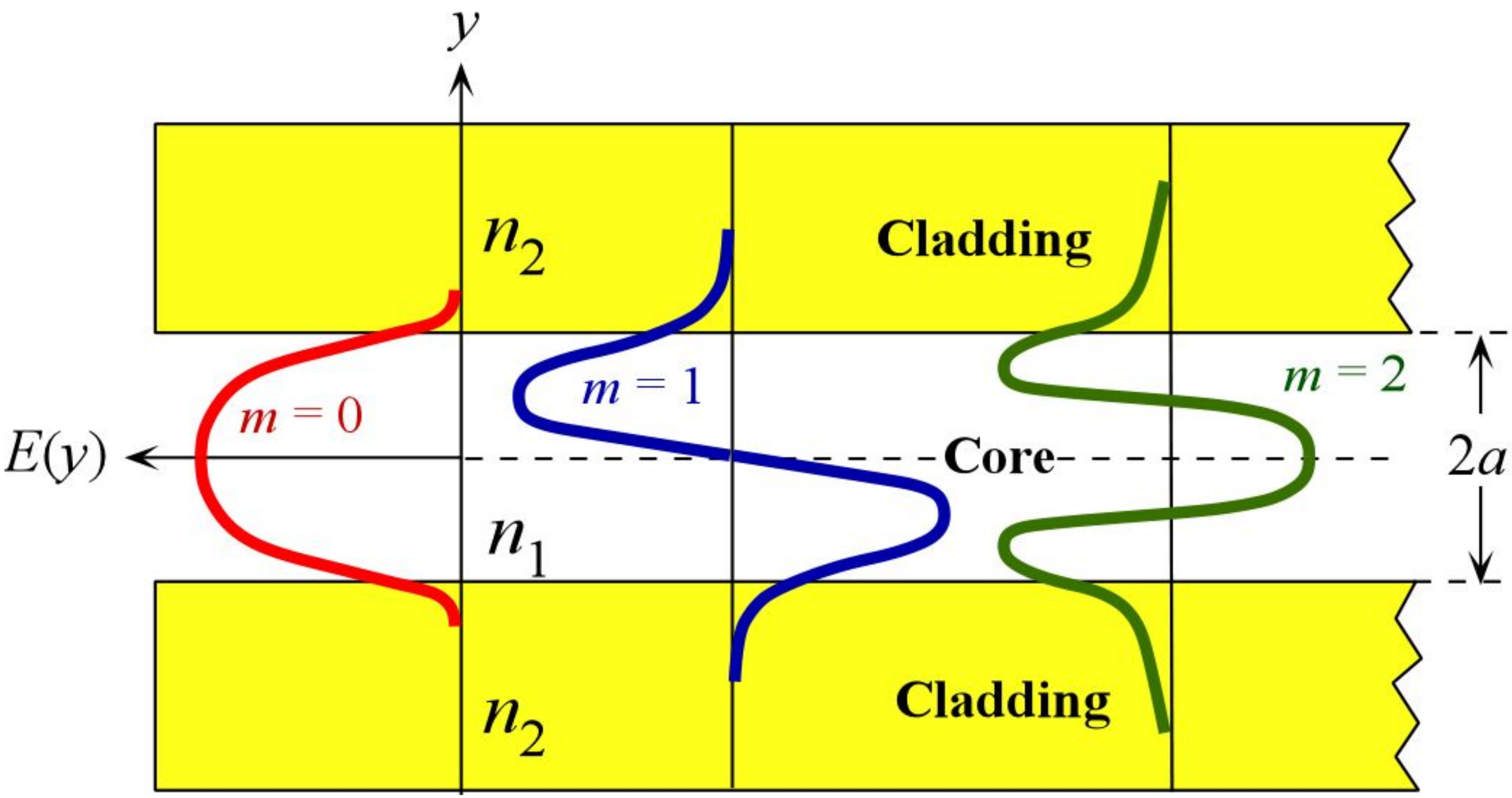
Mode Field Pattern



Left: The upward and downward traveling waves have equal but opposite wave vectors κ_m and interfere to set up a standing electric field pattern across the guide.

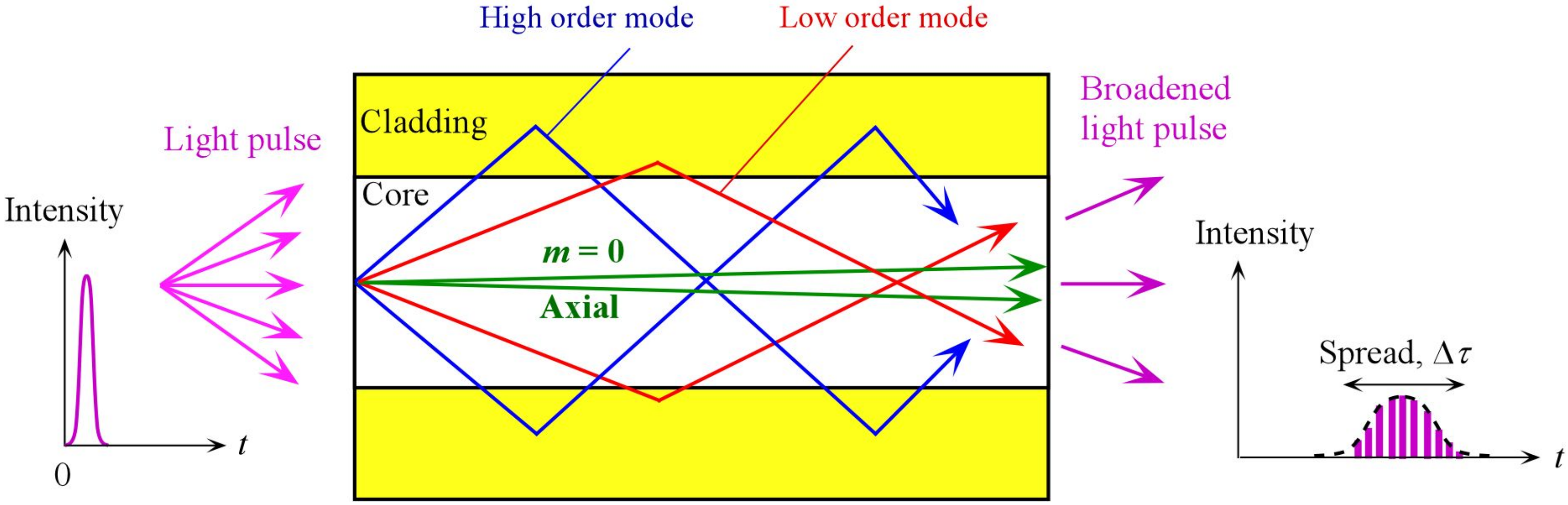
Right: The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest θ. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide

Modes in a Planar Waveguide



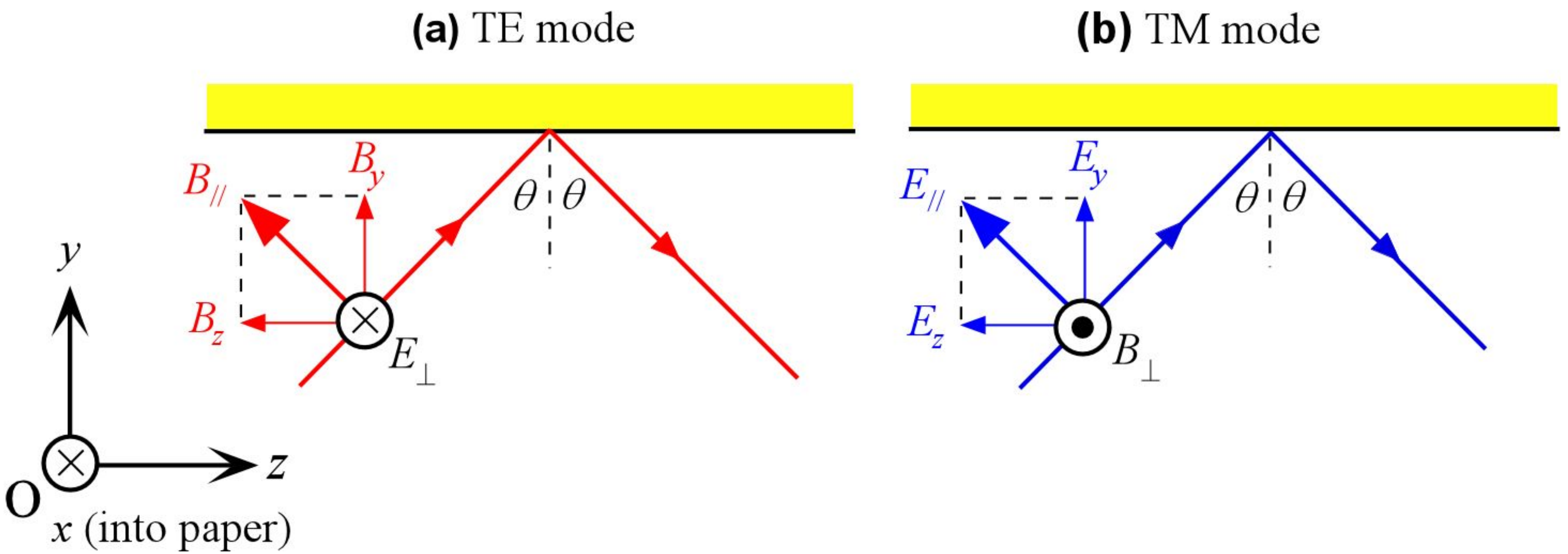
The electric field patterns of the first three modes ($m = 0, 1, 2$) traveling wave along the guide. Notice different extents of field penetration into the cladding

Intermode (Intermodal or Modal) Dispersion



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

TE and TM Modes



$E_\perp$ is along x, so that $E_\perp = E_x$

$B_\perp$ is along $-x$, so that $B_\perp = -B_x$

Possible modes can be classified in terms of (a) transverse electric field (TE) and (b) transverse magnetic field (TM). Plane of incidence is the paper.

V-Number

All waveguides are characterized by a parameter called the ***V*-number** or **normalized frequency**

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

$V < \pi/2$, $m = 0$ is the only possibility and only the fundamental mode ($m = 0$) propagates along the dielectric slab waveguide: a **single mode** planar waveguide.

$\lambda = \lambda_c$ for $V = \pi/2$ is the **cut-off wavelength**, and above this wavelength, only one-mode, the fundamental mode will propagate.

Example on Waveguide Modes

Consider a planar dielectric guide with a core thickness 20 μm, n_1 = 1.455, n_2 = 1.440, light wavelength of 900 nm. Find the modes

TIR phase change φ_m for TE mode

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

TE mode

Waveguide condition

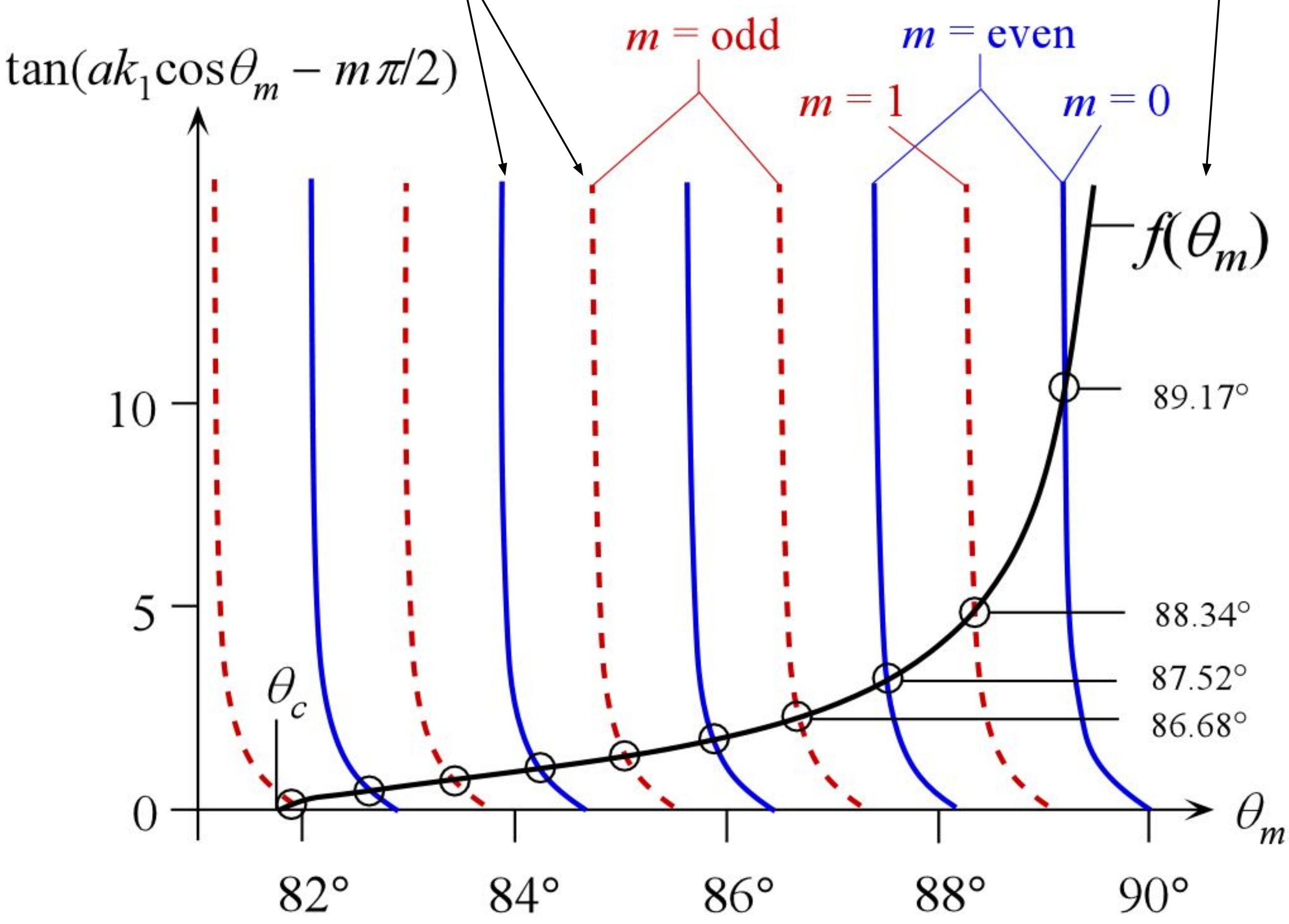
$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Waveguide condition

$$\phi_m = 2ak_1\cos\theta_m - m\pi$$

$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f(\theta_m)$$

TE mode



$$\frac{1}{\delta_m} = \alpha_m = \frac{2\pi n_2 \left[\left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_m - 1 \right]^{1/2}}{\lambda}$$

Mode m, incidence angle θ_m and penetration δ_m for a planar dielectric waveguide with $d = 2a = 20\ \mu m$, $n_1 = 1.455$, $n_2 = 1.440$ ($\lambda = 900$ nm)

m	0	1	2	3	4	5	6	7	8	9
θ_m	89.2°	88.3°	87.5°	86.7°	85.9°	85.0°	84.2°	83.4°	82.6°	81.9°
δ_m (μm)	0.691	0.702	0.722	0.751	0.793	0.866	0.970	1.15	1.57	3.83

$m = 0$
Fundamental mode

Highest mode

Critical angle $\theta_c = \arcsin(n_2/n_1) = 81.77°$

Number of Modes *M*

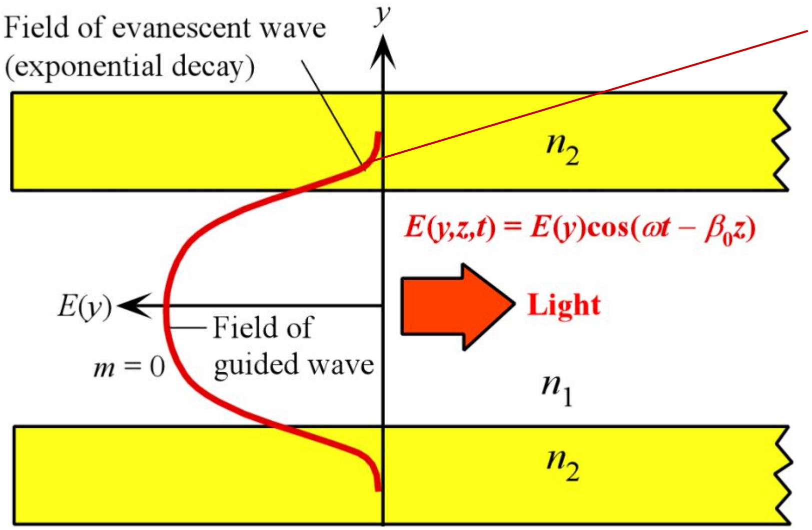
Waveguide condition $\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$

One mode when $V < \pi/2$

Multimode when $V > \pi/2$

$$M = \text{Int}\left(\frac{2V}{\pi}\right) + 1$$

Mode Field Width $2w_o$



$$E_{\text{cladding}}(y') = E_{\text{cladding}}(0)\exp(-\alpha_{\text{cladding}}y')$$

$$\alpha_{\text{cladding}} = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

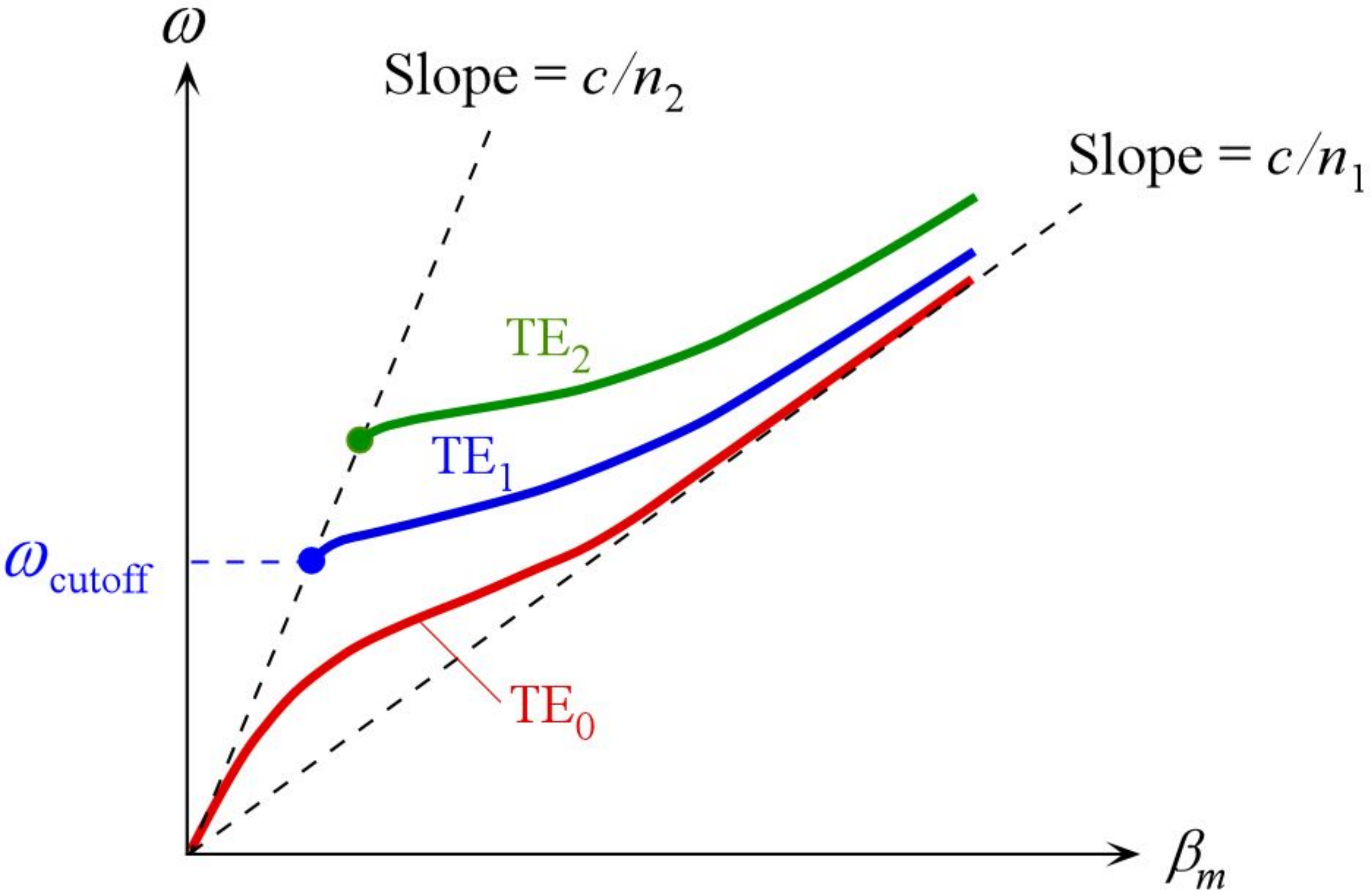
$$\approx \frac{2\pi}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{V}{a}$$

$$\therefore \quad 2w_o \approx 2a\frac{(V+1)}{V}$$

Mode Field Width $2w_o$

Note: The MFW definition here is semiquantitive. A more rigorous approach needs to consider the optical power in the mode and how much of this penetrates the cladding. See optical fibers section.

Waveguide Dispersion Curve

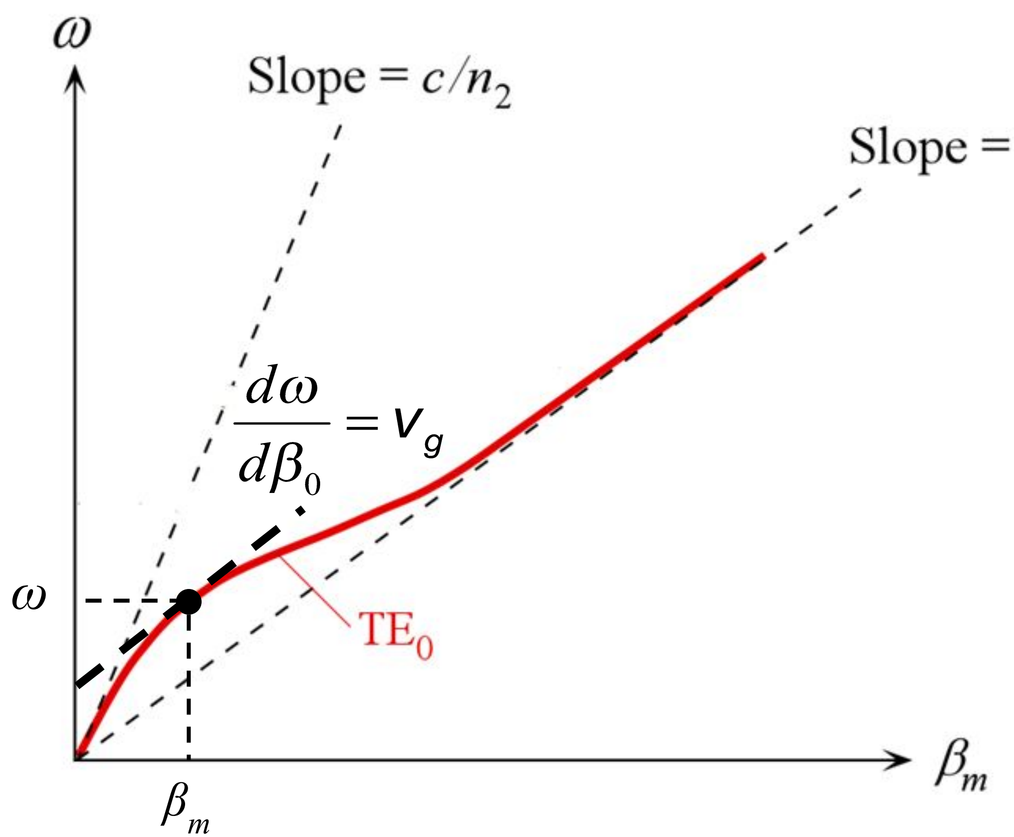


The slope of ω vs. β is the group velocity v_g

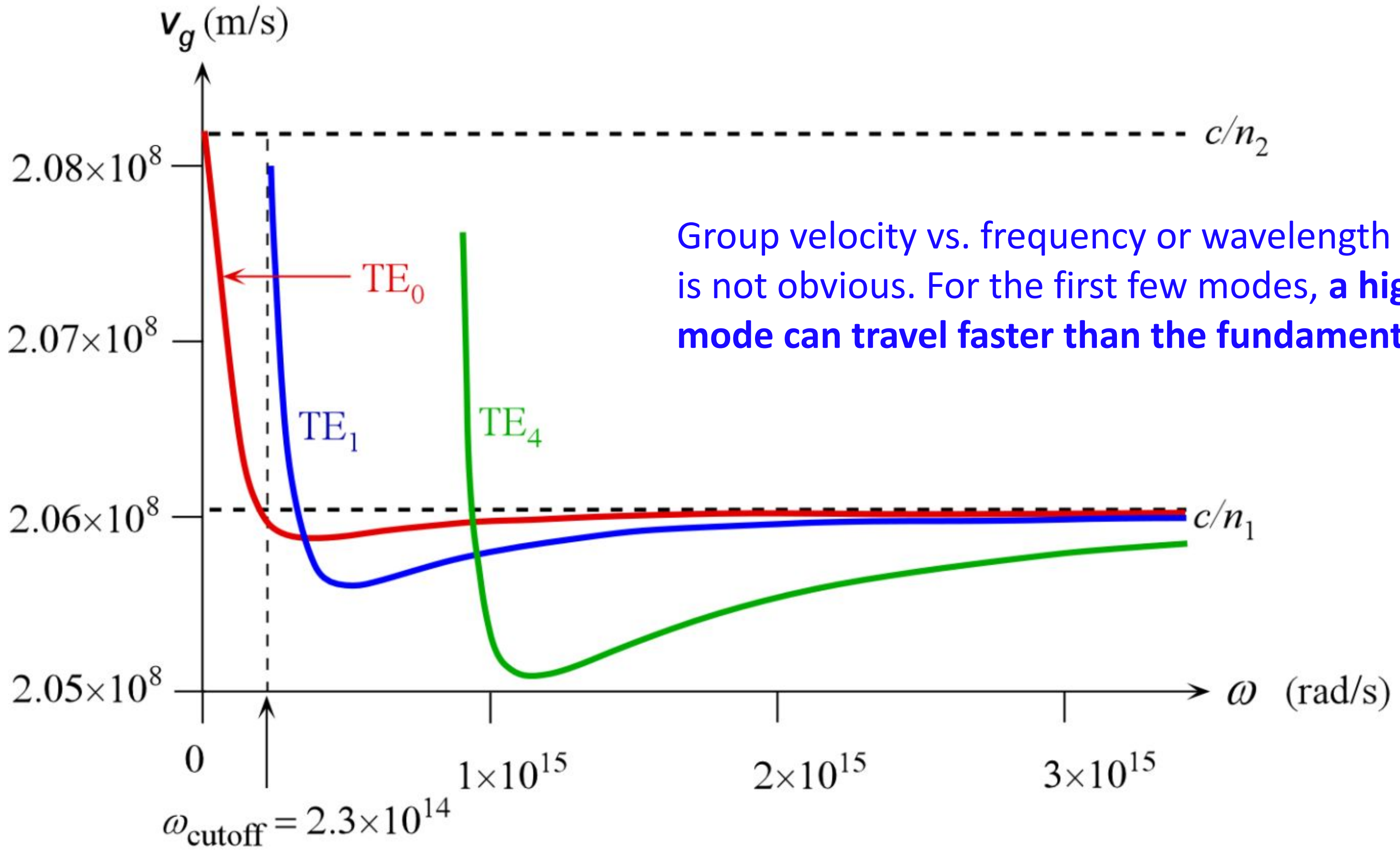
Waveguide Dispersion Curve

Slope = Group Velocity

The slope of ω vs. β is the group velocity v_g



Mode Group Velocities from Dispersion Diagram

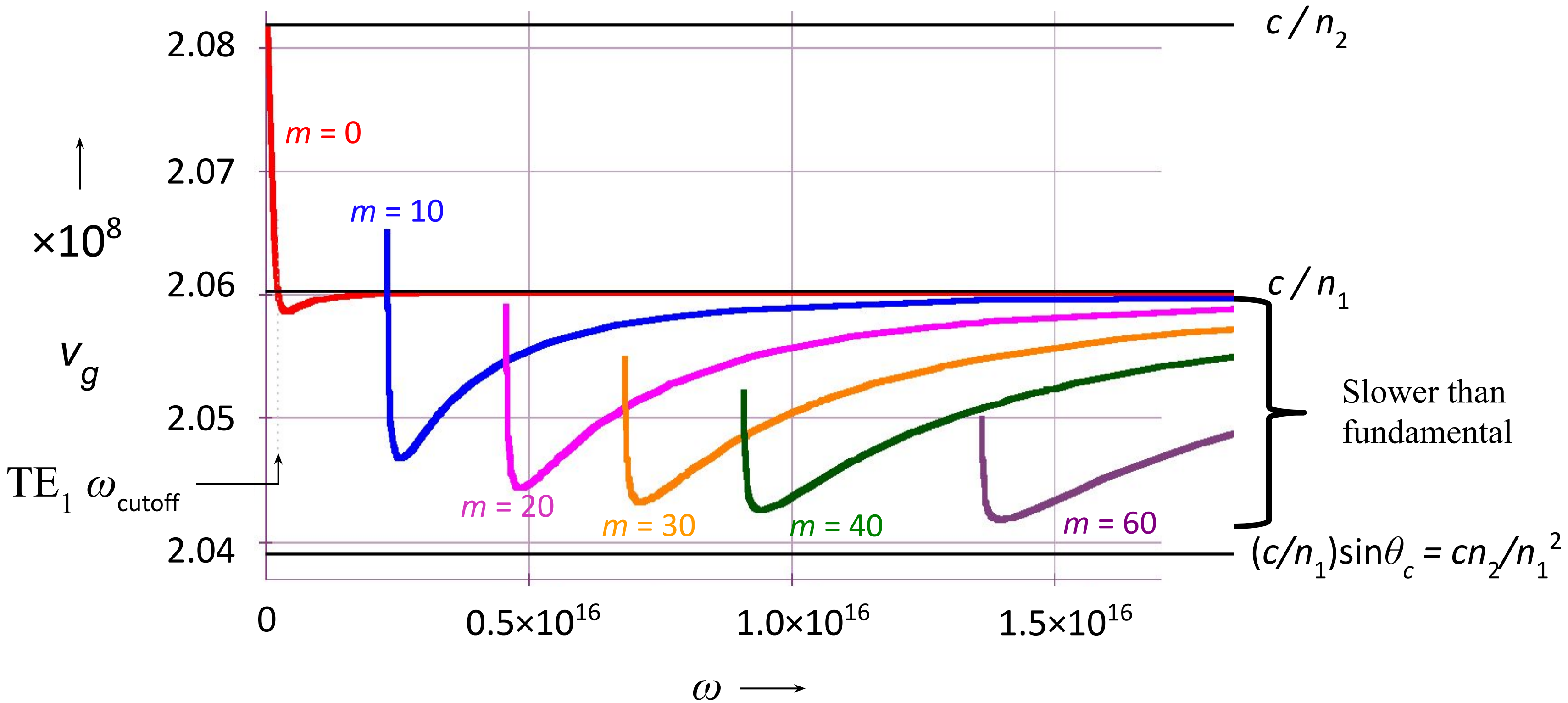


Group velocity vs. frequency or wavelength behavior is not obvious. For the first few modes, **a higher mode can travel faster than the fundamental**.

The group velocity v_g vs. ω for a planar dielectric guide with a core thickness $(2a) = 20$ µm, $n_1 = 1.455$, $n_2 = 1.440$. TE_0, TE_1 and TE_4

Not in the textbook

A Planar Dielectric Waveguide with Many Modes



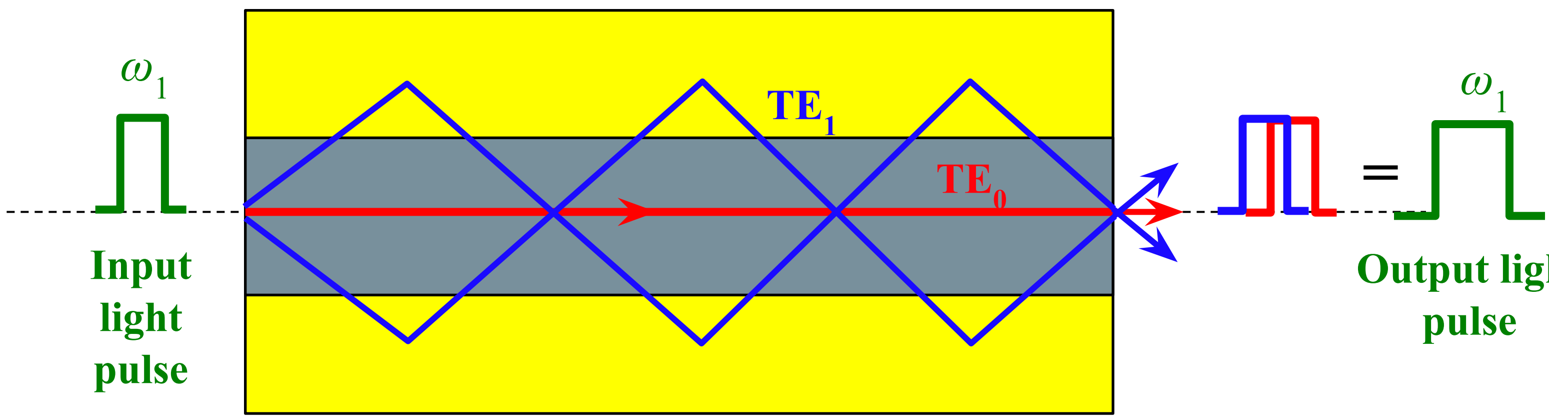
The group velocity v_g vs. ω for a planar dielectric guide

Core thickness ($2a$) = 20 μm, n_1 = 1.455, n_2 = 1.440

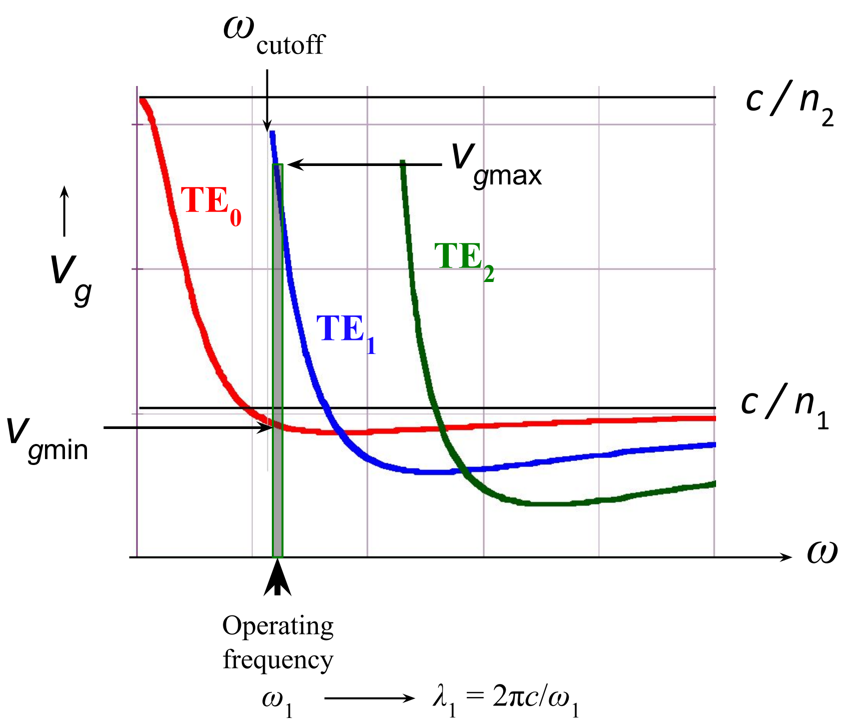
[Calculations by the author]

Not in the textbook

Dispersion in the Planar Dielectric Waveguide with TE_0 and TE_1 (Near cut-off)



Broadened pulse



$$v_{g\max} \approx c/n_2$$

$$v_{g\min} \approx c/n_1$$

$$\Delta\tau = \frac{L}{v_{g\min}} - \frac{L}{v_{g\max}}$$

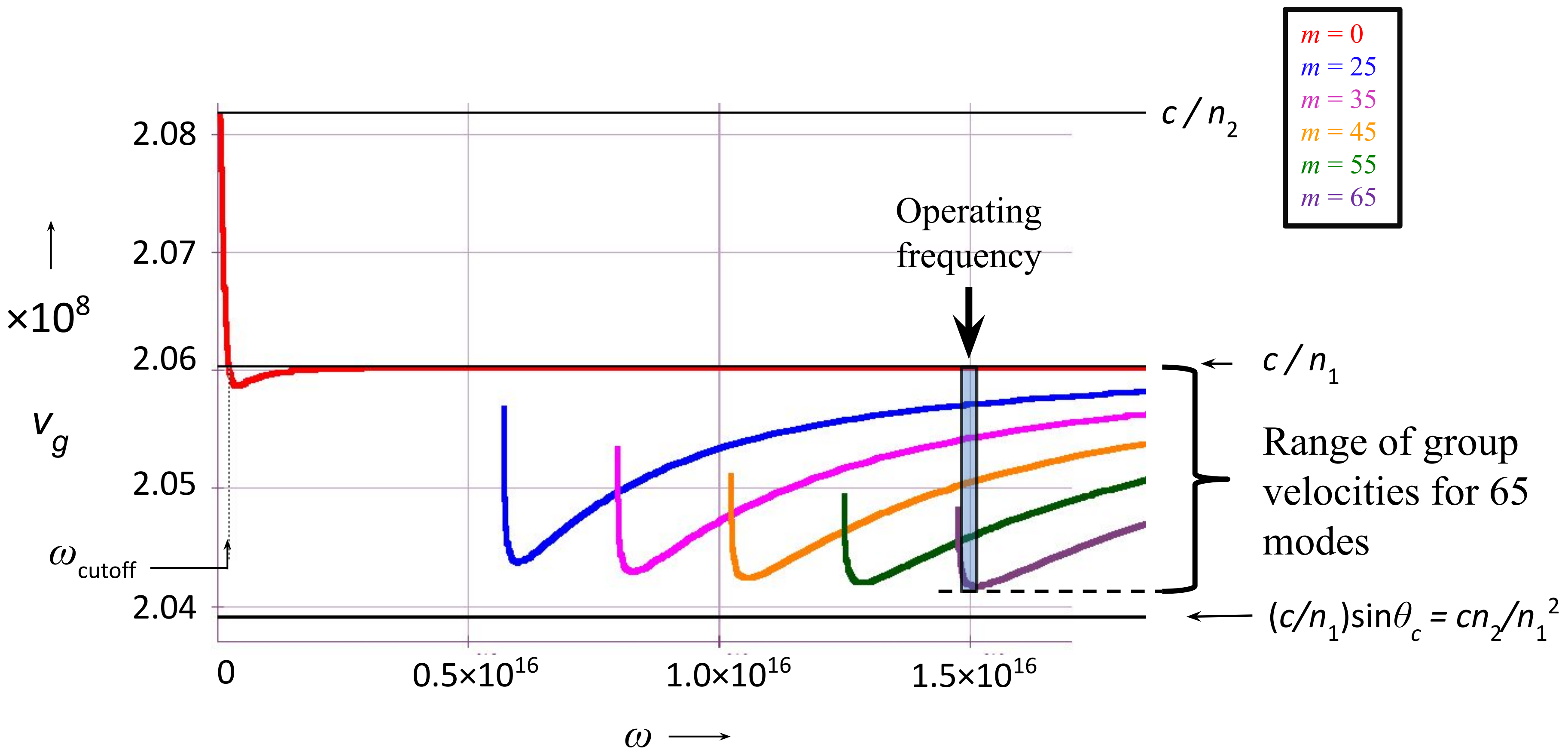
Spread in arrival times

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

Dispersion

Not in the textbook

A Planar Dielectric Waveguide with Many Modes

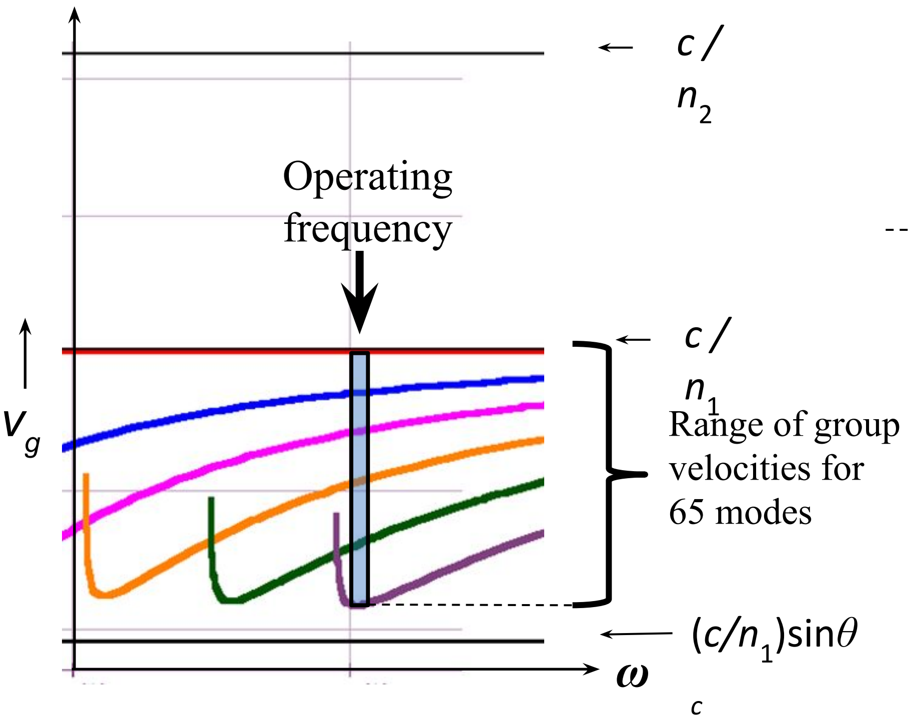


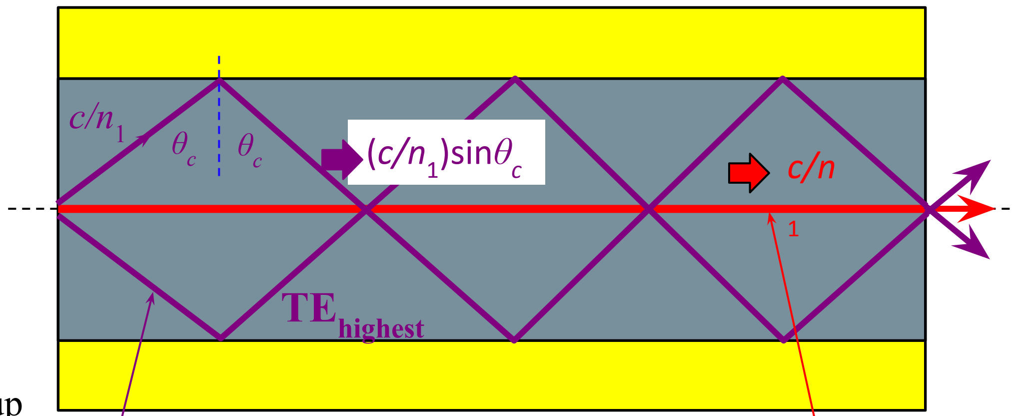
Multimode operation in which many modes propagate with different group velocities

v_g vs. ω for a planar dielectric guide with a core thickness $(2a)$ = 20 μm, n_1 = 1.455, n_2 = 1.440
[Calculations by the author]

Not in the textbook

Dispersion in the Planar Dielectric Waveguide with Many Modes Far from Cutoff





$$v_{g\min} \approx \frac{c}{n_1}\sin\theta_c = \frac{c}{n_1}\left(\frac{n_2}{n_1}\right)$$

$$v_{g\max} \approx \frac{c}{n_1}$$

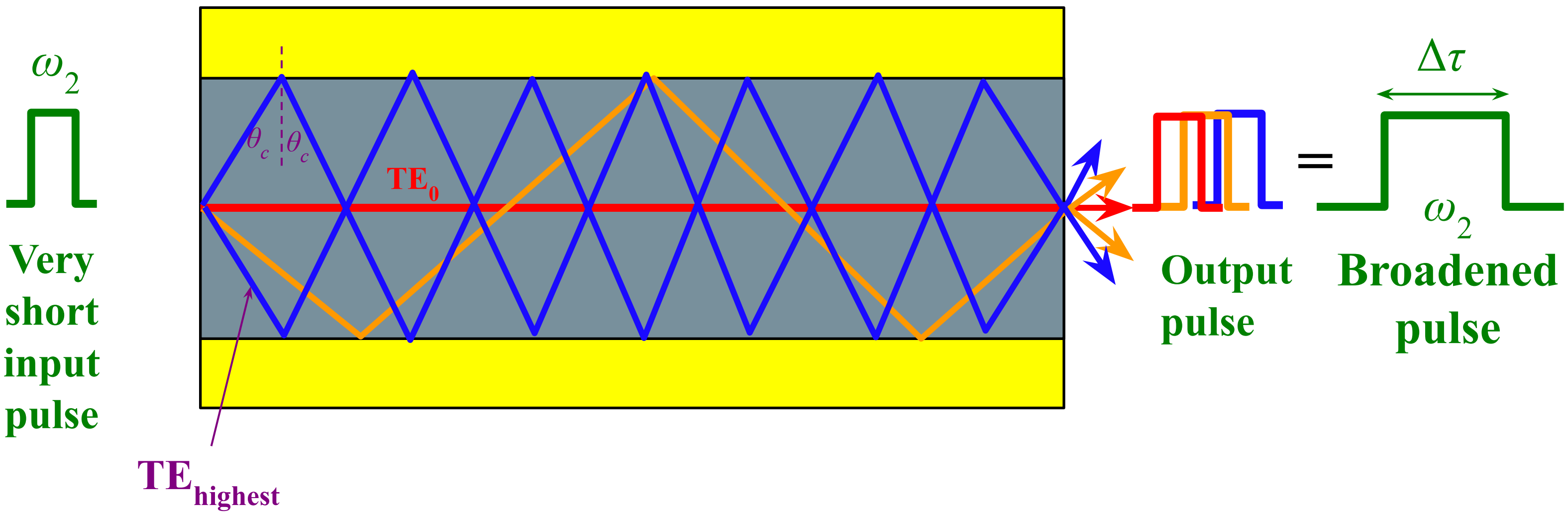
$$\frac{\Delta\tau}{L} = \frac{1}{v_{g\min}} - \frac{1}{v_{g\max}} \Rightarrow \frac{\Delta\tau}{L} = \frac{n_1^2}{cn_2} - \frac{n_1}{c} = \frac{1}{c}\left[\frac{(n_1 - n_2)n_1}{n_2}\right] \approx \frac{(n_1 - n_2)}{c}$$

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

(Since n_1 and n_2 are only slightly different.)

Not in the textbook

Dispersion in the Planar Dielectric Waveguide Many Modes

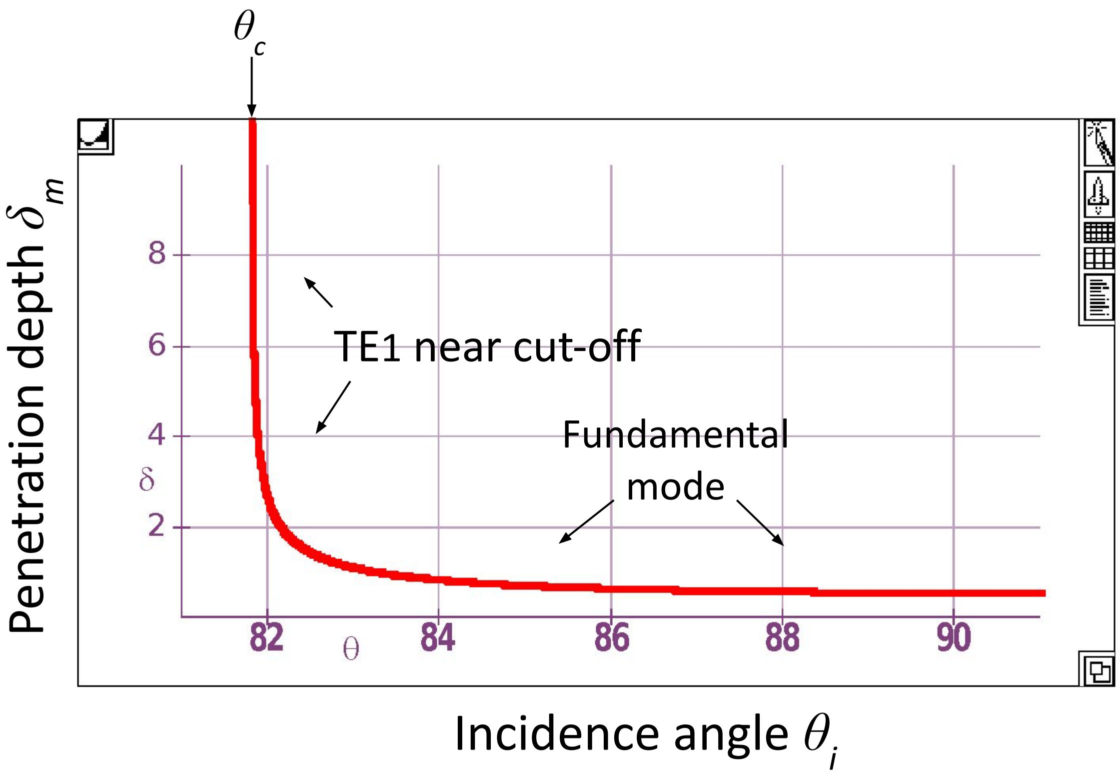


$$\frac{\Delta\tau}{L} = \frac{1}{v_{g\min}} - \frac{1}{v_{g\max}} \quad \Rightarrow \quad \frac{\Delta\tau}{L} \approx \frac{(n_1 - n_2)}{c}\left(\frac{n_1}{n_2}\right)$$

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

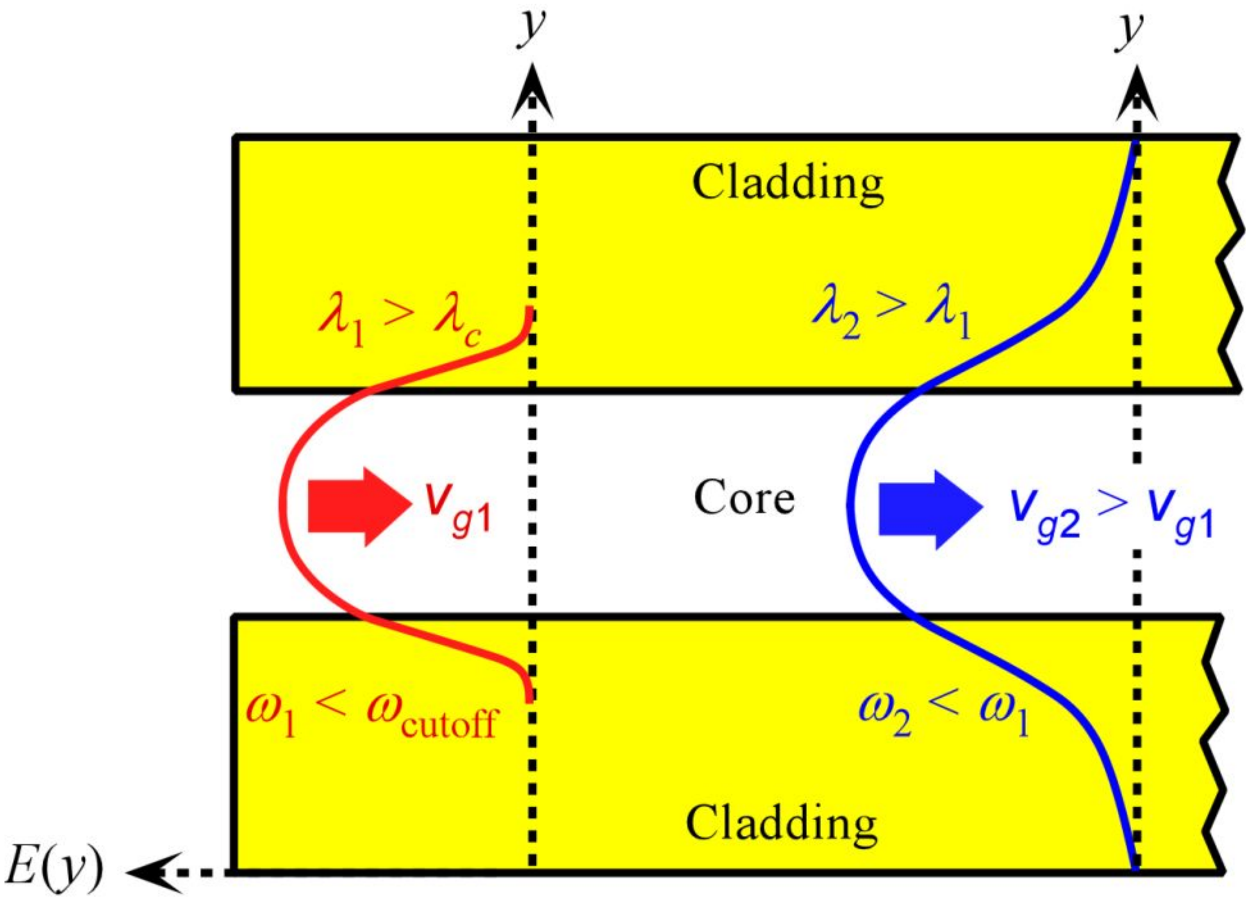
(Since $n_1/n_2 \approx 1$)

How can a higher mode such as TE_1 or TE_2 travel faster than the fundamental near cut-off?



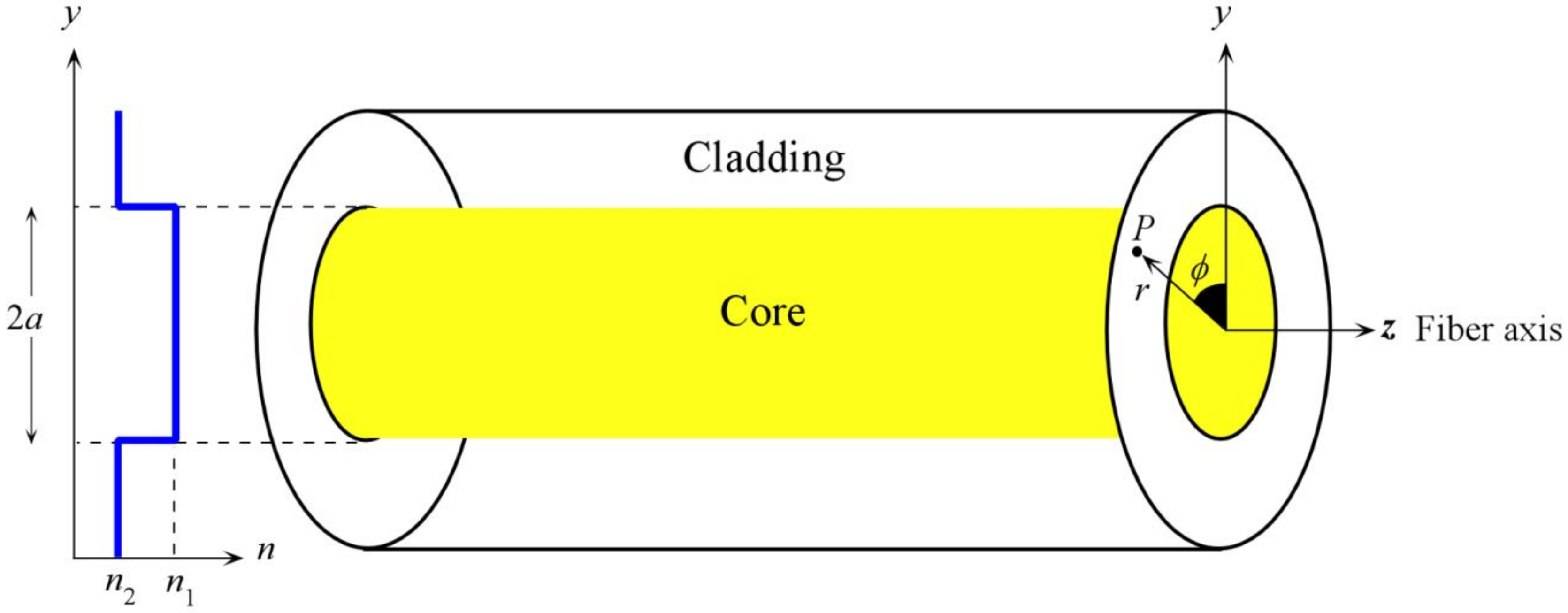
The mode TE_1 penetrates into the cladding where its velocity is higher than in the core. If penetration is large, as near cut-off, TE_1 group velocity along the guide can exceed that of TE_0.

Group Velocity and Wavelength: Fundamental Mode



The electric field of TE_0 mode extends more into the cladding as the wavelength increases. As more of the field is carried by the cladding, the group velocity increases.

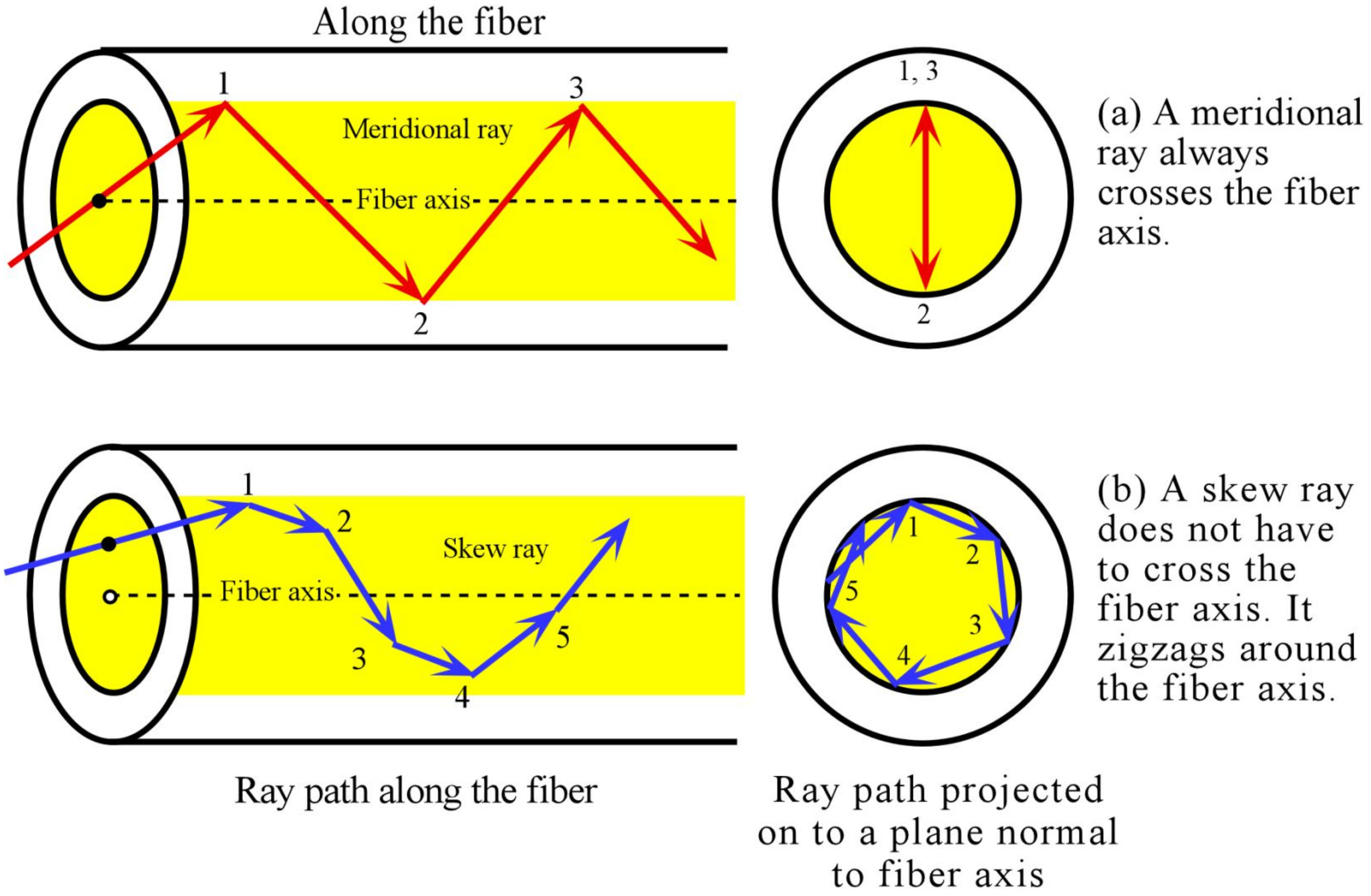
Optical Fibers



The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. The coordinates *r*, φ, z are used to represent any point *P* in the fiber. Cladding is normally much thicker than shown.

Meridional ray enters the fiber through the fiber axis and hence also crosses the fiber axis on each reflection as it zigzags down the fiber. It travels in a plane that contains the fiber axis.

Skew ray enters the fiber off the fiber axis and zigzags down the fiber without crossing the axis



Modes LP_{lm}

Weakly guiding modes in fibers

Δ << 1 weakly guiding fibers

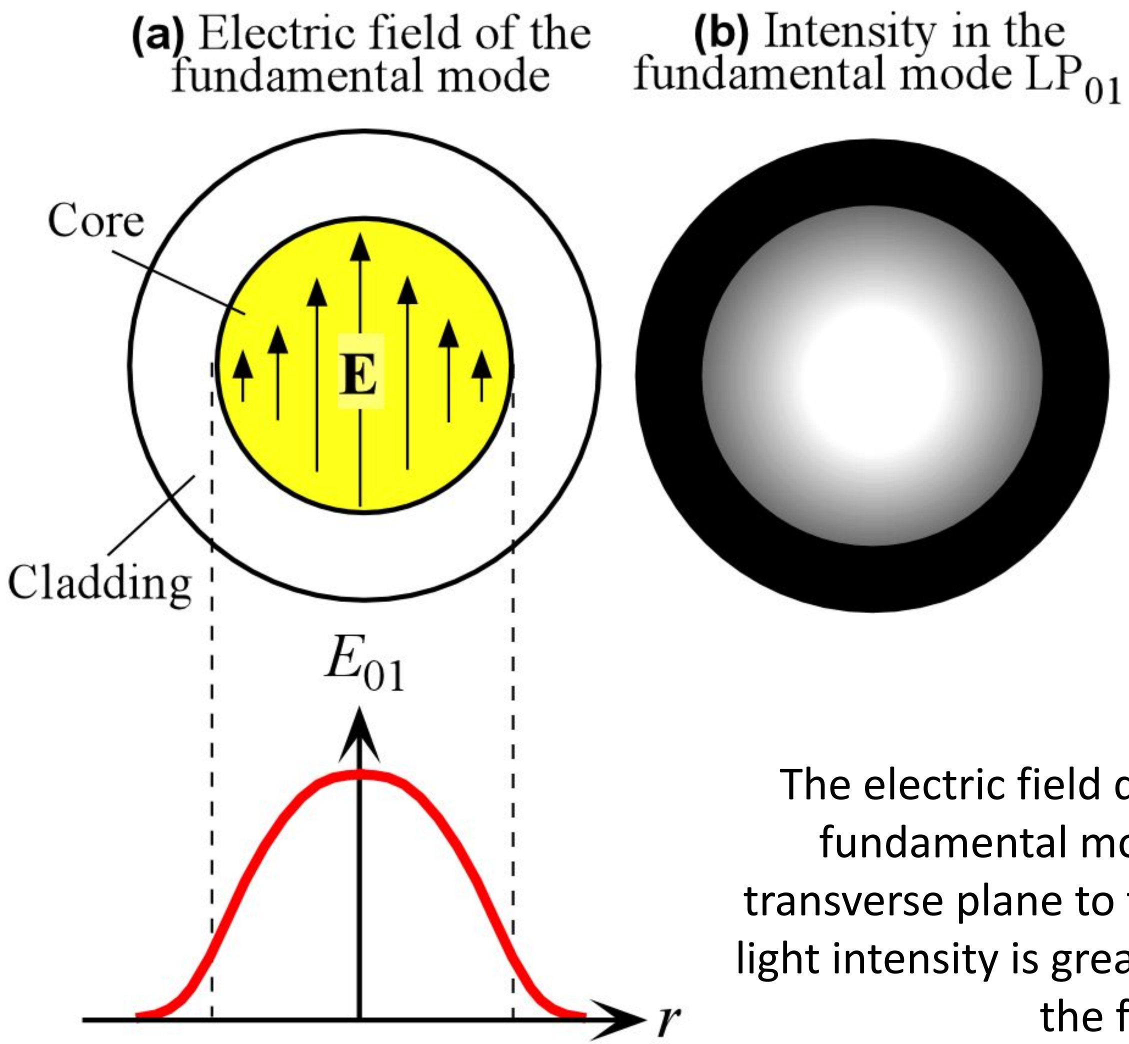
$$E_{\mathrm{LP}} = E_{lm}(r,\varphi)\ \exp j(\omega t - \beta_{lm} z)$$

$E_{lm}(r,\varphi)$: **Field Pattern**

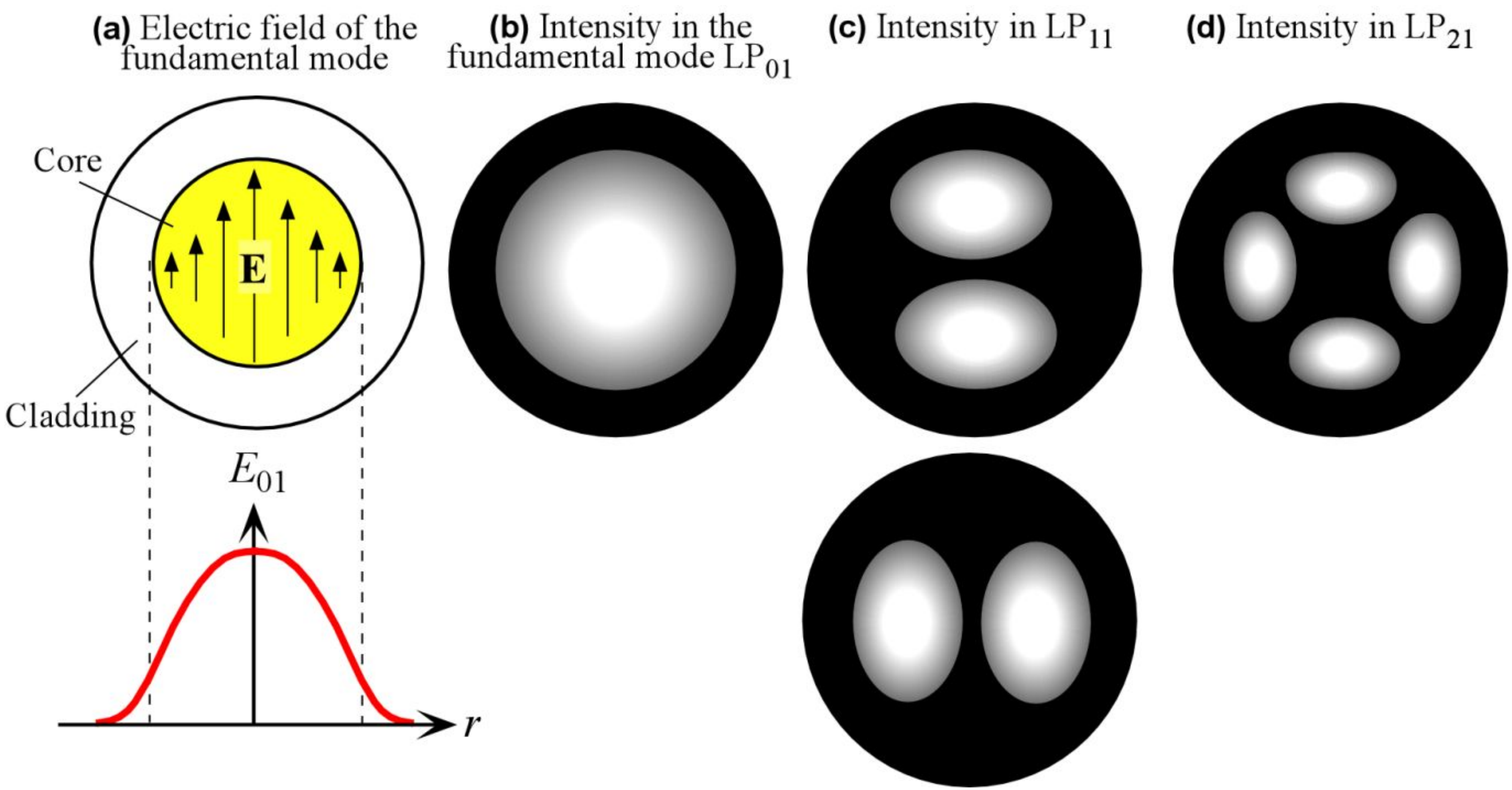
$\exp j(\omega t - \beta_{lm} z)$: **Traveling wave**

E and B are 90° to each other and *z*

Fundamental Mode is the LP_{01} mode: *l* = 0 and *m* = 1



The electric field distribution of the fundamental mode, LP_{01}, in the transverse plane to the fiber axis *z*. The light intensity is greatest at the center of the fiber



The electric field distribution of the fundamental mode in the transverse plane to the fiber axis *z*. The light intensity is greatest at the center of the fiber. Intensity patterns in LP_{01}, LP_{11} and LP_{21} modes. (a) The field in the fundamental mode. (b)-(d) Indicative light intensity distributions in three modes, LP_{01}, LP_{11} and LP_{21}.

LP_{lm}

$$E_{\mathrm{LP}} = E_{lm}(r,\varphi)\ \exp j(\omega t - \beta_{lm} z)$$

m = number of maxima along r starting from the core center. Determines the reflection angle θ

$2l$ = number of maxima around a circumference

l - radial mode number
l - extent of helical propagation, i.e. the amount of skew ray contribution to the mode.

Optical Fiber Parameters

$n = (n_1 + n_2)/2$ = **average refractive index**

Δ = **normalized index difference**

$\Delta = (n_1 - n_2)/n_1 \approx (n_1^2 - n_2^2)/2$

***V*-number** $$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi a}{\lambda}\left(2n_1 n\Delta\right)^{1/2}$$

$V < 2.405$ **only 1 mode exists. Fundamental mode**

$V < 2.405$ **or** $\lambda > \lambda_c$ **Single mode fiber**

$V > 2.405$ **Multimode fiber**

Number of modes $$M \approx \frac{V^2}{2}$$

Modes in an Optical Fiber

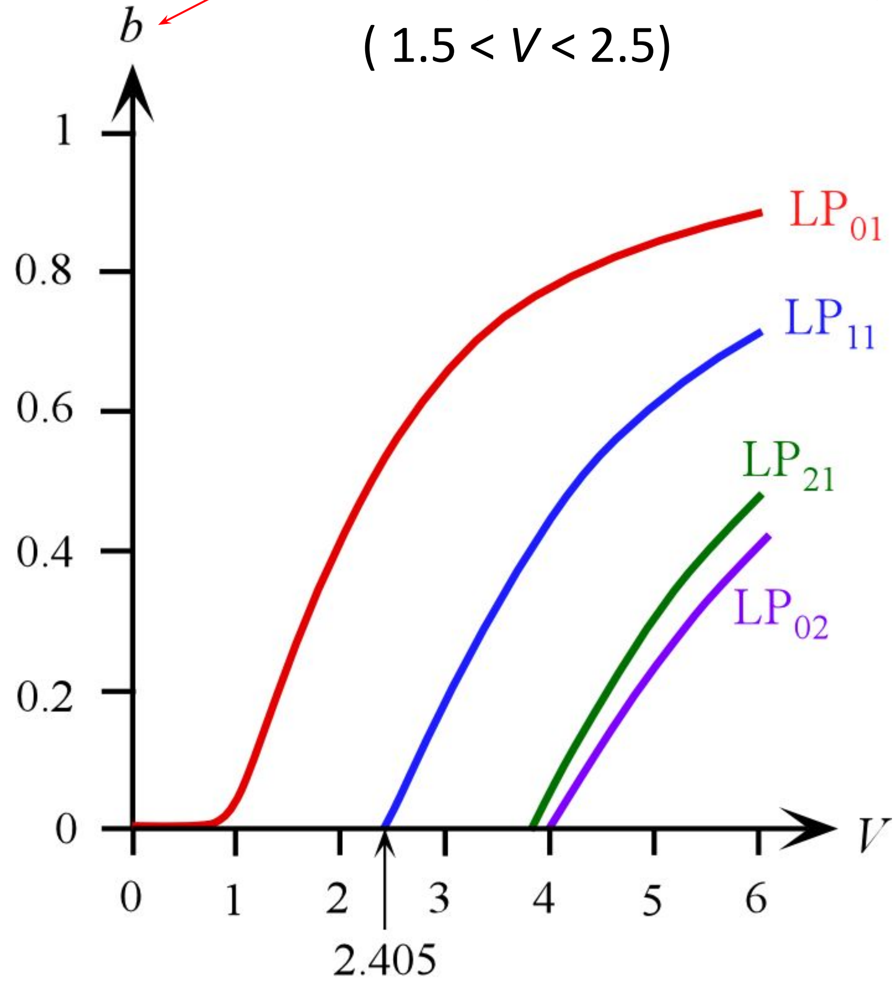
Normalized propagation constant

$$b = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$k = 2\pi/\lambda$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$

(1.5 < V < 2.5)



Normalized propagation constant *b* vs. *V*-number for a step-index fiber for various LP modes

Group Velocity and Group Delay

Consider a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. *What are the group velocity and group delay at this wavelength*?

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \qquad 1.5 < V < 2.5$$

$$b = \frac{(\beta / k) - n_2}{n_1 - n_2} \Rightarrow \beta = n_2 k[1 + b\Delta]$$

$k = 2\pi/\lambda = 4{,}188{,}790\ \text{m}^{-1}$ and $\omega = 2\pi c/\lambda = 1.255757\times10^{15}\ \text{rad s}^{-1}$

$V = (2\pi a/\lambda)(n_1^{\,2} - n_2^{\,2})^{1/2} = 1.910088$

$b = 0.3860859$, and $\beta = 6.044796\times10^{6}\ \text{m}^{-1}$.

Increase wavelength by 0.1% and recalculate. Values in the table

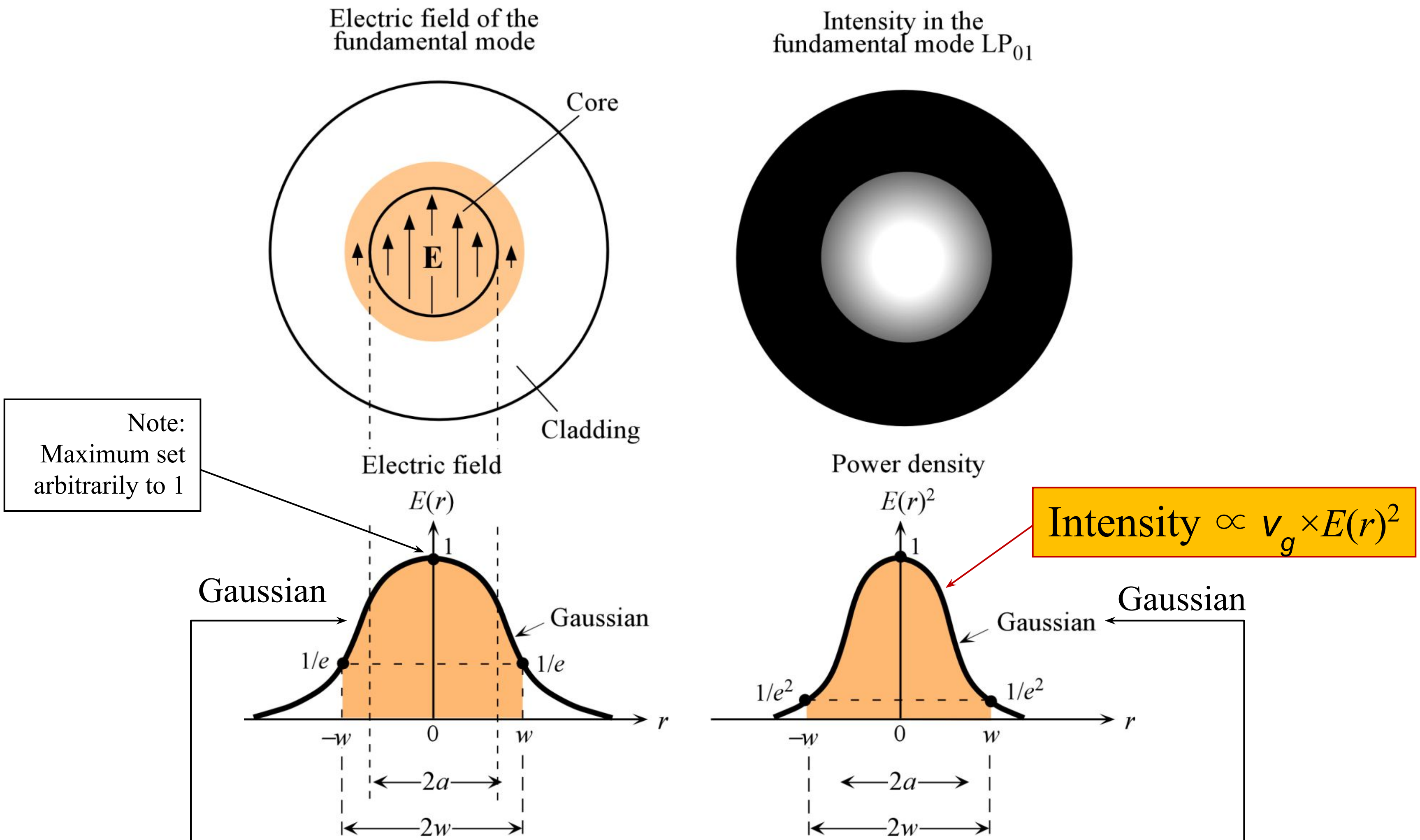
Group Velocity and Group Delay

Calculation →	V	k (m^{-1})	ω (rad s^{-1})	b	β (m^{-1})
λ = 1.500000 μm	1.910088	4188790	1.255757×10^{15}	0.3860859	6.044818×10^{6}
λ' = 1.50150 μm	1.908180	4184606	1.254503×10^{15}	0.3854382	6.038757×10^{6}

$$V_g = \frac{d\omega}{d\beta} = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.254503 - 1.255757)\times10^{15}}{(6.038757 - 6.044818)\times10^{6}} \approx 2.07\times10^{8}\ \mathrm{m\,s^{-1}}$$

The group delay τ_g over 1 km is 4.83 μs

Mode Field Diameter (2w)

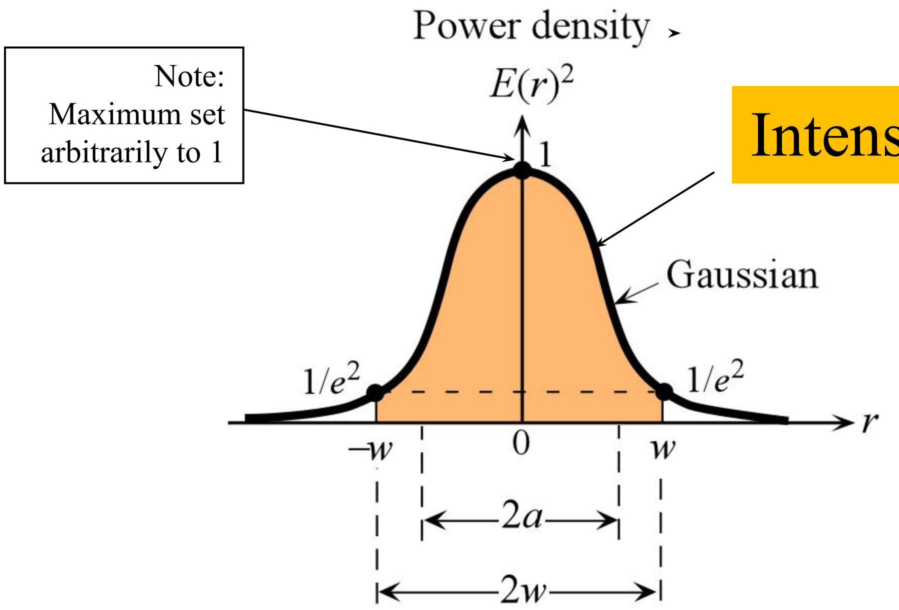


$$E(r) = E(0)\exp[-(r/w)^2]$$

$$E(r)^2 = E(0)^2\exp[-2(r/w)^2]$$

Mode Field Diameter

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$



Intensity $\propto v_g \times E(r)^2$

$2w$ = Mode Field Diameter (MFD)

Marcuse MFD Equation

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$

$0.8 < V < 2.5$

$$2w \approx (2a)(2.6V)$$

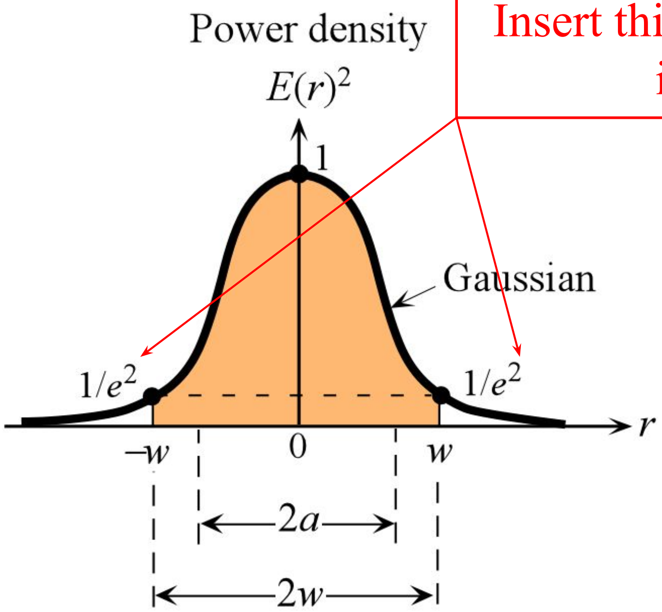
$1.6 < V < 2.4$

Correction note p113

Applies to print version only; e-version is correct

Insert this 2 as superscript on e in Figure 2.16

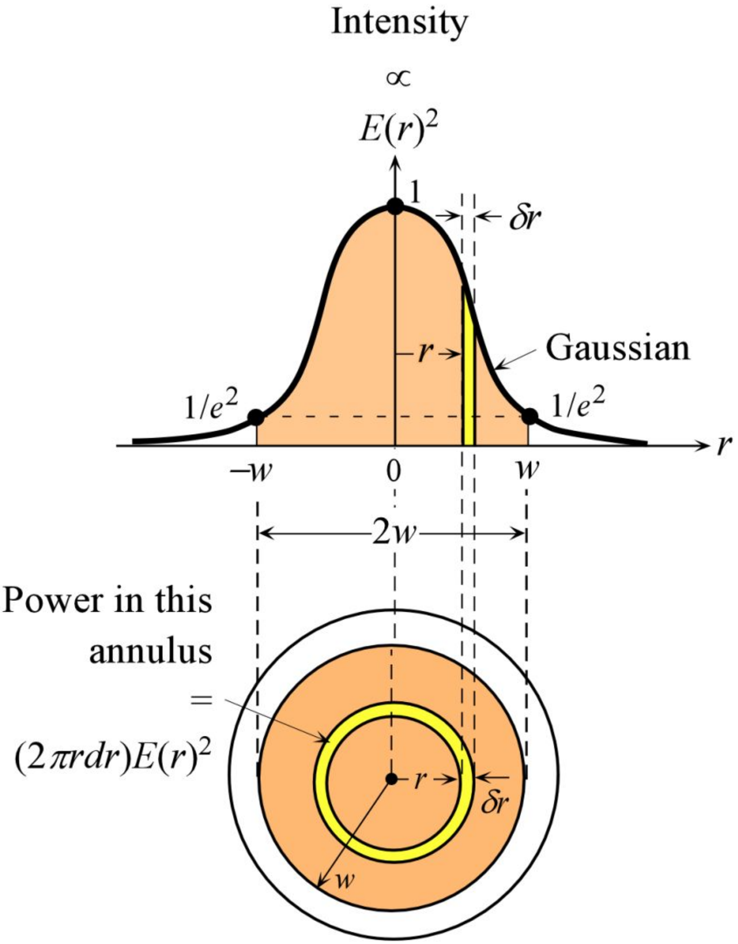
Insert this 2 in Equation (2.3.7)



$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$

Not in the textbook

Mode Field Diameter (2*w*)



$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$

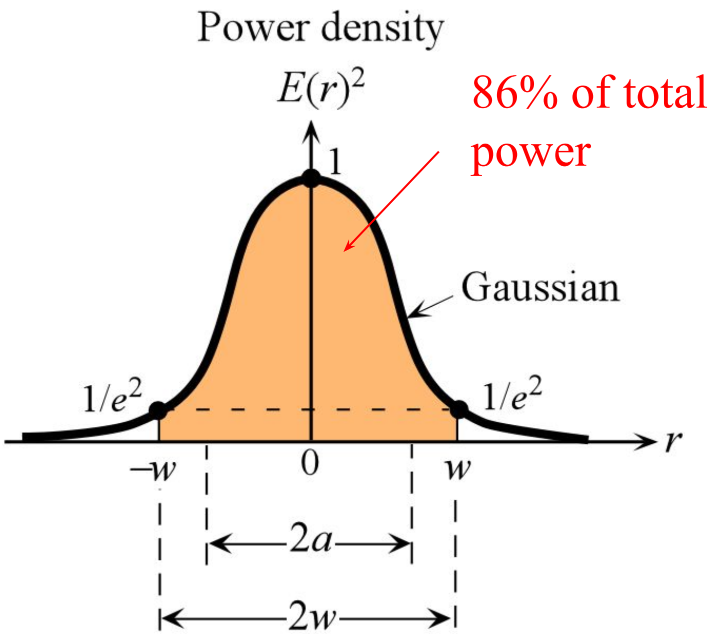
and

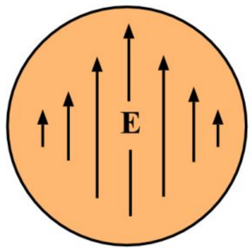
$$\text{Intensity} \propto v_g \times E(r)^2$$

Area of a circular thin strip (annulus) with radius r is $2\pi r dr$. Power passing through this strip is proportional to $E(r)^2(2\pi r)dr$

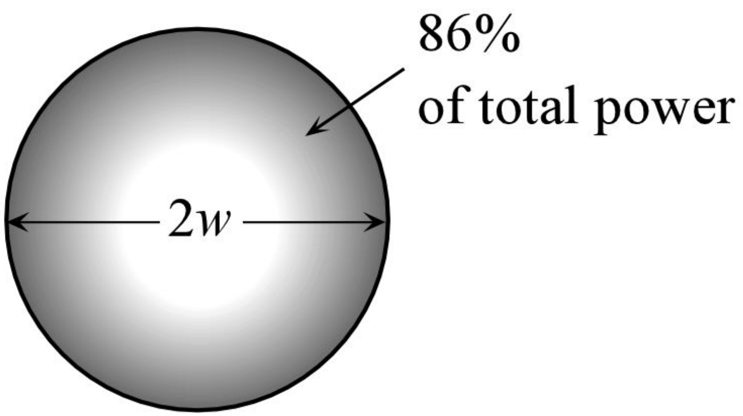
$$\text{Fraction of optical power within MFD} = \frac{\int_0^w E(r)^2 2\pi r dr}{\int_0^\infty E(r)^2 2\pi r dr} = 0.865$$

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$





Mode Field Diameter (2*w*)



Fraction of optical power within MFD = 86 %

This is the same as the fraction of optical power within a radius *w* from the axis of a Gaussian beam (See Chapter 1)

Example: A multimode fiber

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 μm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

Solution

Substitute, a = 50 μm, λ = 0.850 μm, n_1 = 1.468, n_2 = 1.447 into the expression for the V-number,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi 50/0.850)(1.468^2 - 1.447^2)^{1/2}$$

$$= 91.44.$$

Since $V >> 2.405$, the number of modes is

$$M \approx V^2/2 = (91.44)^2/2 = 4181$$

which is large.

Example: A single mode fiber

What should be the core radius of a single mode fiber which has a core of $n_1 = 1.4680$, cladding of $n_2 = 1.447$ and it is to be used with a source wavelength of 1.3 μm?

Solution

For single mode propagation, $V \leq 2.405$. We need,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

or

$$[2\pi a/(1.3\ \mu\text{m})](1.468^2 - 1.447^2)^{1/2} \leq 2.405$$

which gives $\boldsymbol{a \leq 2.01\ \mu\text{m}}$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; a is comparable to λ which means that the geometric ray picture, strictly, cannot be used to describe light propagation.

Example: Single mode cut-off wavelength

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 μm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the V-number and the mode field diameter (MFD) for operation at λ = 1.31 μm?

Solution

For single mode operation,

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$

Substituting for a, n_1 and n_2 and rearranging we get,

$\lambda > [2\pi(4.1\ \mu\text{m})(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251\ \mu\text{m}$

Wavelengths shorter than 1.251 μm give multimode propagation.

At λ = 1.31 μm,

$V = 2\pi[(4.1\ \mu\text{m})/(1.31\ \mu\text{m})](1.4532^2 - 1.4485^2)^{1/2} = 2.30$

Mode field diameter MFD

Solution (continued)

Mode field diameter MFD from the Marcuse Equation is

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$
$$= 2(4.1)[0.65 + 1.62(2.30)^{-3/2} + 2.88(2.30)^{-6}]$$

2*w* = 9.30 μm

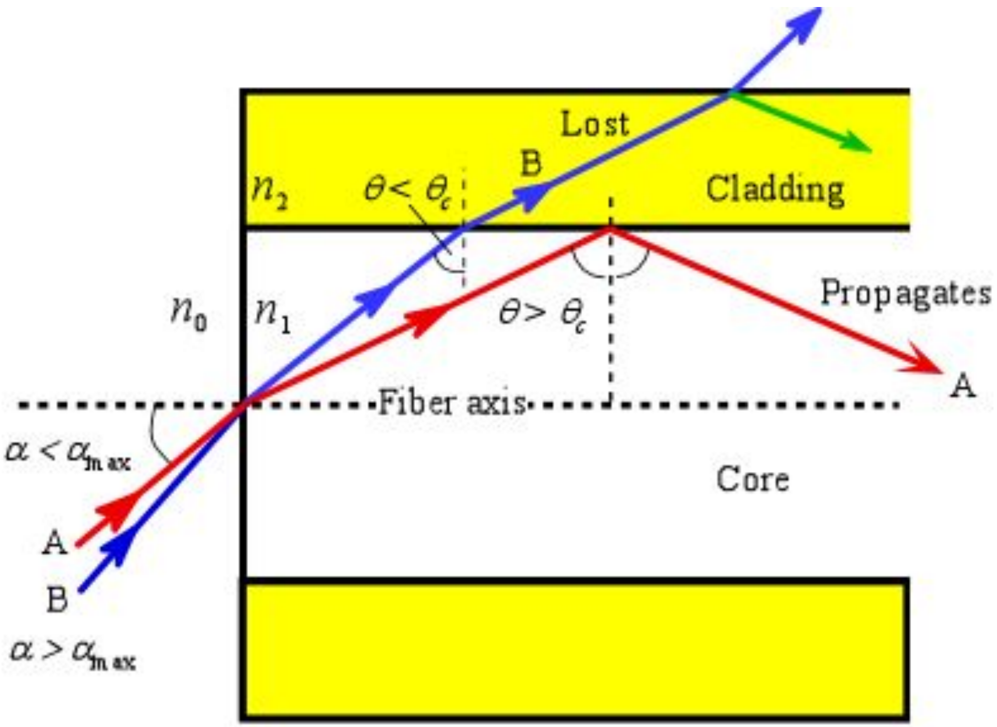
86% of total power is within this diameter

2*w* = (2*a*)(2.6/*V*) = 2(4.1)(2.6/2.30) = 9.28 μm

2*w* = 2*a*[(*V*+1)/*V*] = 11.8 μm

This is for a planar waveguide, and the definition is different than that for an optical fiber

Numerical Aperture NA



Maximum acceptance angle α_{max} is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{max}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{max}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin \alpha_{max} = \frac{\left(n_1^2 - n_2^2\right)^{1/2}}{n_o} = \frac{\text{NA}}{n_o}$$

$$V = \frac{2\pi a}{\lambda}\text{NA}$$

$2\alpha_{max}$ = **total acceptance angle**

NA is an important factor in light launching designs into the optical fiber.

Example: A multimode fiber and total acceptance angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Solution

Normalized refractive index

$\Delta = (n_1 - n_2) / n_1 = \mathbf{0.0135 \text{ or } 1.35\%}$

The numerical aperture is

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} = \mathbf{0.2425 \text{ or } 25.3\%}$$

From, $\sin\alpha_{max} = NA/n_o = 0.2425/1$

Acceptance angle $\alpha_{max} = \mathbf{14°}$

Total acceptance angle $2\alpha_{max} = \mathbf{28°}$

V-number in terms of the numerical aperture can be written as,

$$V = (2\pi a/\lambda)NA = [(2\pi 50\ \mu m)/(0.85\ \mu m)](0.2425) = 89.62$$

The number of modes, $M \approx V^2/2 = \mathbf{4016}$

Example: A single mode fiber

A typical single mode optical fiber has a core of diameter 8 μm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 μm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency λ_c of the fiber?

Solution

The numerical aperture

$NA = (n_1^2 - n_2^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$NA \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} = \mathbf{0.113 \text{ or } 11.3\ \%}$

The acceptance angle is given by

$\sin\alpha_{max} = NA/n_o = 0.113/1$ or $\alpha_{max} = \mathbf{6.5}°$, and $2\alpha_{max} = \mathbf{13°}$

The condition for single mode propagation is $V \leq 2.405$ which corresponds to a minimum wavelength λ_c is given by

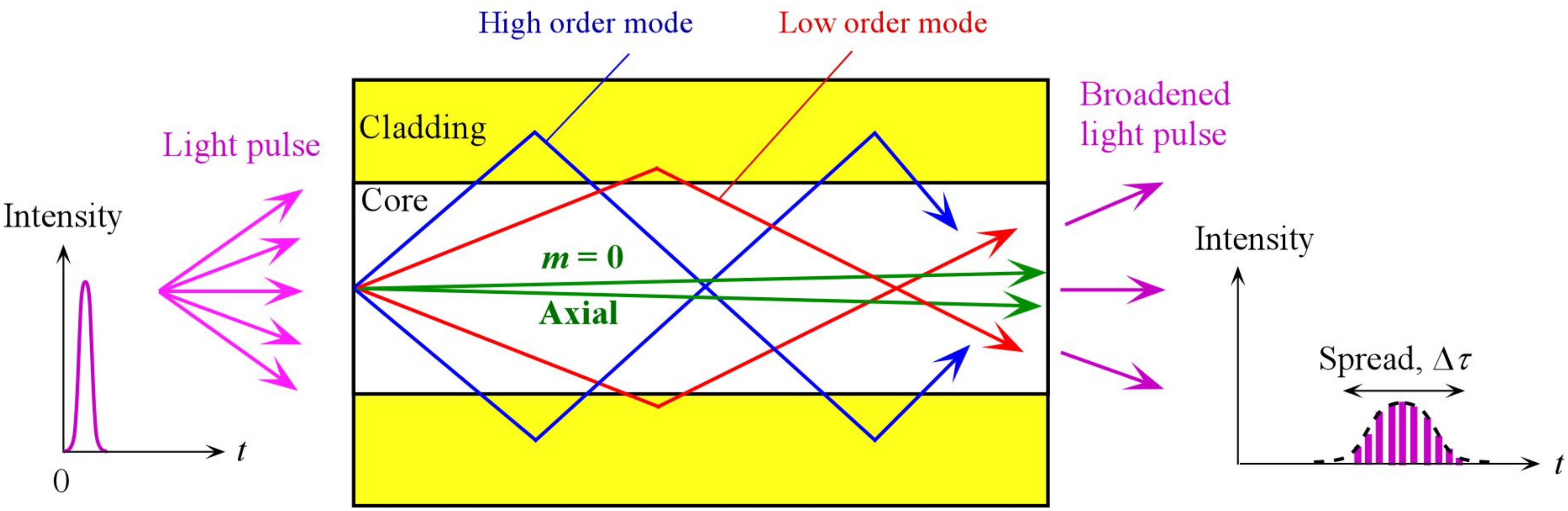
$\lambda_c = [2\pi a NA]/2.405 = [(2\pi)(4\ \mu m)(0.113)]/2.405 = \mathbf{1.18\ \mu m}$

Wavelengths shorter than 1.18 μm will result in multimode operation.

Dispersion = Spread of Information

- **Intermode (Intermodal) Dispersion: Multimode fibers only**
- **Material Dispersion**
  Group velocity depends on N_g and hence on λ
- **Waveguide Dispersion**
  Group velocity depends on waveguide structure
- **Chromatic Dispersion**
  Material dispersion + Waveguide Dispersion
- **Polarization Dispersion**
- **Profile Dispersion**
  Like material and waveguide dispersion. Add all 3
  Material + Waveguide + Profile
- **Self phase modulation dispersion**

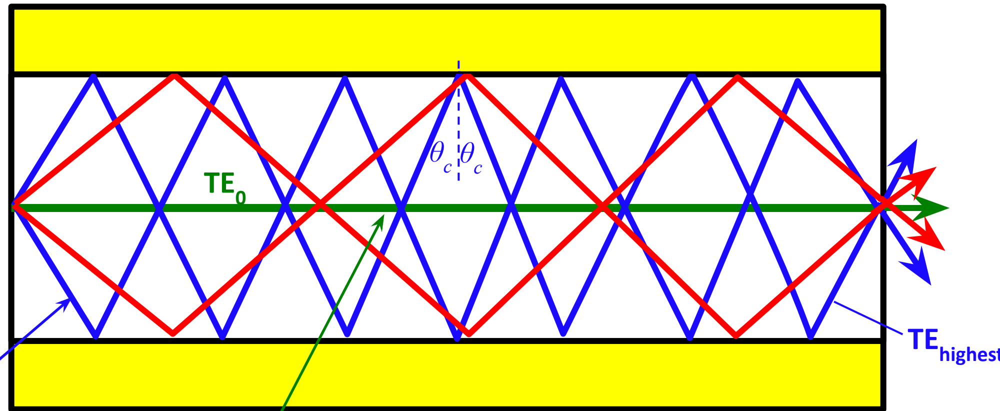
Intermode Dispersion (MMF)



$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

(Since n_1 and n_2 are only slightly different)

Intermode Dispersion (MMF)



$$v_{g\min} \approx \frac{c}{n_1}\sin\theta_c = \frac{c}{n_1}\left(\frac{n_2}{n_1}\right)$$

$$v_{g\max} \approx \frac{c}{n_1}$$

$$\Delta\tau = \frac{L}{v_{g\min}} - \frac{L}{v_{g\max}}$$

$$\frac{\Delta\tau}{L} = \frac{(n_1 - n_2)}{c}\left(\frac{n_1}{n_2}\right)$$

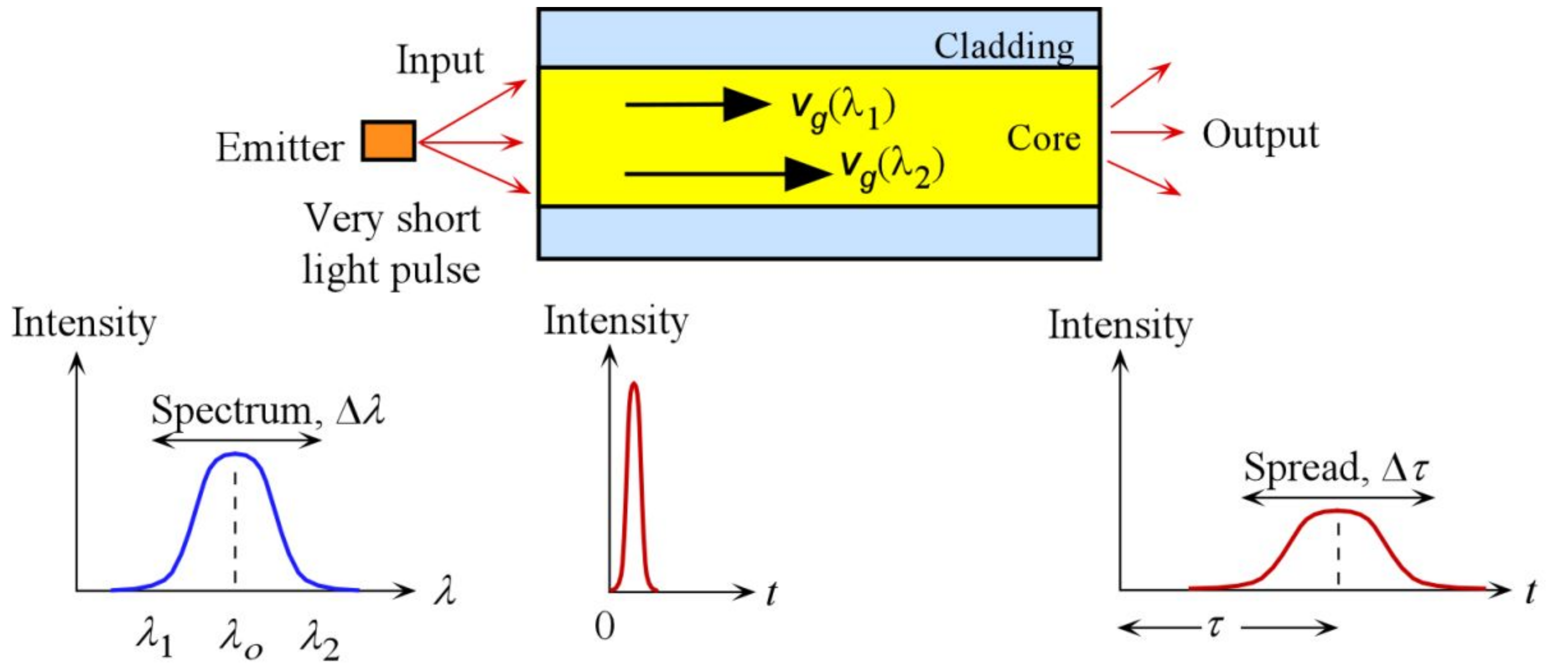
$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

$\Delta\tau/L \approx 10 - 50$ ns / km

Depends on length!

Intramode Dispersion (SMF)

Dispersion in the fundamental mode



Group Delay $\tau = L / v_g$

Group velocity v_g depends on

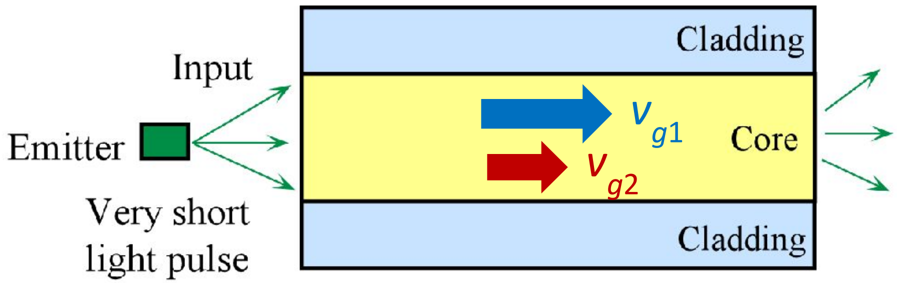
Refractive index = $n(\lambda)$ **Material Dispersion**

V-number = $V(\lambda)$ **Waveguide Dispersion**

$\Delta = (n_1 - n_2)/n_1 = \Delta(\lambda)$ **Profile Dispersion**

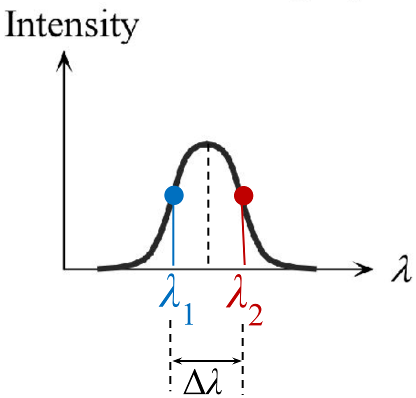
Intramode Dispersion (SMF)

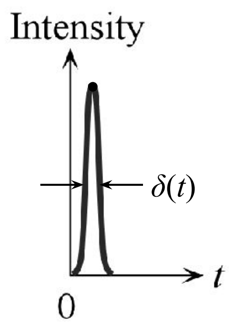
Chromatic dispersion in the fundamental mode

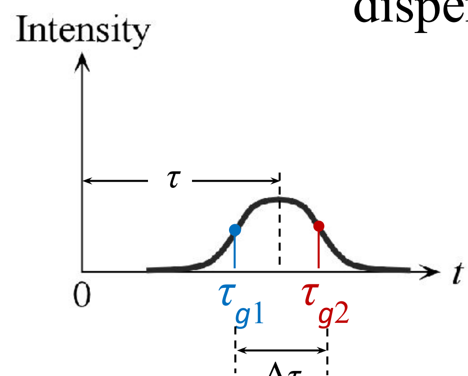


$$\Delta\tau = DL\Delta\lambda$$

Output pulse dispersed







$$\Delta\lambda = \lambda_2 - \lambda_1$$

Chromatic spread

$$D = \frac{\Delta\tau}{L\Delta\lambda}$$

OR

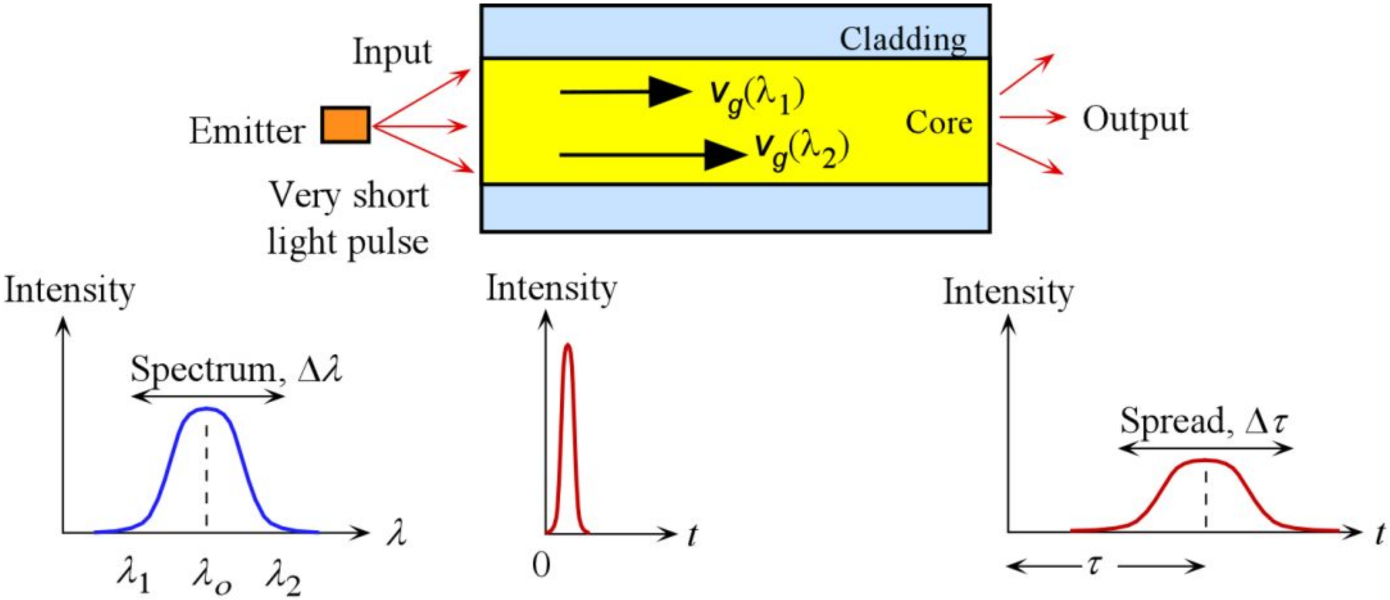
$$D = \frac{d\tau}{Ld\lambda}$$

$$\Delta\tau = \tau_{g1} - \tau_{g2}$$

Dispersion

Definition of Dispersion Coefficient

Material Dispersion



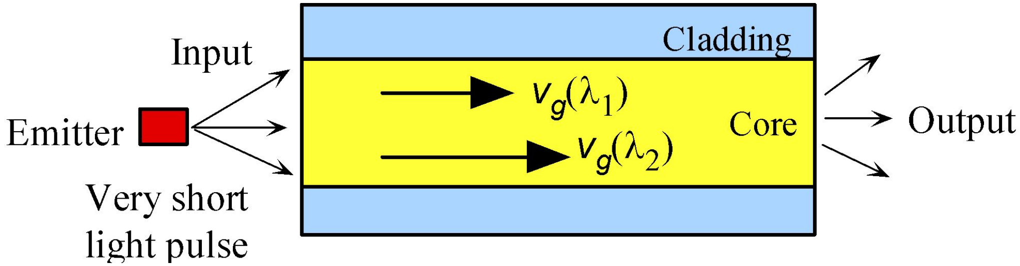
Emitter emits a spectrum $\Delta\lambda$ of wavelengths.

Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

$$\frac{\Delta\tau}{L} = D_m \Delta\lambda$$

D_m = Material dispersion coefficient, ps nm^{-1} km^{-1}

Material Dispersion



$$v_g = c / N_g$$

Group velocity — Depends on the wavelength

$$\frac{\Delta\tau}{L} = D_m \Delta\lambda$$

D_m = Material dispersion coefficient, ps nm^{-1} km^{-1}

$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$

Wave guide dispersion

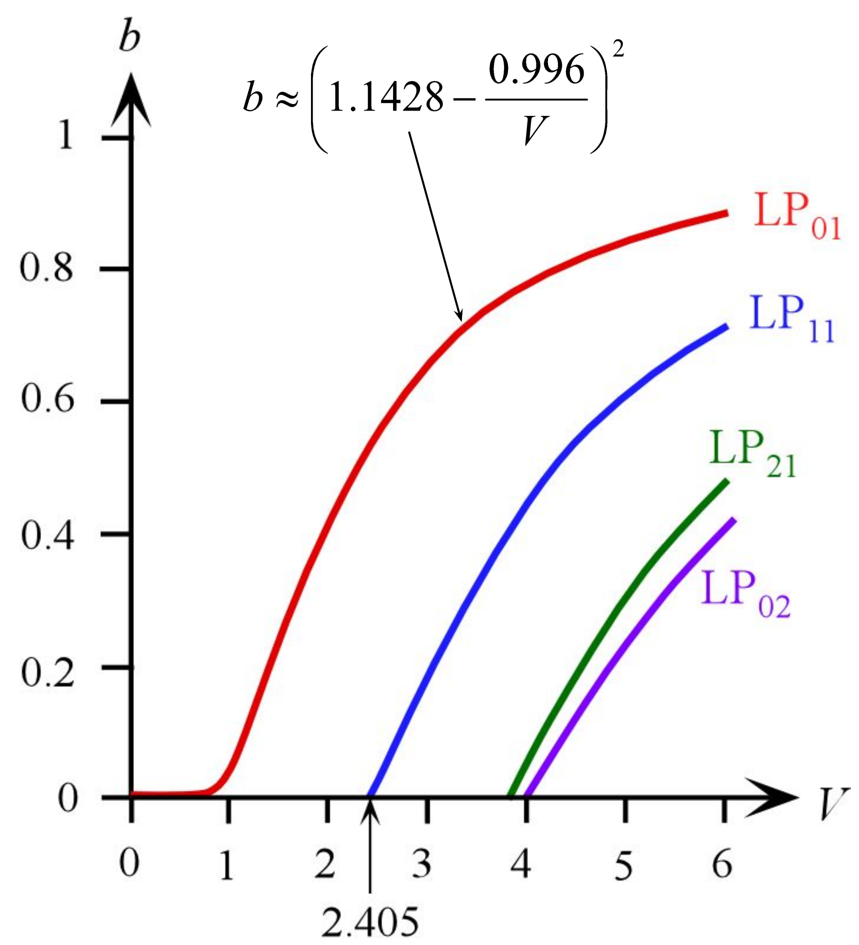
b hence *β* depend on *V* and hence on *λ*

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

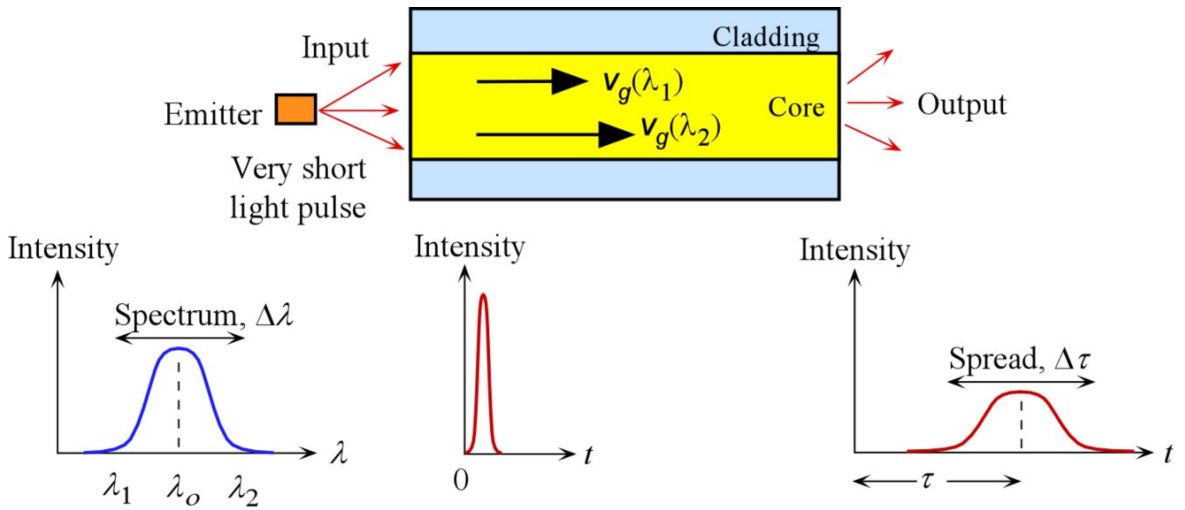
Normalized propagation constant

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$k = 2\pi/\lambda$$



Waveguide Dispersion



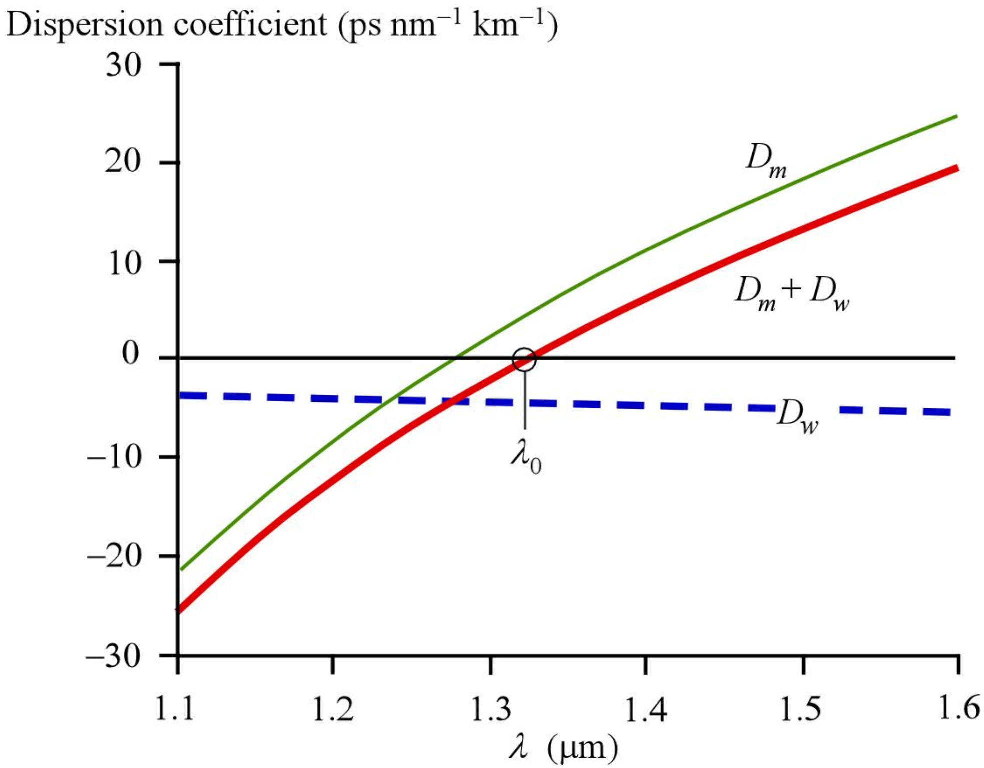
Waveguide dispersion The group velocity $v_g(01)$ of the fundamental mode depends on the *V*-number, which itself depends on the source wavelength λ, even if n_1 and n_2 were constant. Even if n_1 and n_2 were wavelength independent (no material dispersion), we will still have waveguide dispersion by virtue of $v_g(01)$ depending on V and V depending inversely on λ. Waveguide dispersion arises as a result of the guiding properties of the waveguide which imposes a nonlinear ω vs. β_{lm} relationship.

$$\frac{\Delta\tau}{L} = D_w \Delta\lambda$$

D_w = waveguide dispersion coefficient

D_w depends on the waveguide structure, ps nm^{-1} km^{-1}

Chromatic Dispersion

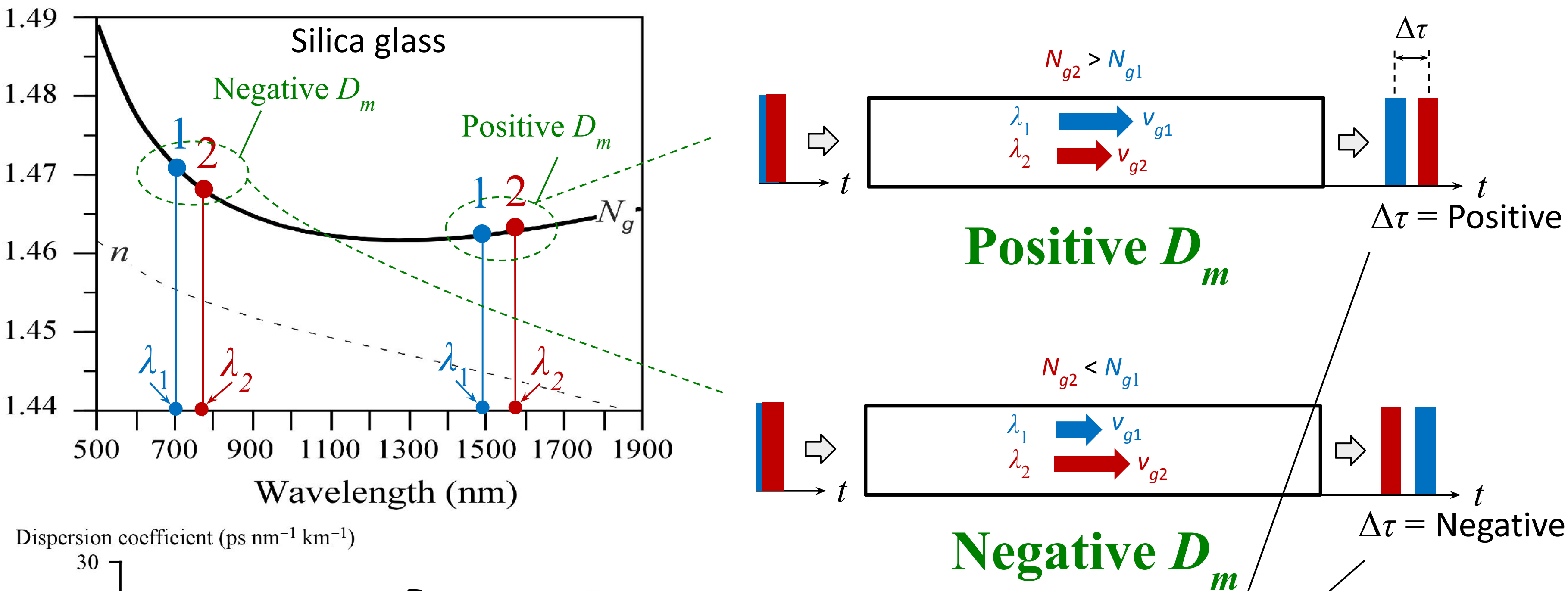


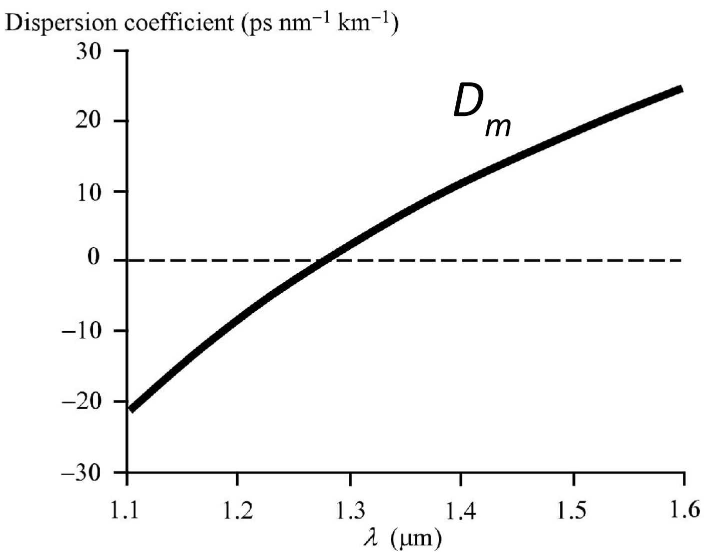
Material dispersion coefficient (D_m) for the core material (taken as SiO_2), waveguide dispersion coefficient (D_w) (a = 4.2 µm) and the total or chromatic dispersion coefficient D_{ch} (= D_m + D_w) as a function of free space wavelength, λ

Chromatic = Material + Waveguide

$$\frac{\Delta\tau}{L} = (D_m + D_w)\Delta\lambda$$

What do Negative and Positive D_m mean?

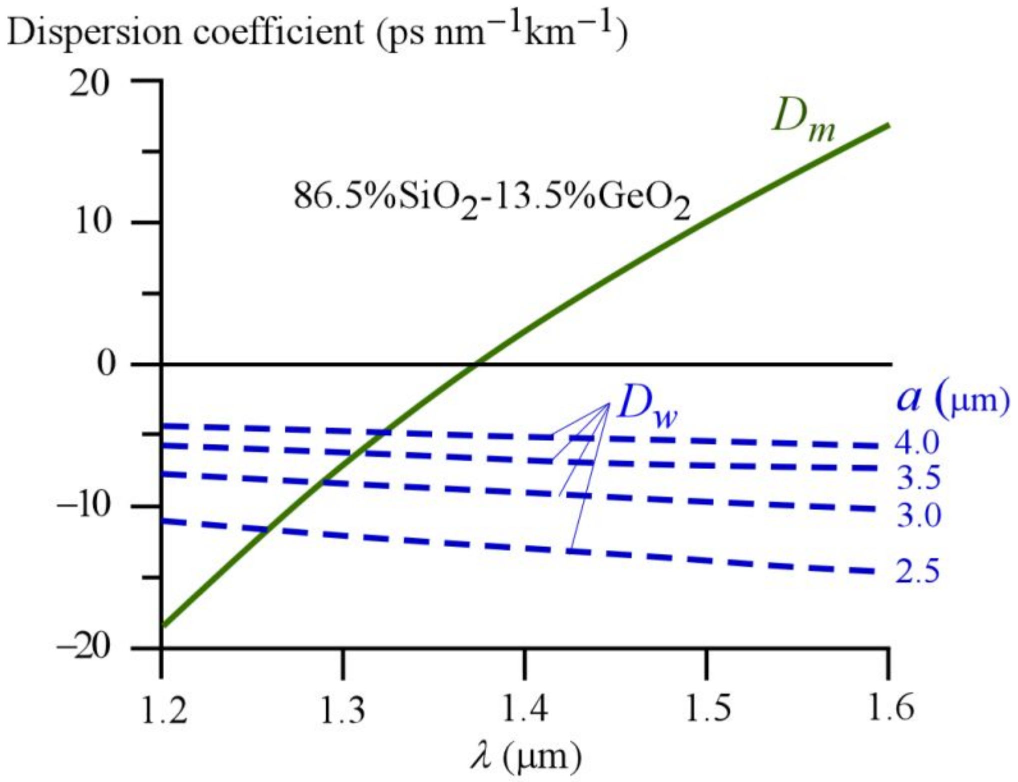




$$D_m = \frac{\Delta\tau}{L\Delta\lambda}$$

$\Delta\lambda = \lambda_2 - \lambda_1$

Waveguide Dimension and Chromatic Dispersion



$$D_w = \frac{n_2 \Delta}{c\lambda}\left[V \frac{d^2(bV)}{dV^2}\right]$$

$$D_w \approx -\frac{0.025\lambda}{a^2 c n_2}$$

$$D_w(\mathrm{ps\,nm^{-1}\,km^{-1}}) \approx -\frac{83.76\,\lambda(\mu\mathrm{m})}{[a(\mu\mathrm{m})]^2 n_2}$$

Waveguide dispersion depends on the guide properties

Profile Dispersion

Group velocity $v_g(01)$ of the fundamental mode depends on Δ, refractive index difference.

Δ may not be constant over a range of wavelengths: $\Delta = \Delta(\lambda)$

$$\frac{\Delta\tau}{L} = D_p \Delta\lambda$$

D_p = *Profile dispersion coefficient*

$D_p < 0.1 \text{ ps nm}^{-1} \text{ km}^{-1}$

Can generally be ignored

NOTE

Total intramode (chromatic) dispersion coefficient D_{ch}

$$D_{ch} = D_m + D_w + D_p$$

where D_m, D_w, D_p are material, waveguide and profile dispersion coefficients respectively

Chromatic Dispersion

$$D_{ch} = D_m + D_w + D_p$$

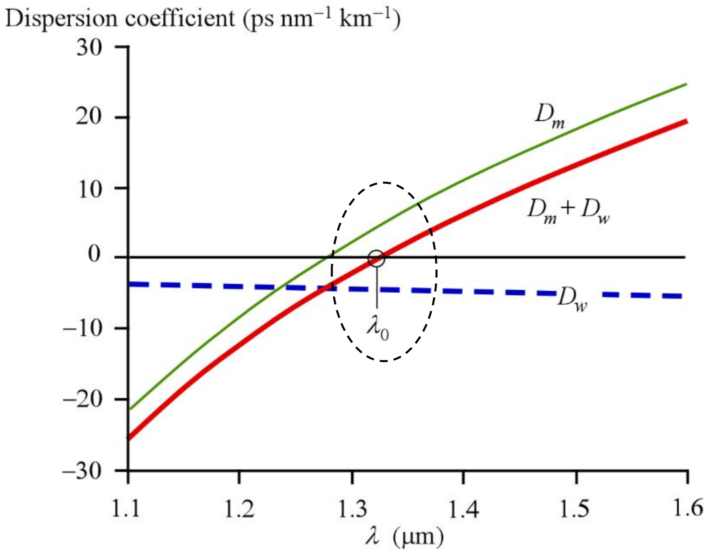
$$\frac{\Delta\tau}{L} = D_{ch}\Delta\lambda$$

S_0 = Chromatic dispersion slope at λ_0

Chromatic dispersion is zero at $\lambda = \lambda_0$

$$D_{ch} = \frac{S_0\lambda}{4}\left[1 - \left(\frac{\lambda_0}{\lambda}\right)^4\right]$$

Is dispersion really zero at λ_0?



The cause of $\Delta\tau$ is the wavelength spread $\Delta\lambda$ at the input

$$\Delta\tau = f(\Delta\lambda)$$

$$\Delta\tau = \tau(\lambda) - \tau(\lambda_0) = \left[\frac{d\tau}{d\lambda}\right]_{\lambda_0}(\Delta\lambda) + \frac{1}{2!}\left[\frac{d^2\tau}{d\lambda^2}\right]_{\lambda_0}(\Delta\lambda)^2 + \cdots$$

$$\frac{d\Delta\tau}{L d\lambda} = D_{ch}$$

$$\frac{dD_{ch}}{d\lambda} = S_0$$

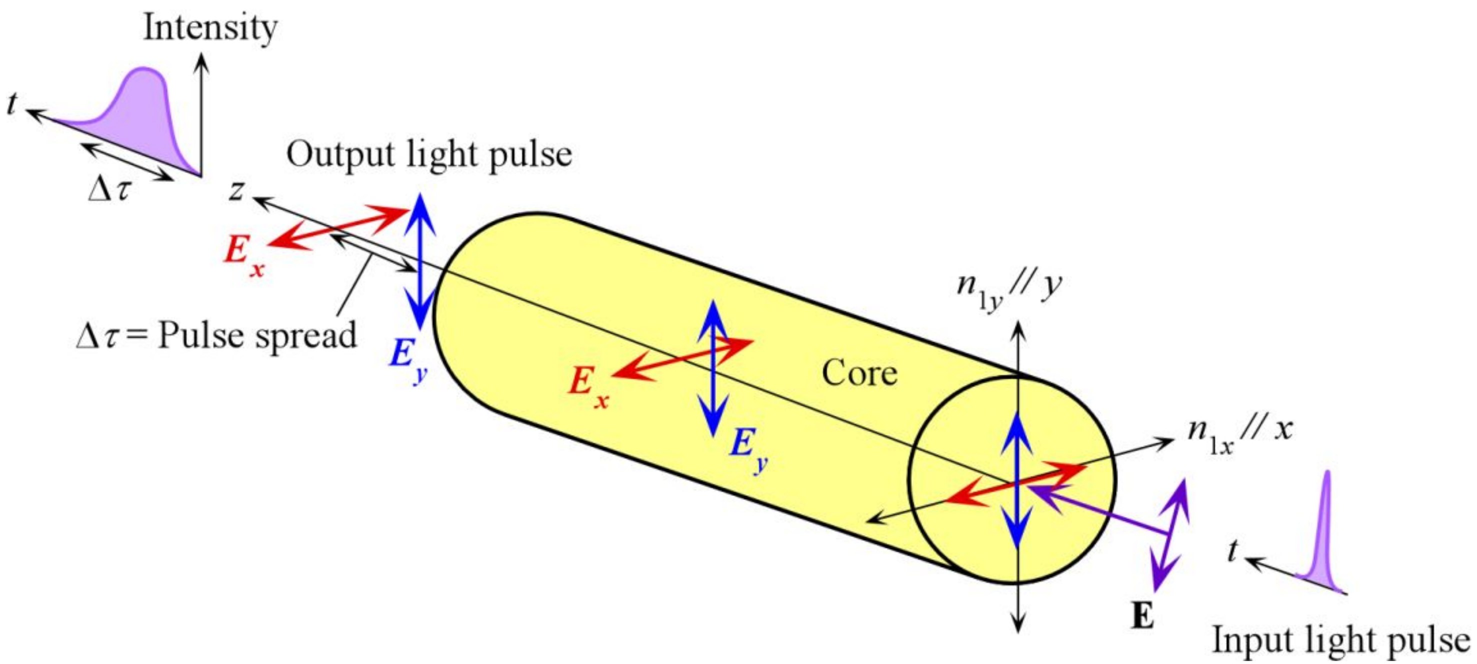
$$= 0$$

$$\Delta\tau = [L D_{ch}(\lambda_0)]\Delta\lambda + \frac{1}{2}\left[\frac{d}{d\lambda}\left(\frac{d\tau}{d\lambda}\right)\right]_{\lambda_0}(\Delta\lambda)^2$$

$$\Delta\tau = \frac{L}{2}S_0(\Delta\lambda)^2 = \frac{1\,\text{km}}{2}\left(0.090\,\text{ps}\,\text{nm}^{-2}\,\text{km}^{-1}\right)\left(2\,\text{nm}\right)^2 = 1.01\,\text{ps}$$

Polarization Dispersion

n different in different directions due to induced strains in fiber in manufacturing, handling and cabling. $\delta n/n < 10^{-6}$



$$\Delta\tau = D_{\text{PMD}}\sqrt{L}$$

D_{PMD} = Polarization dispersion coefficient

Typically $D_{\text{PMD}} = 0.1 - 0.5$ ps nm^{-1} km$^{-1/2}$

Self-Phase Modulation Dispersion : Nonlinear Effect

At sufficiently high light intensities, the refractive index of glass n' is

$n' = n + CI$

where C is a constant and I is the light intensity. **The intensity of light modulates its own phase.**

$$\frac{\Delta\tau}{L} \approx \frac{C\Delta I}{c}$$

What is the optical power that will give $\Delta\tau/L \approx$ 0.1 ps km^{-1}?

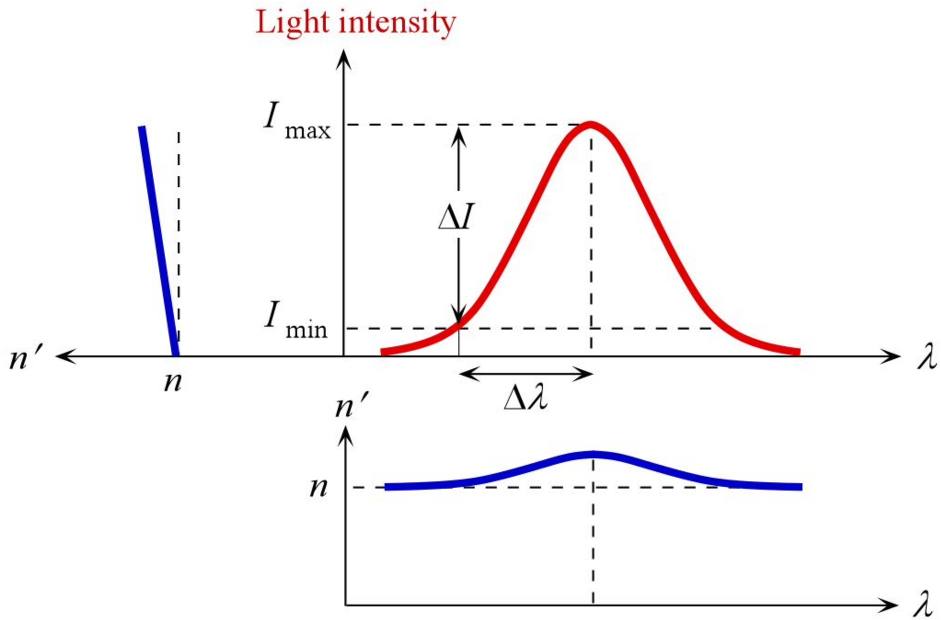
Take $C = 10^{-14}$ cm^2 W^{-1}

$\therefore\ \Delta I \approx (c/C)(\Delta\tau/L) = 3\times10^6$ W cm^{-2}

or $\Delta n \approx 3\times10^{-6}$

Given $2a \approx 10$ μm, $A \approx 7.85\times10^{-7}$ cm^2

$\therefore$ Optical power ≈ 2.35 W in the core



In many cases, this dispersion will be less than other dispersion mechanisms

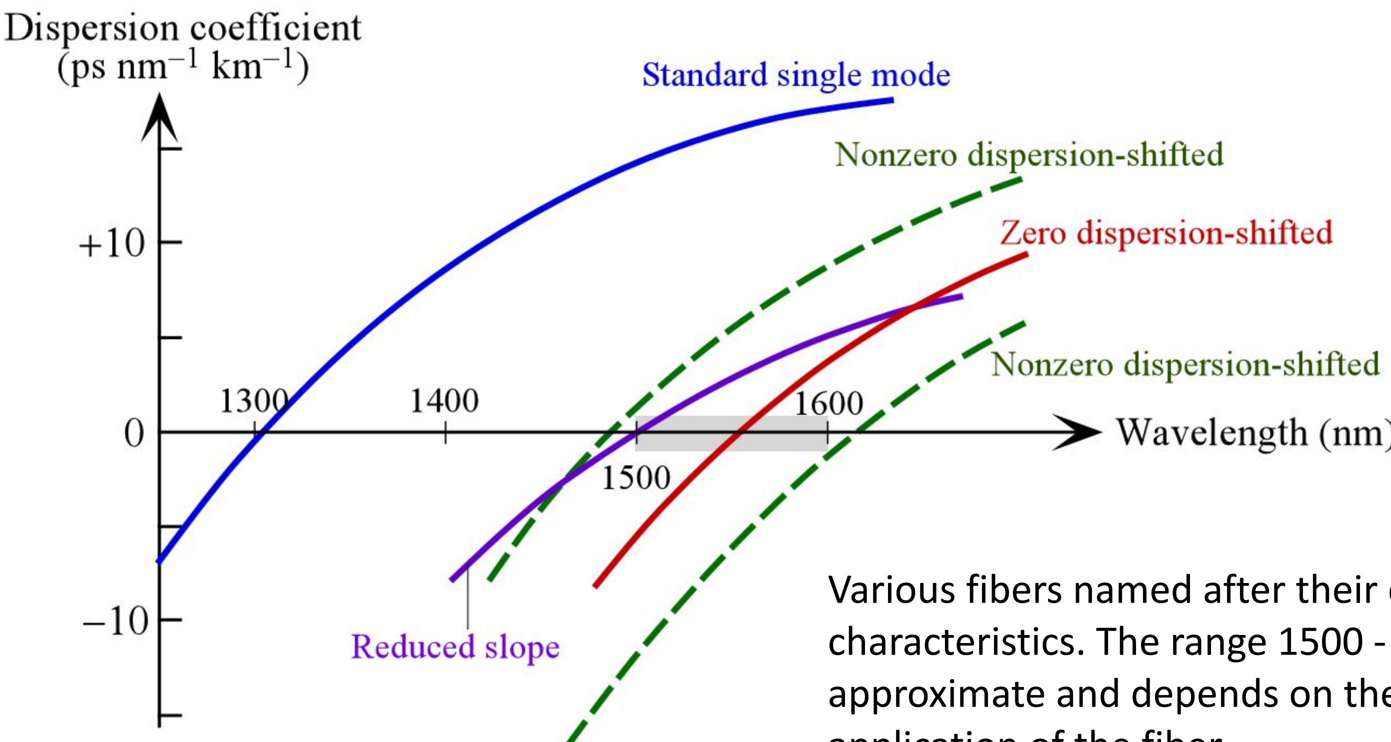
Nonzero Dispersion Shifted Fiber

For Wavelength Division Multiplexing (WDM) avoid 4 wave mixing: cross talk.

We need dispersion not zero but very small in Er-amplifer band (1525-1620 nm)

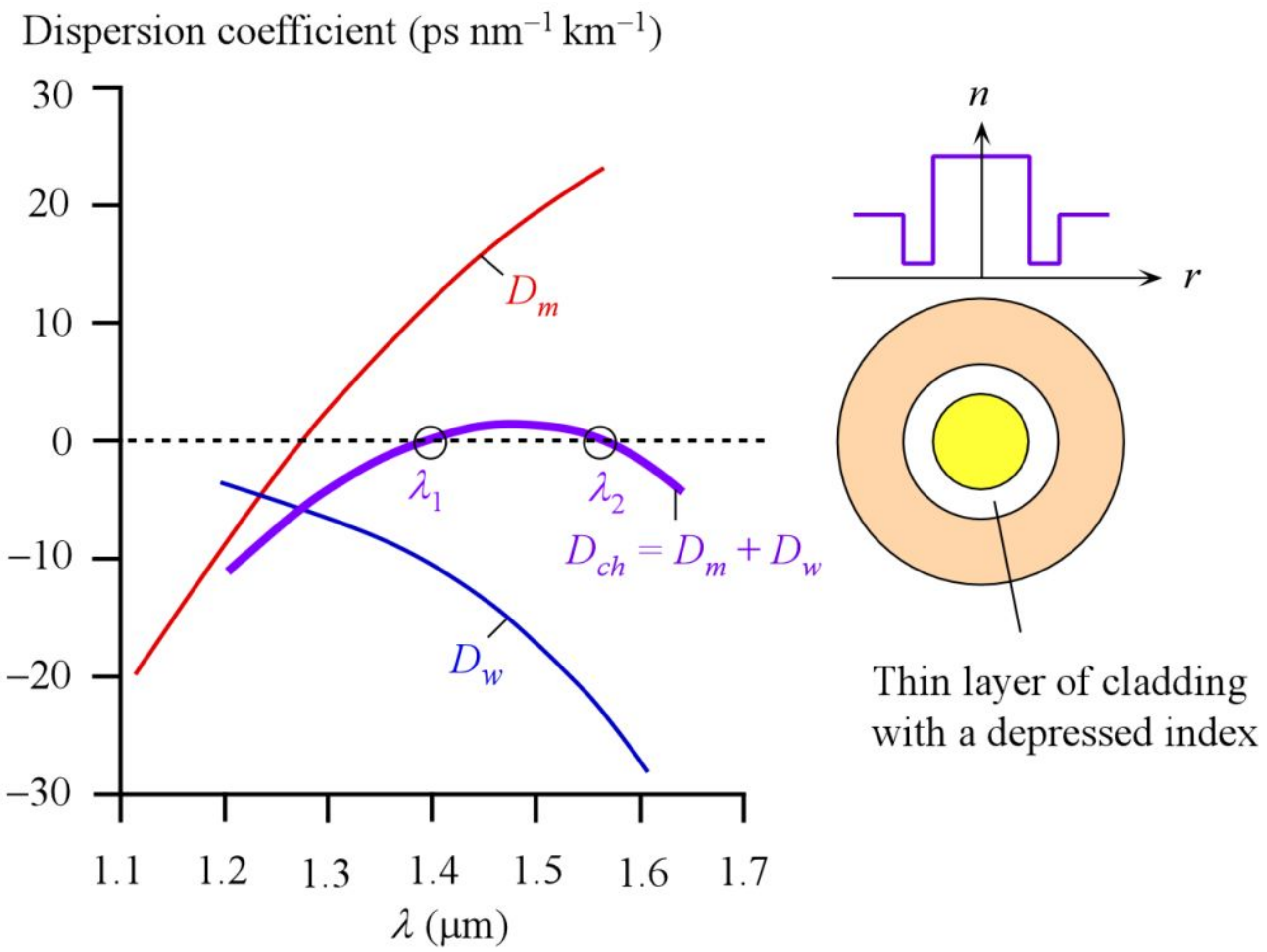
$D_{ch} = 0.1 - 6$ ps nm^{-1} km^{-1}.

Nonzero dispersion shifted fibers



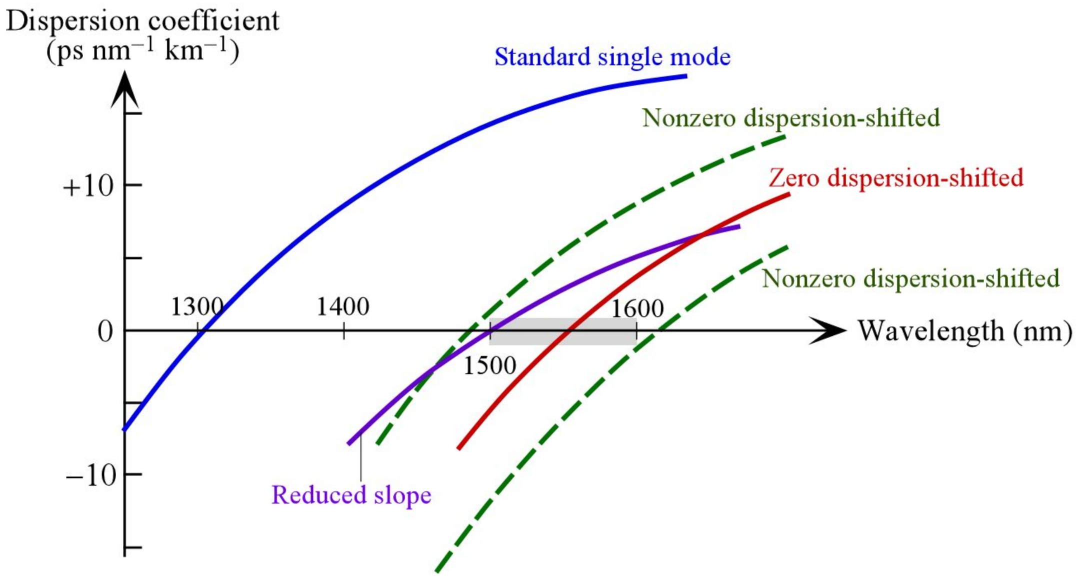
Various fibers named after their dispersion characteristics. The range 1500 - 1600 nm is only approximate and depends on the particular application of the fiber.

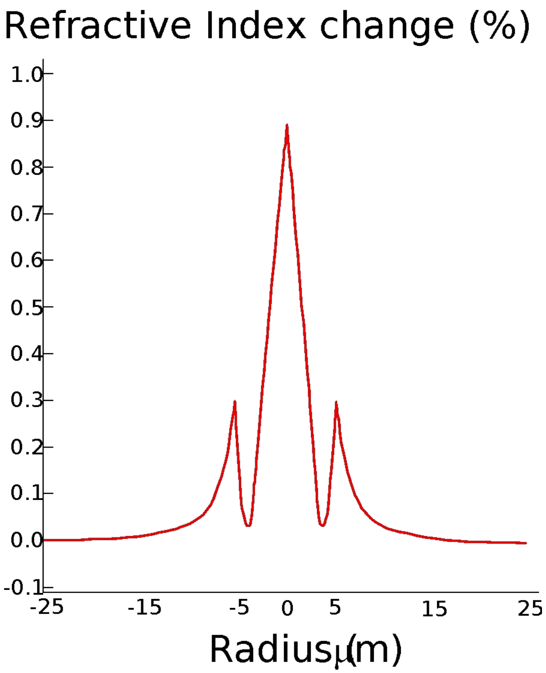
Dispersion Flattened Fiber



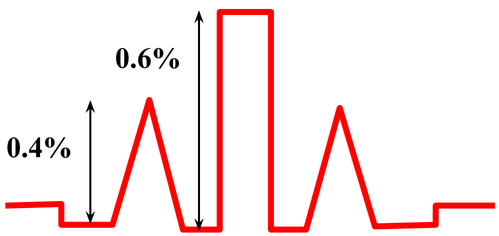
Dispersion flattened fiber example. The material dispersion coefficient (D_m) for the core material and waveguide dispersion coefficient (D_w) for the doubly clad fiber result in a flattened small chromatic dispersion between λ_1 and λ_2.

Nonzero Dispersion Shifted Fiber: More Examples





Nonzero dispersion shifted fiber (Corning)



Fiber with flattened dispersion slope (schematic)

Commercial Fibers for Optical Communications

Fiber	D_{ch} ps nm^{-1} km^{-1}	S_0 ps nm^{-2} km^{-1}	D_{PMD} ps km$^{-1/2}$	**Some attributes**
Standard single mode, ITU-T G.652	17 (1550 nm)	≤ 0.093	< 0.5 (cabled)	D_{ch} = 0 at $\lambda_0 \approx$ 1312 nm, MFD = 8.6 - 9.5 μm at 1310 nm. $\lambda_c \leq$1260 nm.
Non-zero dispersion shifted fiber, ITU-T G.655	0.1 − 6 (1530 nm)	< 0.05 at 1550 nm	< 0.5 (cabled)	For 1500 - 1600 nm range. WDM application MFD = 8 − 11 μm.
Non-zero dispersion shifted fiber, ITU-T G.656	2 − 14	< 0.045 at 1550 nm	< 0.20 (cabled)	For 1460 - 1625 nm range. DWDM application. MFD = 7 − 11 μm (at 1550 nm). Positive D_{ch}. λ_c <1310 nm
Corning SMF28e+ (Standard SMF)	18 (1550 nm)	0.088	< 0.1	Satisfies G.652. $\lambda_0 \approx$ 1317 nm, MFD = 9.2 μm (at 1310 nm), 10.4 μm (at 1550 nm); $\lambda_c \leq$ 1260 nm.
OFS TrueWave RS Fiber	2.6 - 8.9	0.045	0.02	Satisfies G.655. Optimized for 1530 nm - 1625nm. MFD = 8.4 μm (at 1550 nm); $\lambda_c \leq$1260 nm.
OFS REACH Fiber	5.5 -8.9	0.045	0.02	Higher performance than G.655 specification. Satisfies G.656. For DWDM from 1460 to 1625 nm. $\lambda_0 \leq$ 1405 nm. MFD = 8.6 μm (at 1550 nm)

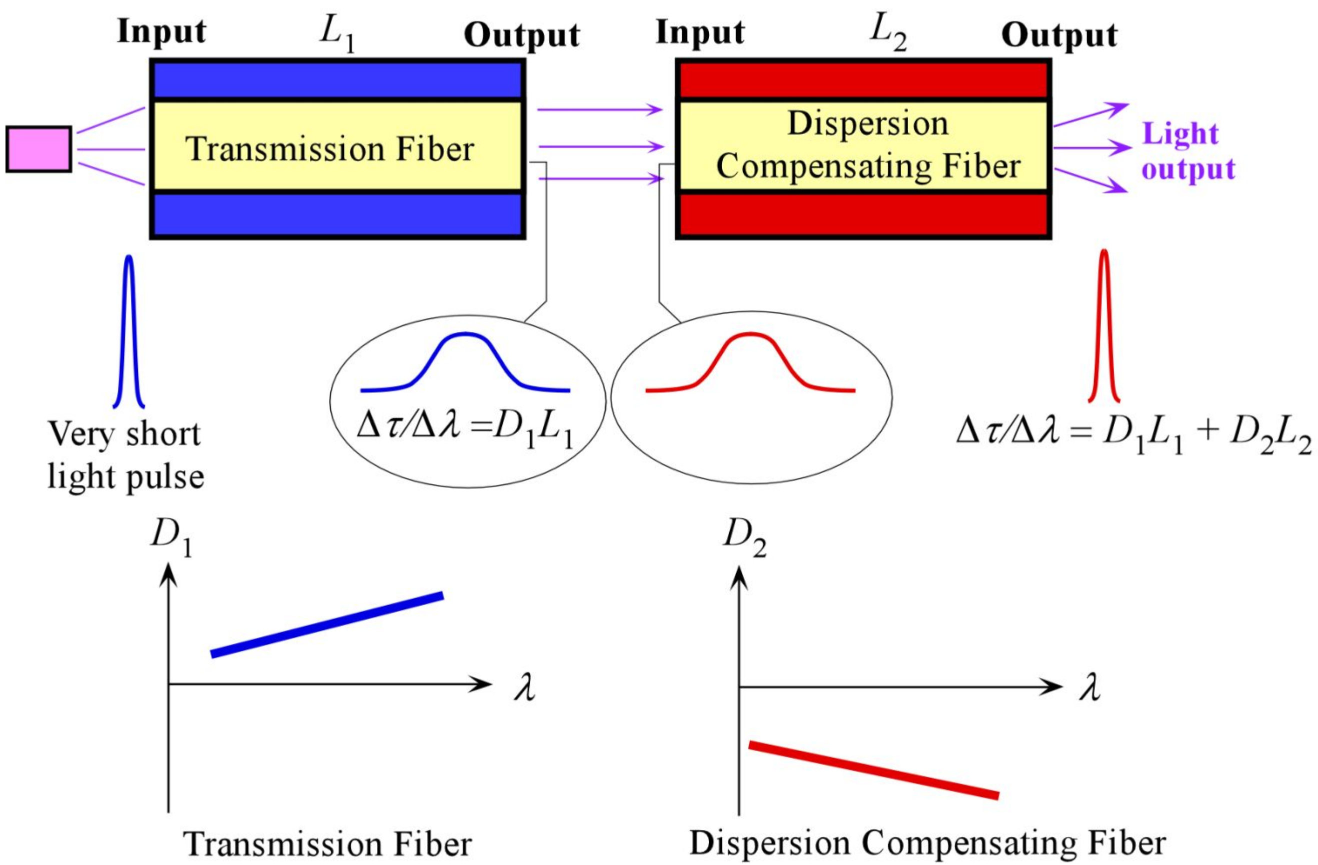
Single Mode Fibers: Selected Examples

TABLE 2.3 Selected single-mode fibers

Fiber	D_{ch}(ps nm^{-1} km^{-1})	S_0 (ps nm^{-2} km^{-1})	D_{PMD} (ps km$^{-1/2}$)	Some attributes
Standard single mode, ITU-T G.652	17 (1550 nm)	≤0.093	<0.5 (cabled)	$D_{ch} = 0$ at $\lambda_0 \approx 1312$ nm. MFD = 8.6–9.5 μm at 1310 nm. $\lambda_c \leq 1260$ nm.
Nonzero dispersion-shifted fiber, ITU-T G.655	0.1–6 (1530 nm)	<0.05 at 1550 nm	<0.5 (cabled)	For 1500–1600 nm range. WDM application. MFD = 8–11 μm.
Nonzero dispersion-shifted fiber, ITU-T G.656	2–14	<0.045 at 1550 nm	<0.20 (cabled)	For 1460–1625 nm range. DWDM application. MFD = 7–11 μm (at 1550 nm). Positive D_{ch}. $\lambda_c < 1310$ nm
Corning SMF28e+ (Standard SMF)	18 (1550 nm)	0.088	<0.1	Satisfies G.652. $\lambda_0 \approx 1317$ nm, MFD = 9.2 μm (at 1310 nm), 10.4 μm (at 1550 nm); $\lambda_c \leq 1260$ nm.
OFS TrueWave RS Fiber	2.6–8.9	0.045	0.02	Satisfies G.655. Optimized for 1530–1625 nm. MFD = 8.4 μm (at 1550 nm); $\lambda_c \leq 1260$ nm.
OFS REACH Fiber	5.5–8.9	0.045	0.02	Higher performance than G.655 specification. Satisfies G.656. For DWDM from 1460 to 1625 nm. $\lambda_0 \leq 1405$ nm. MFD = 8.6 μm (at 1550 nm)

Note: ITU-T is the International Telecommunications Union with the suffix T representing the Telecommunication Standardization Sector in ITU. G.652, G.655, and G.656 are their standards for three single-mode fibers: a standard SMF, nonzero dispersion-shifted fibers for WDM (wavelength division multiplexing), and DWDM (dense WDM) applications, respectively. A few selected commercial SMF properties are also given. λ_0 is the wavelength at which $D_{ch} = 0$.

Dispersion Compensation

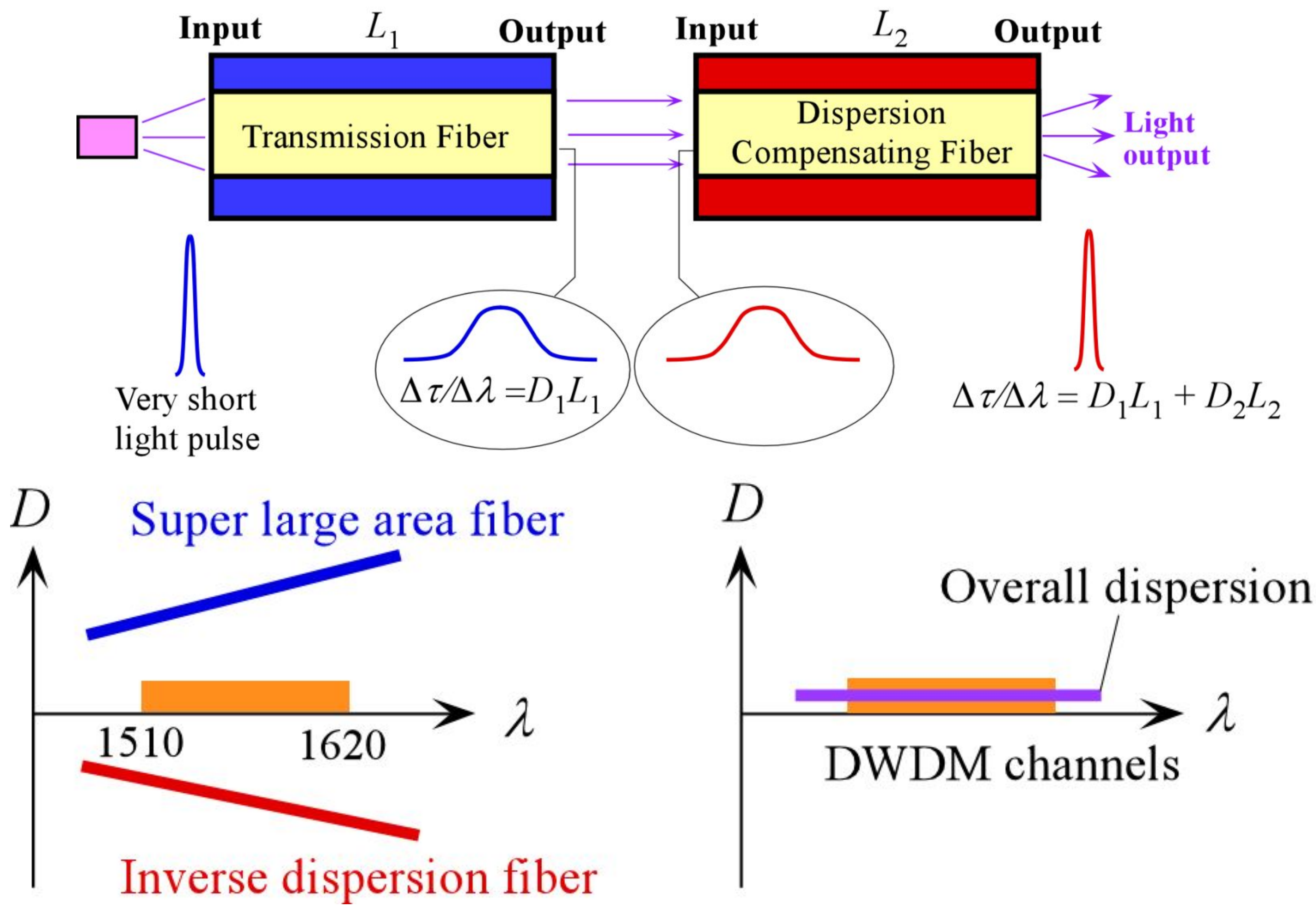


Total dispersion $= D_tL_t + D_cL_c$ **= (10 ps nm^{-1} km^{-1})(1000 km) + (−100 ps nm^{-1} km^{-1})(80 km)**

= 2000 ps/nm for 1080 km

$D_{effective}$ **= 1.9 ps nm^{-1} km^{-1}**

Dispersion Compensation

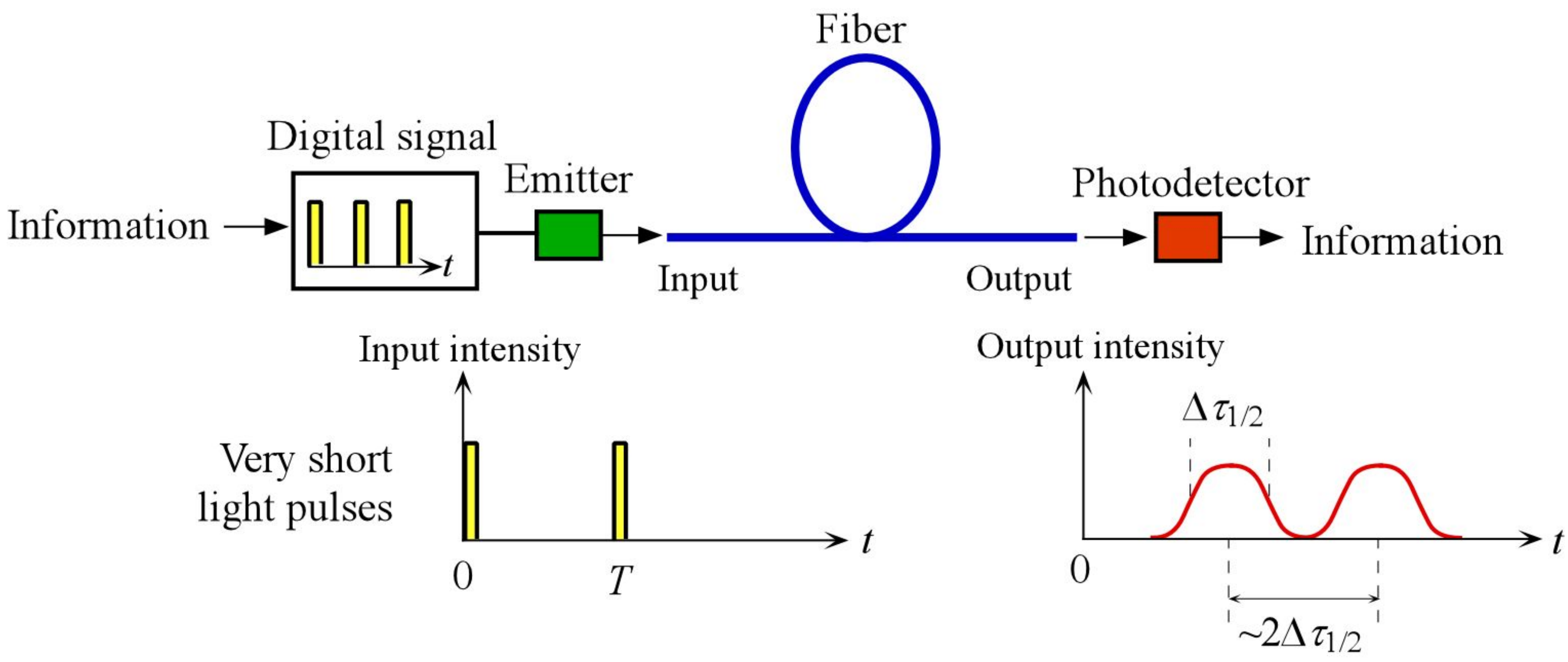


Dispersion *D* vs. wavelength characteristics involved in dispersion compensation. Inverse dispersion fiber enables the dispersion to be reduced and maintained flat over the communication wavelengths.

Dispersion Compensation and Management

- Compensating fiber has higher attenuation.
  Doped core. Need shorter length
- More susceptible to nonlinear effects.
  Use at the receiver end.
- Different cross sections. Splicing/coupling losses.
- Compensation depends on the temperature.
- Manufacturers provide transmission fiber spliced to inverse dispersion fiber for a well defined *D* vs. λ

Dispersion and Maximum Bit Rate

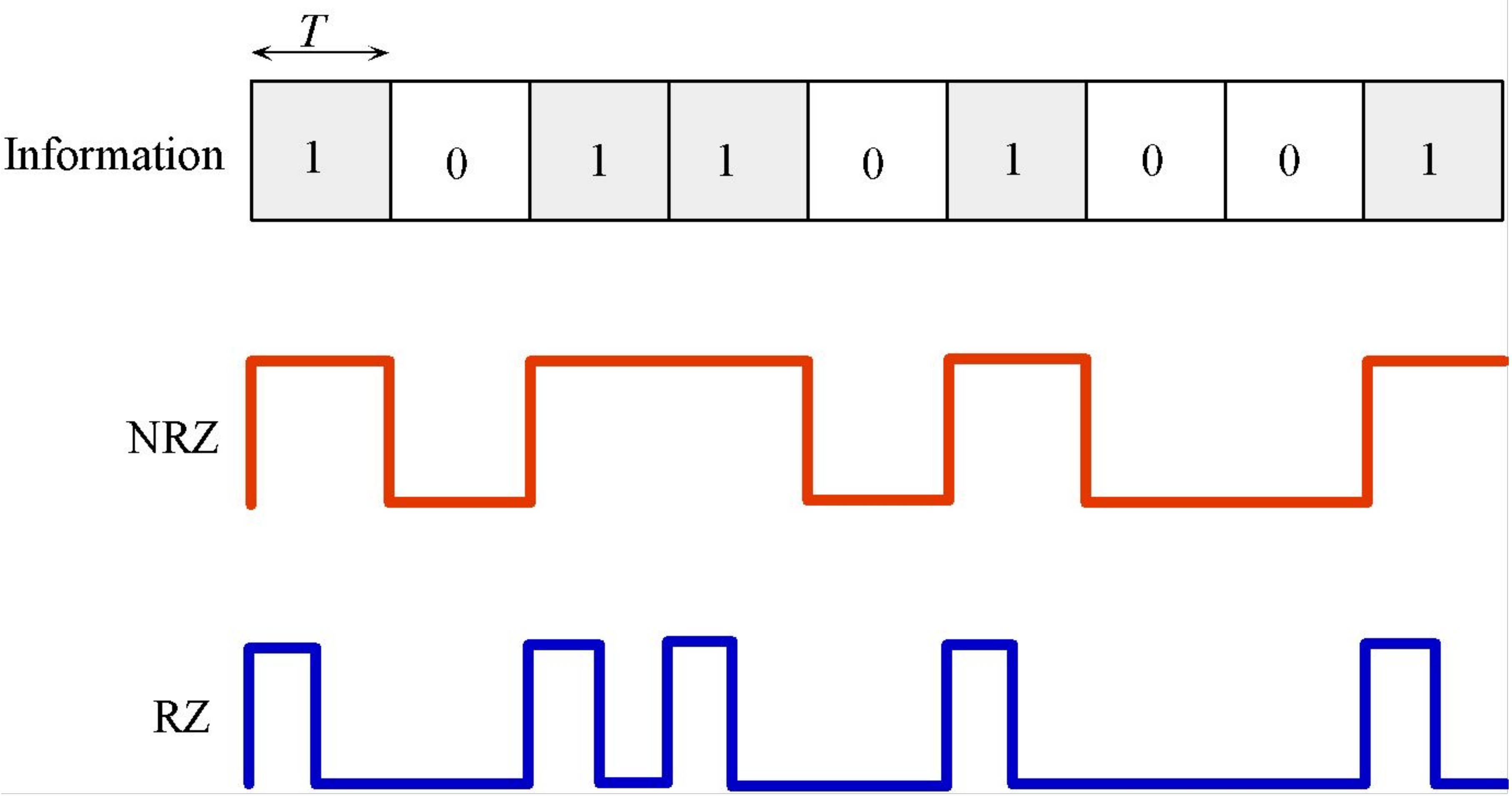


$$B \approx \frac{0.5}{\Delta\tau_{1/2}}$$

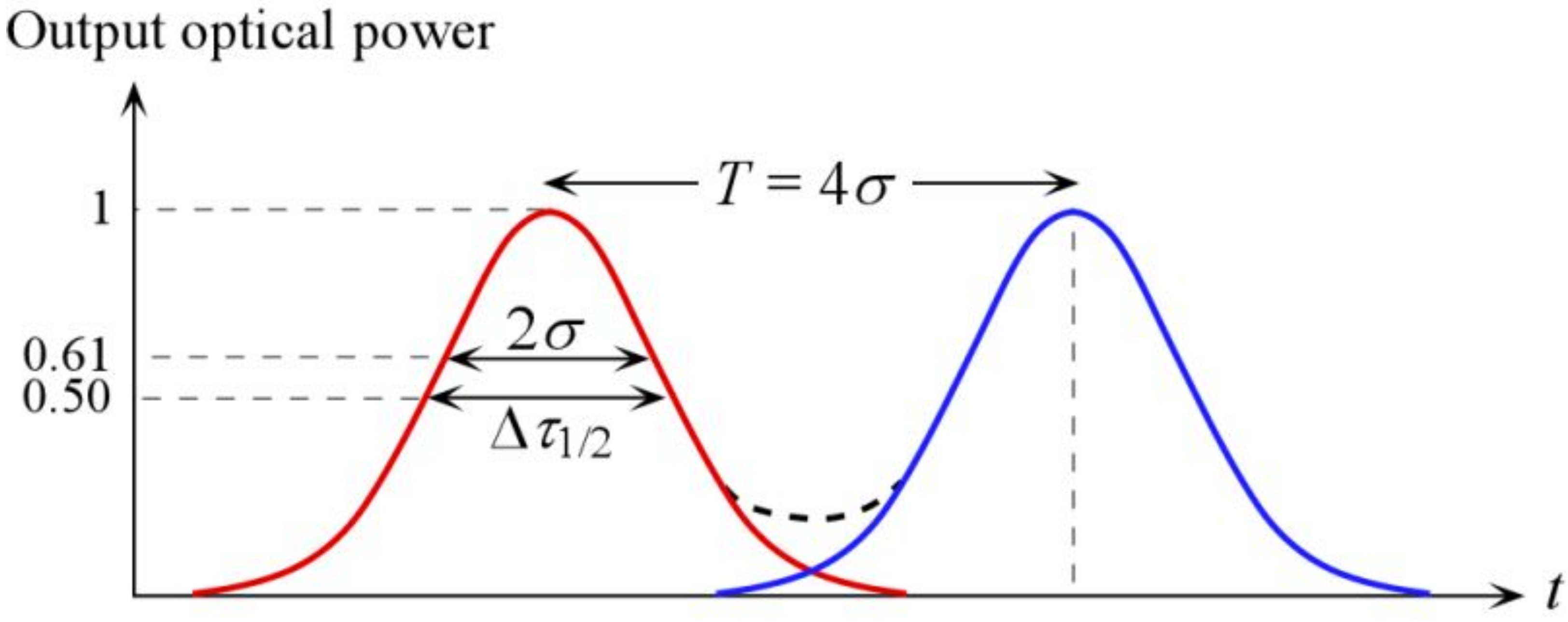
Return-to-zero (RTZ) bit rate or data rate.

Nonreturn to zero (NRZ) bit rate = 2 RTZ bitrate

NRZ and RTZ



Maximum Bit Rate *B*



A Gaussian output light pulse and some tolerable intersymbol interference between two consecutive output light pulses (*y*-axis in relative units). At time *t* = σ from the pulse center, the relative magnitude is $e^{-1/2}$ = 0.607 and full width root mean square (rms) spread is $\Delta\tau_{rms}$ = 2σ. (The RTZ case)

Dispersion and Maximum Bit Rate

Maximum Bit Rate

$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta\tau_{1/2}}$$

Dispersion

$$\frac{\Delta\tau_{1/2}}{L} = D_{ch}\Delta\lambda_{1/2}$$

$$BL \approx \frac{0.59}{\Delta\tau_{1/2}} = \frac{0.59}{|D_{ch}|\Delta\lambda_{1/2}}$$

Bit Rate × Distance is

inversely proportional to dispersion

inversely proportional to line width of laser

(so, we need single frequency lasers!)

Dispersion and Maximum Bit Rate

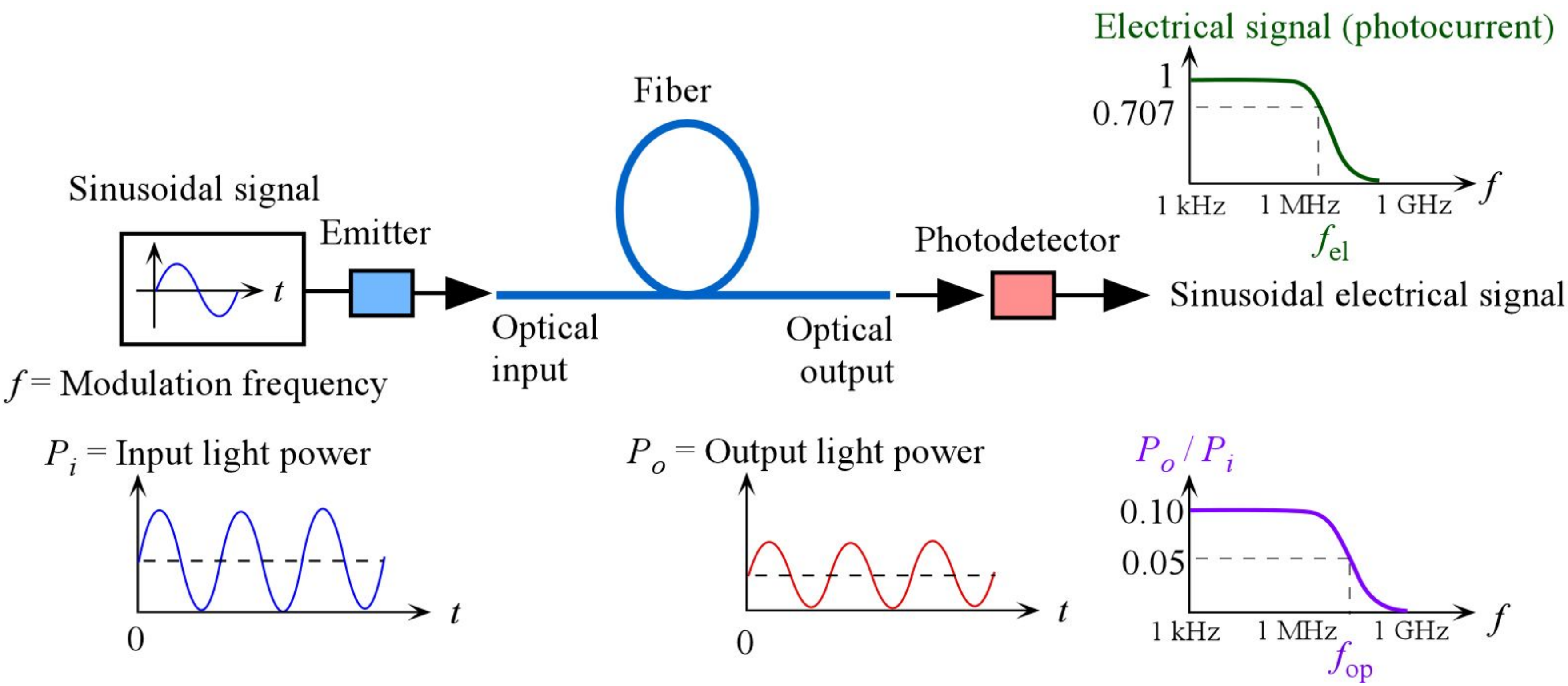
Maximum Bit Rate

$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta\tau_{1/2}}$$

$$\sigma^2 = \sigma^2_{\text{intermodal}} + \sigma^2_{\text{intramodal}}$$

$$(\Delta\tau_{1/2})^2 = (\Delta\tau_{1/2})^2_{\text{intermodal}} + (\Delta\tau_{1/2})^2_{\text{intramodal}}$$

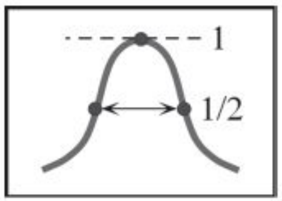
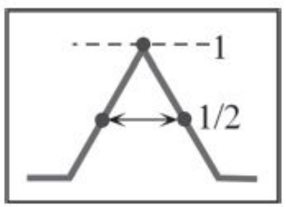
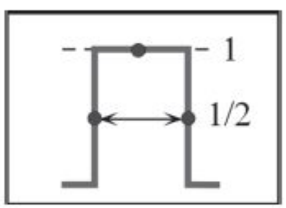
Optical Bandwidth



An optical fiber link for transmitting analog signals and the effect of dispersion in the fiber on the bandwidth, f_{op}.

Pulse Shape and Maximum Bit Rate

TABLE 2.4 Relationships between dispersion parameters, maximum bit rates, and bandwidths

Dispersed pulse shape	Pulse shape and FWHM width, $\Delta\tau_{1/2}$	$\Delta\tau_{1/2}$ FWHM width	B (RZ)	f_{op}
Gaussian with rms deviation σ		$\Delta\tau_{1/2} = 2.353\sigma$ $\sigma = 0.425\Delta\tau_{1/2}$	$0.25/\sigma$	$0.75B = 0.19/\sigma$
Triangular pulse with full-width ΔT		$\Delta\tau_{1/2} = 0.5\Delta T = (6^{1/2})\sigma$ $\sigma = \Delta\tau_{1/2}/2.45 = 0.408\Delta\tau_{1/2}$	$0.25/\sigma$	$0.99B = 0.247/\sigma$
Rectangular with full-width ΔT		$\sigma = 0.289\Delta T = 0.289\Delta\tau_{1/2}$ $\Delta\tau_{1/2} = \Delta T = (2)(3^{1/2})\sigma$	$<1/\Delta T$	$0.69B = 0.17/\sigma$

(*Source:* Data from J. Gowar, *Optical Communication Systems*, 2nd Edition (Prentice Hall, Pearson Education, 1993) Chapter 1.)

Note: RZ = Return-to-zero pulses.

Example: Bit rate and dispersion

Consider an optical fiber with a chromatic dispersion coefficient 8 ps km^{-1} nm^{-1} at an operating wavelength of 1.5 μm. Calculate the bit rate distance product (*BL*), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta\lambda_{1/2}$ of 2 nm is used.

Solution

For FWHP dispersion,

$$\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2} = (8 \text{ ps nm}^{-1} \text{ km}^{-1})(2 \text{ nm}) = 16 \text{ ps km}^{-1}$$

Assuming a Gaussian light pulse shape, the RTZ bit rate × distance product (*BL*) is

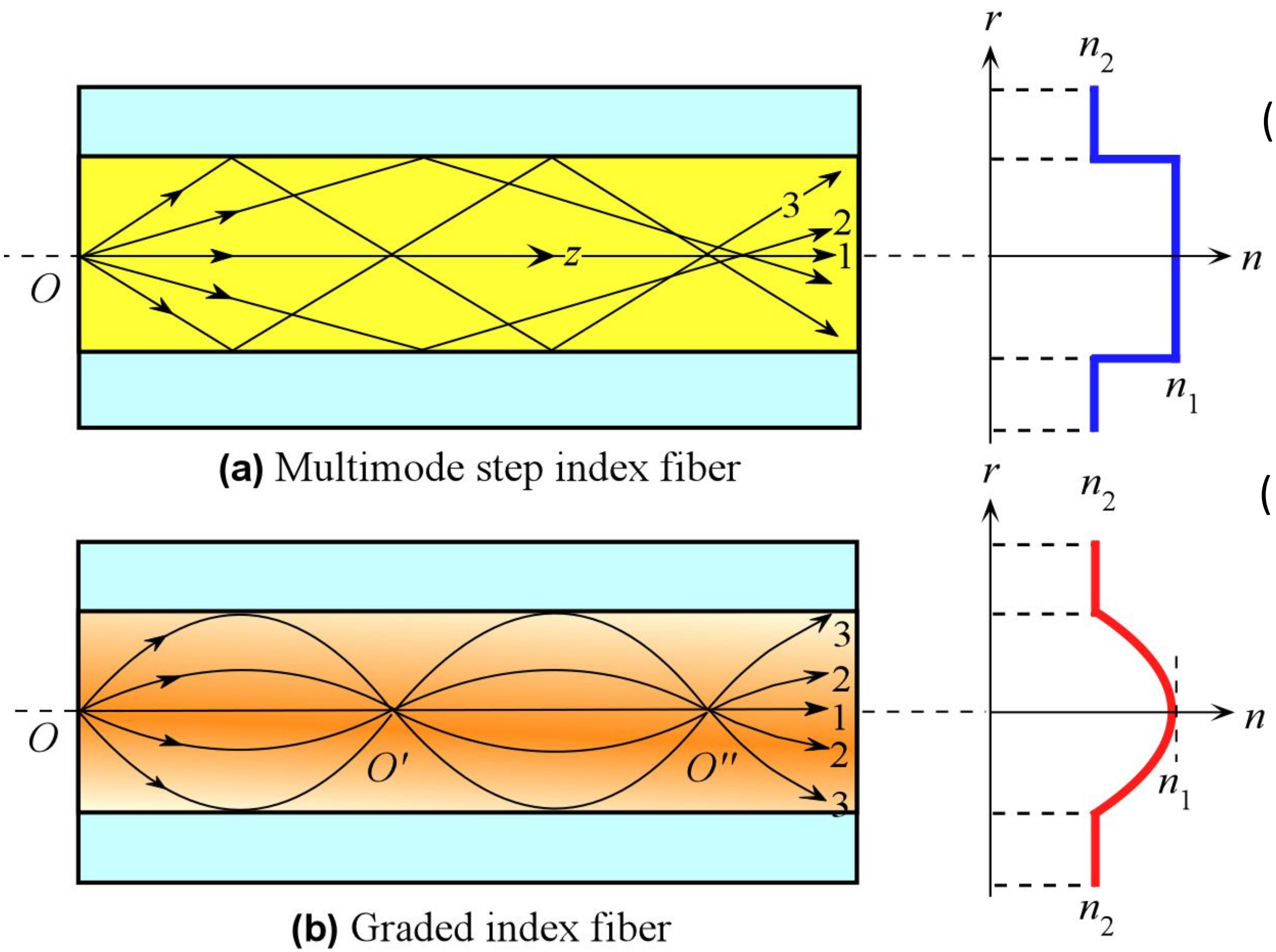
$$BL = 0.59L/\Delta\tau_{1/2} = 0.59/(16 \text{ ps km}^{-1}) = 36.9 \text{ Gb s}^{-1} \text{ km}$$

The optical and electrical bandwidths for a 10 km fiber are

$$f_{\text{op}} = 0.75B = 0.75(36.9 \text{ Gb s}^{-1} \text{ km}) / (10 \text{ km}) = 2.8 \text{ GHz}$$

$$f_{\text{el}} = 0.70f_{op} = 1.9 \text{ GHz}$$

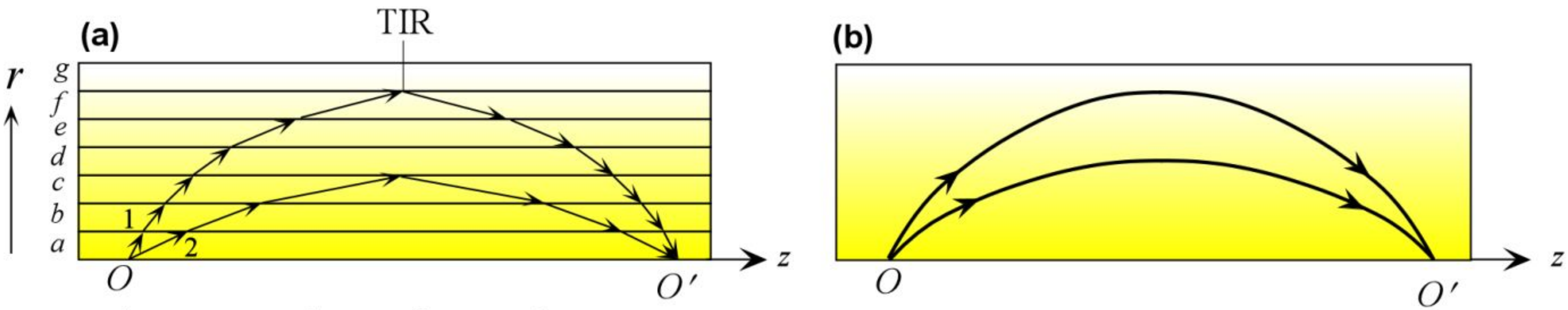
Graded Index (GRIN) Fiber



(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

Graded Index (GRIN) Fiber



n decreases step by step from one layer to next upper layer

Continuous decrease in *n* gives a ray path changing continuously.

(a) A ray in thinly stratifed medium becomes refracted as it passes from one layer to the next upper layer with lower *n* and eventually its angle satisfies TIR.

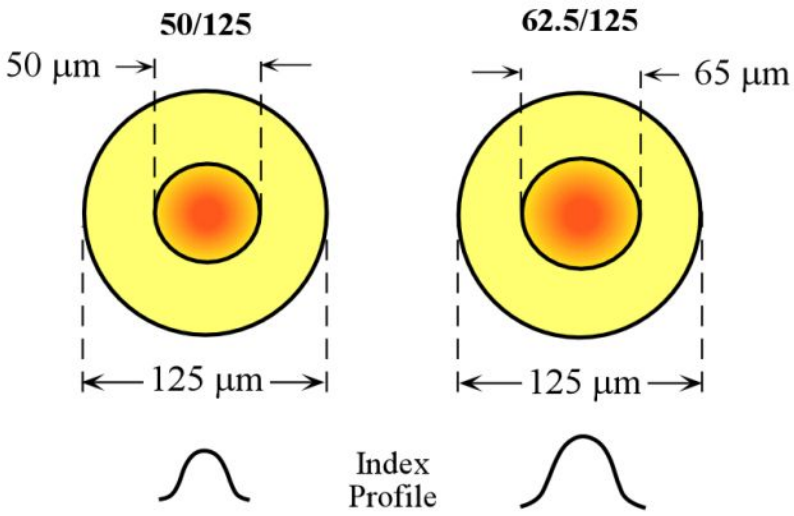
(b) In a medium where *n* decreases continuously the path of the ray bends continuously.

Graded Index (GRIN) Fiber

The refractive index profile can generally be described by a power law with an index γ called the **profile index** (or **the coefficient of index grating**) so that,

$$n = n_1[1 - 2\Delta(r/a)^{\gamma}]^{1/2} \;\; ; \;\; r < a,$$

$$n = n_2 \quad ; r \geq a$$



$$\gamma_o \approx 2 + \delta - \Delta \frac{(4+\delta)(3+\delta)}{5+2\delta} \approx 2$$

Minimum intermodal dispersion

$$\frac{\sigma_{intermode}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2$$

Minimum intermodal dispersion

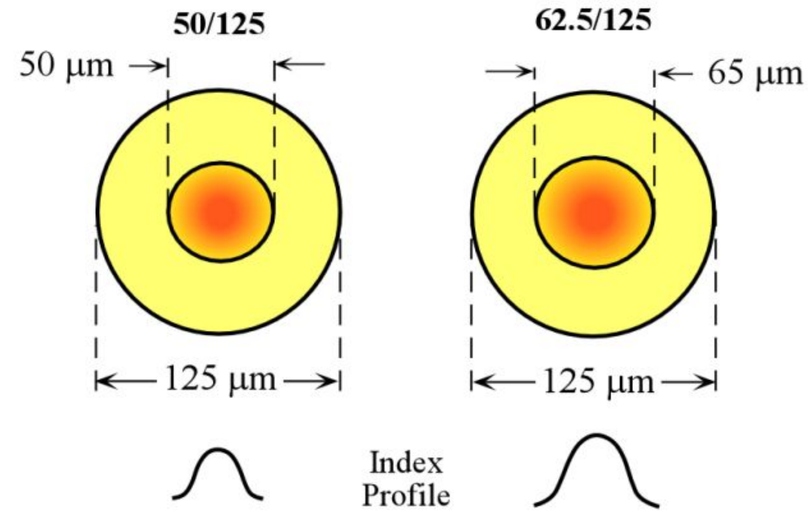
Graded Index (GRIN) Fiber

Minimum intermodal dispersion

$$\gamma_o \approx 2 + \delta - \Delta \frac{(4+\delta)(3+\delta)}{5+2\delta} \approx 2$$

Profile dispersion parameter

$$\delta = -\left(\frac{n_1 \lambda}{N_{g1} \Delta}\right) \frac{d\Delta}{d\lambda}$$

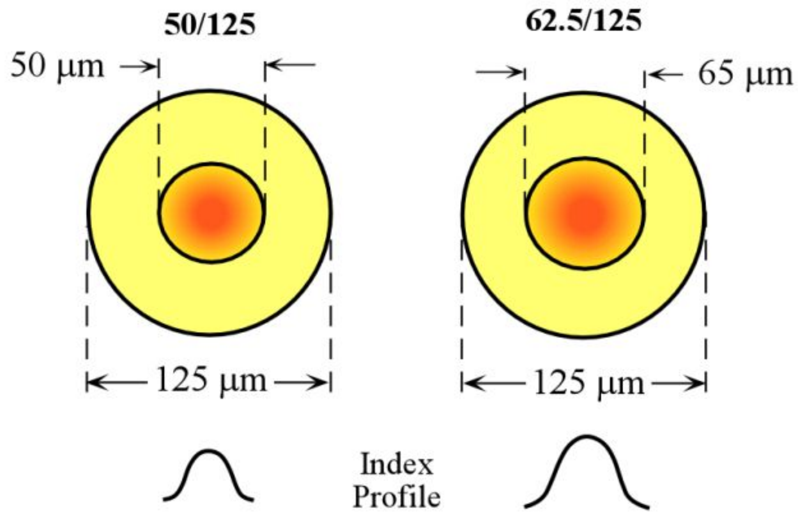


$$\frac{\sigma_{intermode}}{L} \approx \frac{n_1}{20\sqrt{3}c} \Delta^2$$

Minimum intermodal dispersion

Graded Index (GRIN) Fiber

$$\mathrm{NA} = \mathrm{NA}(r) = [n(r)^2 - n_2^2]^{1/2}$$



Effective numerical aperture for GRIN fibers

$$\mathrm{NA}_{\mathrm{GRIN}} \approx (1/2^{1/2})(n_1^2 - n_2^2)^{1/2}$$

Number of modes in a graded index fiber

$$M \approx \left(\frac{\gamma}{\gamma+2}\right)\frac{V^2}{2}$$

Table 2.5
Graded index multimode fibers

d = core diameter (μm), *D* = cladding diameter (μm). Typical properties at 850 nm. VCSEL is a vertical cavity surface emitting laser. α is attenuation along the fiber. OM1, OM3 and OM4 are fiber standards for LAN data links (ethernet). α are reported typical attenuation values. 10G and 40G networks represent data rates of 10 Gb s^{-1} and 40 Gb s^{-1} and correspond to 10 GbE (Gigabit Ethernet) and 40 GbE systems.

MMF d/D	Compliance standard	Source	Typical D_{ch} ps nm^{-1} km^{-1}	Bandwidth MHz·km	NA	α dB km^{-1}	Reach in 10G and 40G networks
50/125	OM4	VCSEL	−100	4700 (EMB)	0.200	< 3	550 m (10G)
				3500 (OFLBW)			150 m (40G)
50/125	OM3	VCSEL	−100	2000 (EMB)	0.200	< 3	300 m (10G)
				500 (OFLBW)			
62.5/125	OM1	LED	−117	200 (OFLBW)	0.275	< 3	33 m (10G)

Example: Dispersion in a GRIN Fiber and Bit Rate

Graded index fiber. Diameter of 50 μm and a refractive index of n_1 = 1.4750, Δ = 0.010.
The fiber is used in LANs at 850 nm with a vertical cavity surface emitting laser (VCSEL) that has very a narrow linewidth that is about 0.4 nm (FWHM). Assume that the chromatic dispersion at 850 nm is −100 ps nm^{-1} km^{-1} as shown in Table 2.5. Assume the fiber has been optimized at 850 nm, and find the minimum rms dispersion. How many modes are there? What would be the upper limit on its bandwidth? What would be the bandwidth in practice?

Solution

Given Δ and n_1, we can find n_2 from

$\Delta = 0.01 = (n_1 - n_2)/n_1 = (1.4750 - n_2)/1.4750.$

$\therefore \quad n_2 = 1.4603.$

The V-number is then

$V = [(2\pi)(25\ \mu m)/(0.850\ \mu m)(1.4750^2 - 1.4603^2)^{1/2} = 38.39$

For the number of modes we can simply take $\gamma = 2$ and use

$M = (V^2/4) = (38.39^2/4) = 368$ modes

The lowest intermodal dispersion for a profile optimized graded index fiber for a 1 km of fiber, L = 1 km, is

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$\frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2 = \frac{1.4750}{20\sqrt{3}(3\times10^8)}(0.010)^2$$

$$= 14.20\times10^{-15} \text{ s m}^{-1} \text{ or } 14.20 \text{ ps km}^{-1}$$

Assuming a triangular output light pulse and the relationship between σ and $\Delta\tau_{1/2}$ given in Table 2.4, the intermodal spread $\Delta\tau_{\text{intermode}}$ (FWHM) in the group delay over 1 km is

$$\Delta\tau_{\text{intermode}} = (6^{1/2})\sigma_{\text{intermode}} = (2.45)(14.20 \text{ ps}) = 34.8 \text{ ps}$$

We also need the material dispersion at the operating wavelength over 1 km, which makes up the intramodal dispersion $\Delta\tau_{\text{intramode}}$ (FWHM)

$$\Delta\tau_{\text{intramode}} = L|D_{ch}|\,\Delta\lambda_{1/2} = (1 \text{ km})(-100 \text{ ps nm}^{-1} \text{ km}^{-1})(0.40 \text{ nm}) = 40.0 \text{ ps}$$

$$\Delta\tau^2 = \Delta\tau^2_{\text{intermode}} + \Delta\tau^2_{\text{intramode}} = (34.8)^2 + (40.0)^2 \Rightarrow \Delta\tau = 53.0 \text{ ps}$$

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$B = \frac{0.25}{\sigma} = \frac{0.25}{0.408\Delta\tau} = \frac{0.61}{\Delta\tau} = \frac{0.61}{(53.0\times10^{-12}\,\mathrm{s})}$$ = **11.5 Gb s^{-1}**

Optical bandwidth f_{op} = 0.99B = **11.4 GHz**

This is the upper limit since we assumed that the graded index fiber is perfectly optimized with $\sigma_{intermode}$ being minimum. Small deviations around the optimum γ cause large increases in $\sigma_{intermode}$, which would sharply reduce the bandwidth.

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{n_1\Delta}{c} = \frac{(1.475)(0.01)}{3\times10^8}$$

$= 4.92\times10^{-11}$ s m^{-1} or 49.2 ns km^{-1}

To calculate the *BL* we use $\sigma_{\text{intermode}} \approx 0.29\Delta\tau$

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{0.29(\Delta\tau / L)} = \frac{0.25}{(0.29)(49.2\times10^{-9}\ \text{s km}^{-1})}$$ = 17.5 Mb s^{-1} km

LANs now use graded index MMFs, and the step index MMFs are used mainly in low speed instrumentation

Example: Dispersion in a graded-index fiber and bit rate

Consider a graded index fiber whose core has a diameter of 50 µm and a refractive index of n_1 = 1.480. The cladding has n_2 = 1.460. If this fiber is used at 1.30 µm with a laser diode that has very a narrow linewidth what will be the bit rate × distance product? Evaluate the *BL* product if this were a multimode step index fiber.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.48 - 1.46)/1.48 = 0.0135$. Dispersion for 1 km of fiber is

$\sigma_{\text{intermode}}/L = n_1\Delta^2/[(20)(3^{1/2})c] = 2.6\times10^{-14}$ s m^{-1} or 0.026 ns km^{-1}.

$BL = 0.25/\sigma_{\text{intermode}}$ = **9.6 Gb s^{-1} km**

We have ignored any material dispersion and, further, we assumed the index variation to perfectly follow the optimal profile which means that in practice *BL* will be worse. (For example, a 15% variation in γ from the optimal value can result in $\sigma_{\text{intermode}}$ and hence *BL* that are more than 10 times worse.)

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be 6.67×10^{-11} s m^{-1} or 66.7 ns km^{-1} and BL = 12.9 **Mb s^{-1} km**

Note: Over long distances, the bit rate × distance product is not constant for multimode fibers and typically $B \propto L^{-\gamma}$ where γ is an index between 0.5 and 1. The reason is that, due to various fiber imperfections, there is mode mixing which reduces the extent of spreading.

Example: Combining intermodal and intramodal dispersions

Consider a graded index fiber with a core diameter of 30 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient D_m at 1300 nm is −7.5 ps nm^{-1} km^{-1}.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$. Modal dispersion for 1 km is

$\sigma_{intermode} = Ln_1\Delta^2/[(20)(3^{1/2})c] = 2.9\times10^{-11}$ s ^{1} or 0.029 ns.

The material dispersion is

$\Delta\tau_{1/2} = LD_m\Delta\lambda_{1/2}$ = (1 km)(7.5 ps nm^{-1} km^{-1})(3 nm) = 0.0225 ns

Assuming a Gaussian output light pulse shape,

$\sigma_{intramode} = 0.425\Delta\tau_{1/2}$ = (0.425)(0.0225 ns) = 0.0096 ns

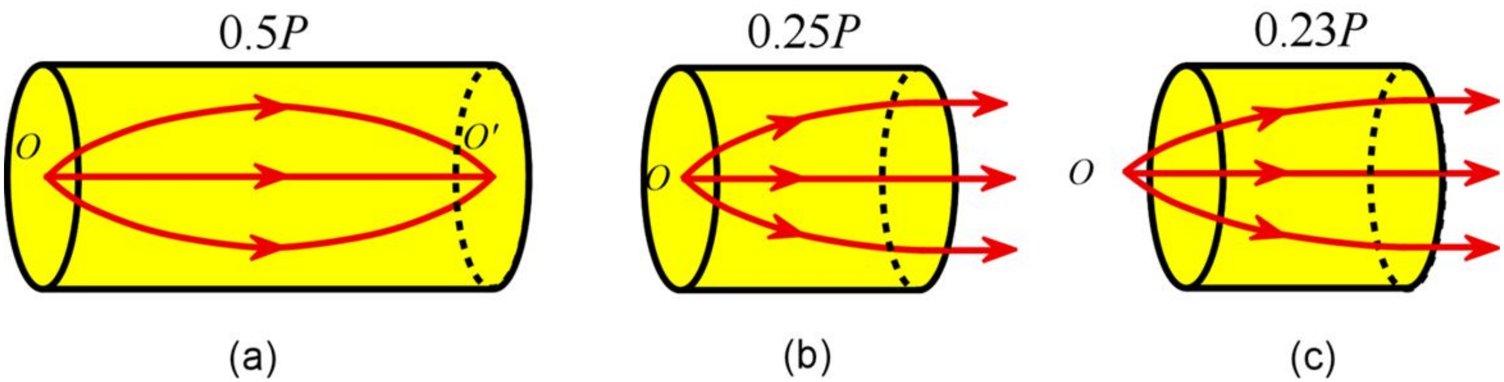
Total dispersion is

$$\sigma_{rms} = \sqrt{\sigma^2_{intermode} + \sigma^2_{intramode}} = \sqrt{0.029^2 + 0.0096^2} = 0.030\ \text{ns}$$

Assume L = 1 km

$B = 0.25/\Delta\tau_{rms}$ = **8.2 Gb**

GRIN Rod Lenses

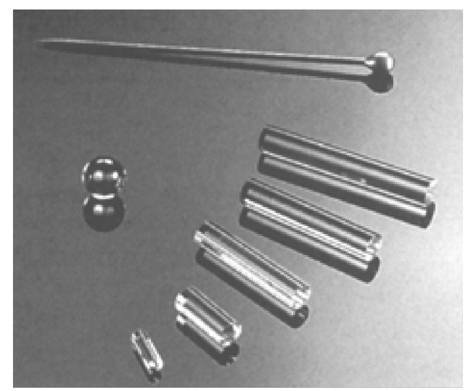


Point *O* is on the rod face center and the lens focuses the rays onto *O′* on to the center of the opposite face.

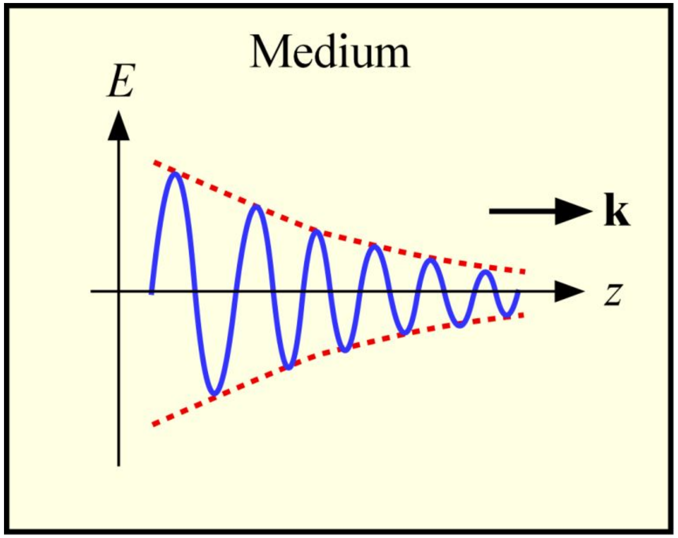
The rays from *O* on the rod face center are collimated out.

***O* is slightly away from the rod face and the rays are collimated out.**

One pitch (*P*) is a full one period variation in the ray trajectory along the rod axis.



Attenuation



The attenuation of light in a medium

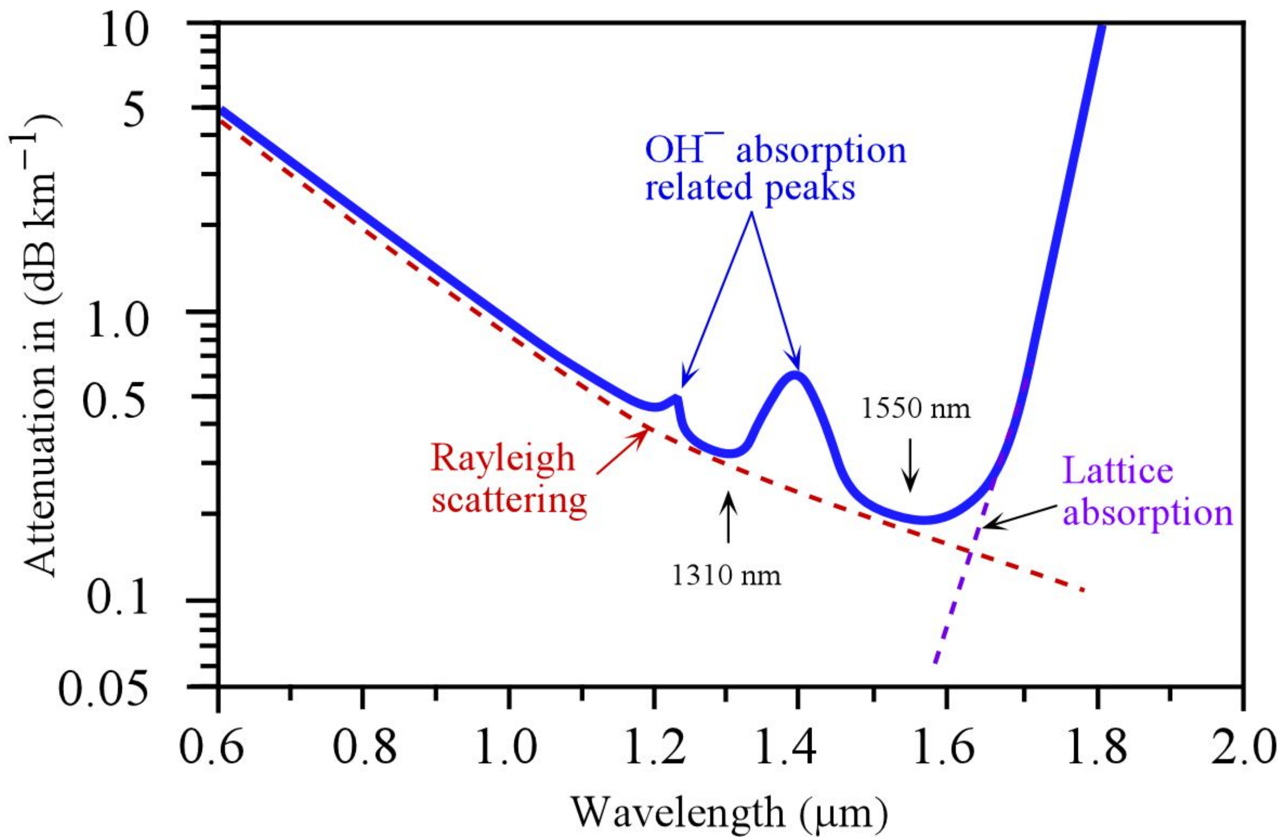
Attenuation = Absorption + Scattering

Attenuation coefficient α is defined as the *fractional decrease in the optical power per unit distance*. α is in m^{-1}.

$$P_{out} = P_{in}\exp(-\alpha L)$$

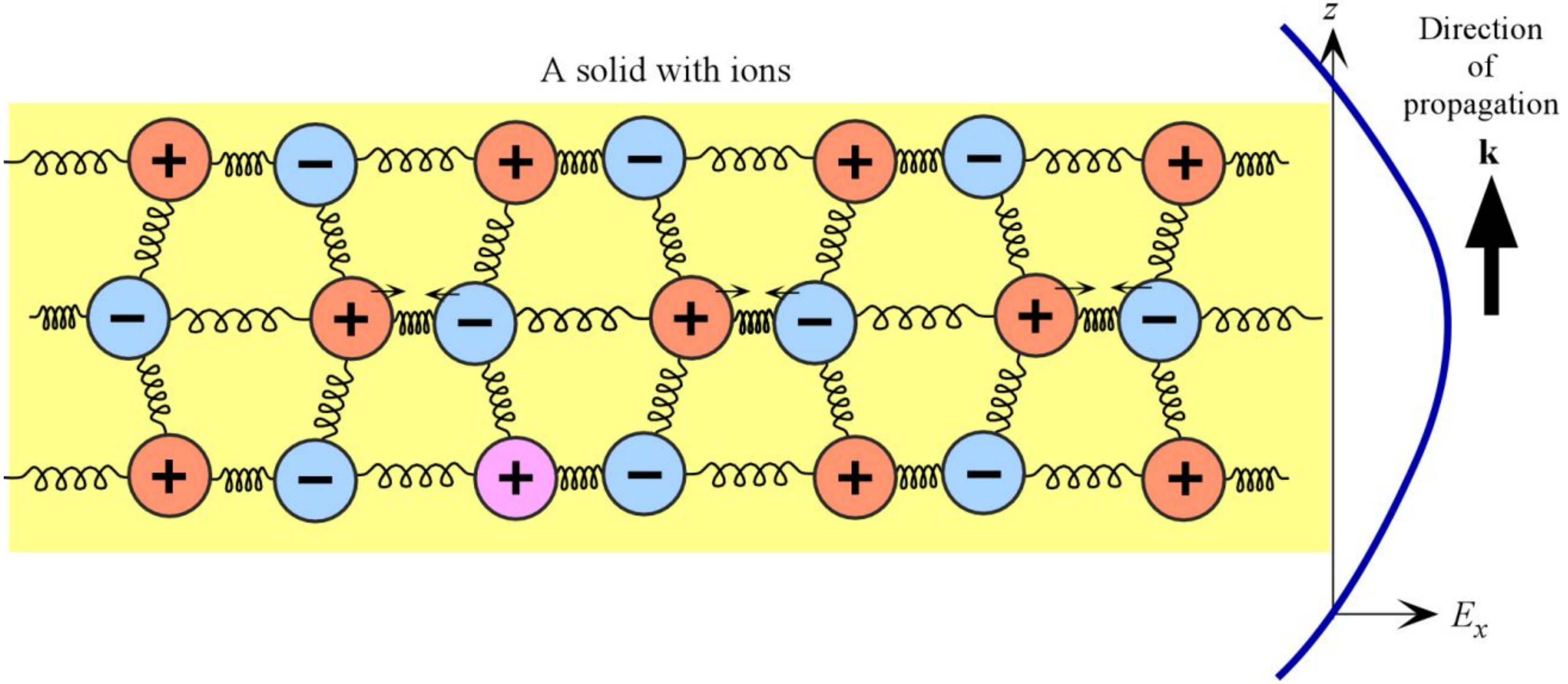
$$\alpha_{dB} = \frac{1}{L} 10\log\left(\frac{P_{in}}{P_{out}}\right) \qquad \alpha_{dB} = \frac{10}{\ln(10)}\alpha = 4.34\alpha$$

Attenuation in Optical Fibers



Attenuation vs. wavelength for a standard silica based fiber.

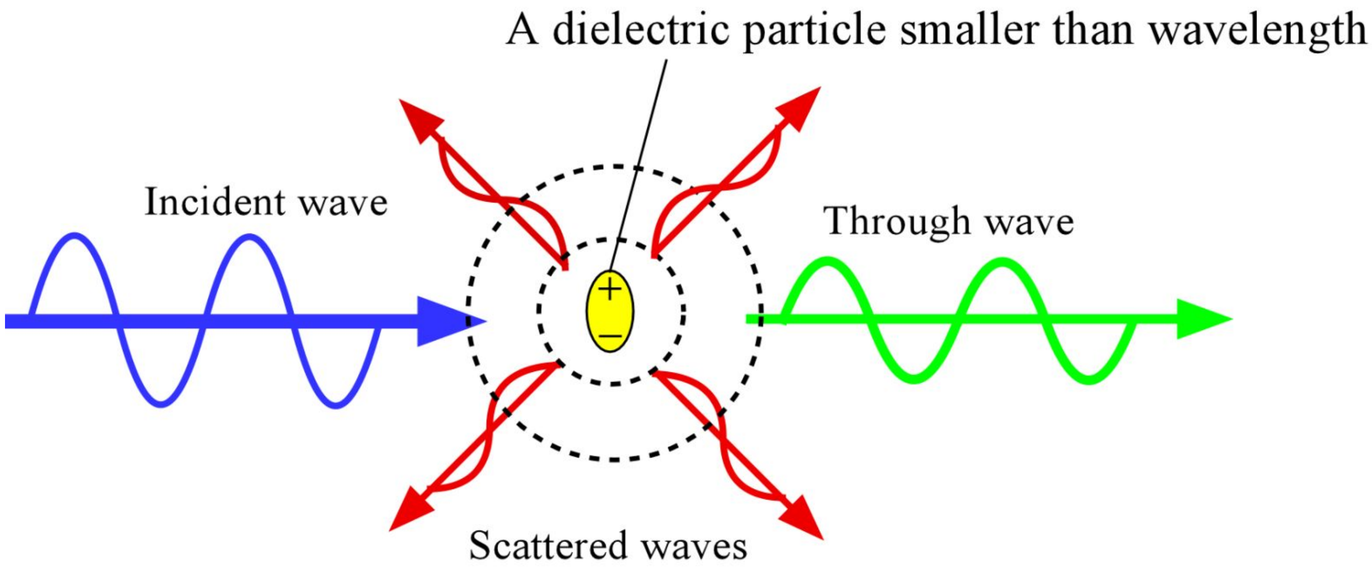
Lattice Absorption (Reststrahlen Absorption)



EM Wave oscillations are coupled to lattice vibrations (phonons), vibrations of the ions in the lattice. Energy is transferred from the EM wave to these lattice vibrations.

This corresponds to "Fundamental Infrared Absorption" in glasses

Rayleigh Scattering



Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2 \beta_T k_B T_f$$

β_T = isothermal compressibility (at T_f)

T_f = *fictive temperature* (roughly the *softening temperature of glass*) where the liquid structure during the cooling of the fiber is frozen to become the glass structure

Example: Rayleigh scattering limit

What is the attenuation due to Rayleigh scattering at around the λ = 1.55 μm window given that pure silica (SiO_2) has the following properties: T_f = 1730°C (softening temperature); $\beta_T = 7\times10^{-11}$ m^2 N^{-1} (at high temperatures); n = 1.4446 at 1.5 μm.

Solution

We simply calculate the Rayleigh scattering attenuation using

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2\beta_T k_B T_f$$

$$\alpha_R \approx \frac{8\pi^3}{3(1.55\times10^{-6})^4}(1.4446^2-1)^2(7\times10^{-11})(1.38\times10^{-23})(1730+273)$$

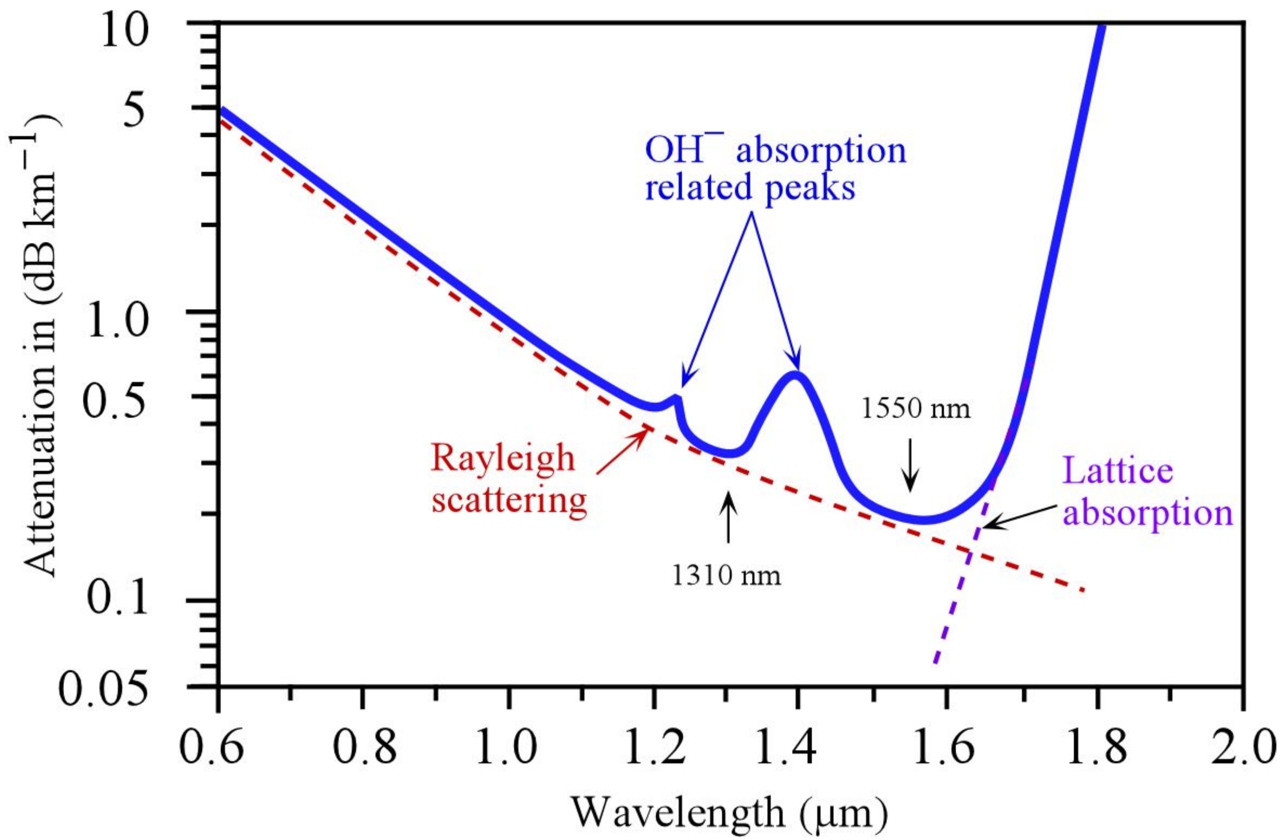
α_R = 3.276×10^{-5} m^{-1} or 3.276×10^{-2} km^{-1}

Attenuation in dB per km is

$\alpha_{dB} = 4.34\alpha_R$ = (4.34)(3.735×10^{-2} km^{-1}) = **0.142 dB km^{-1}**

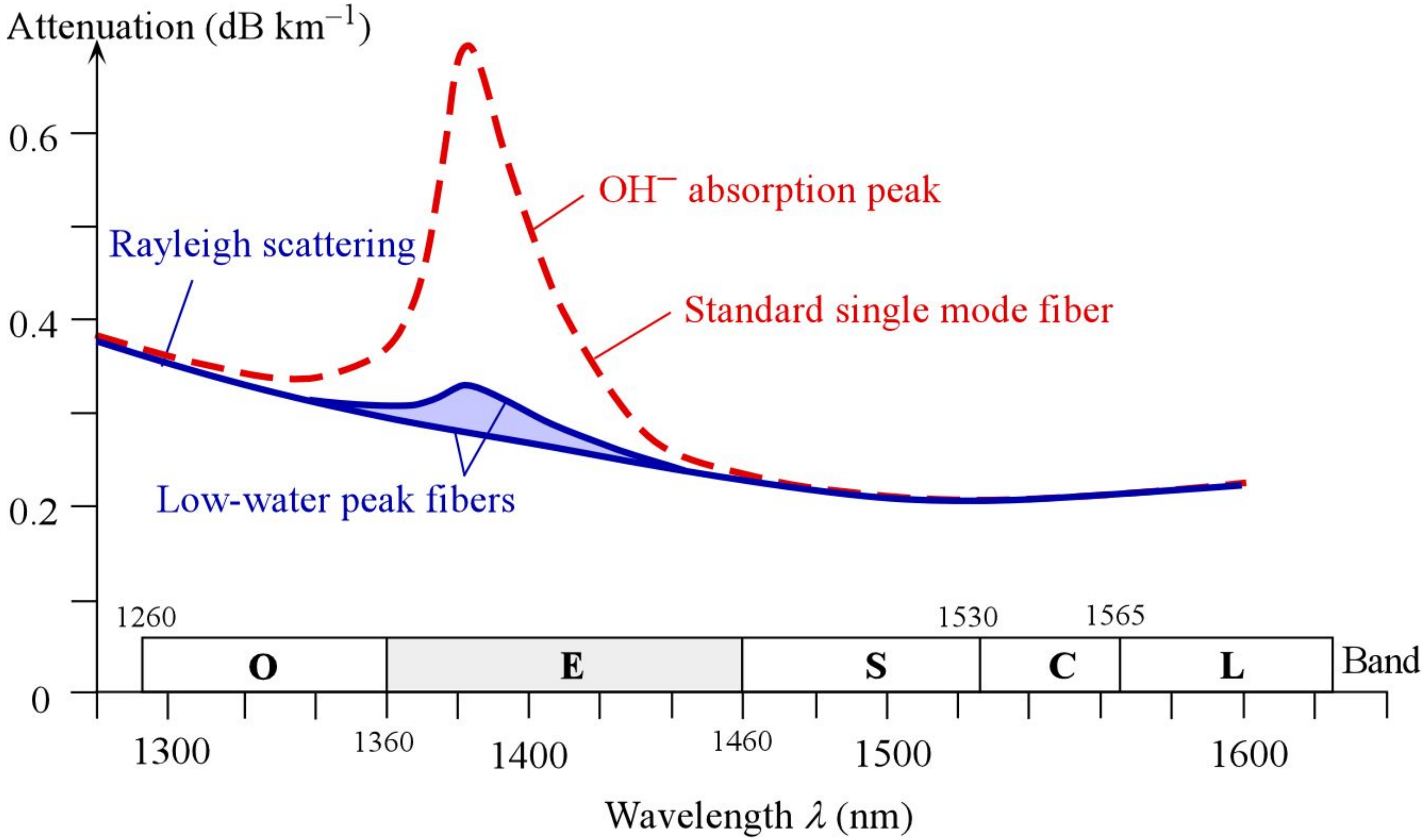
This represents the lowest possible attenuation for a silica glass core fiber at 1.55 μm.

Attenuation in Optical Fibers



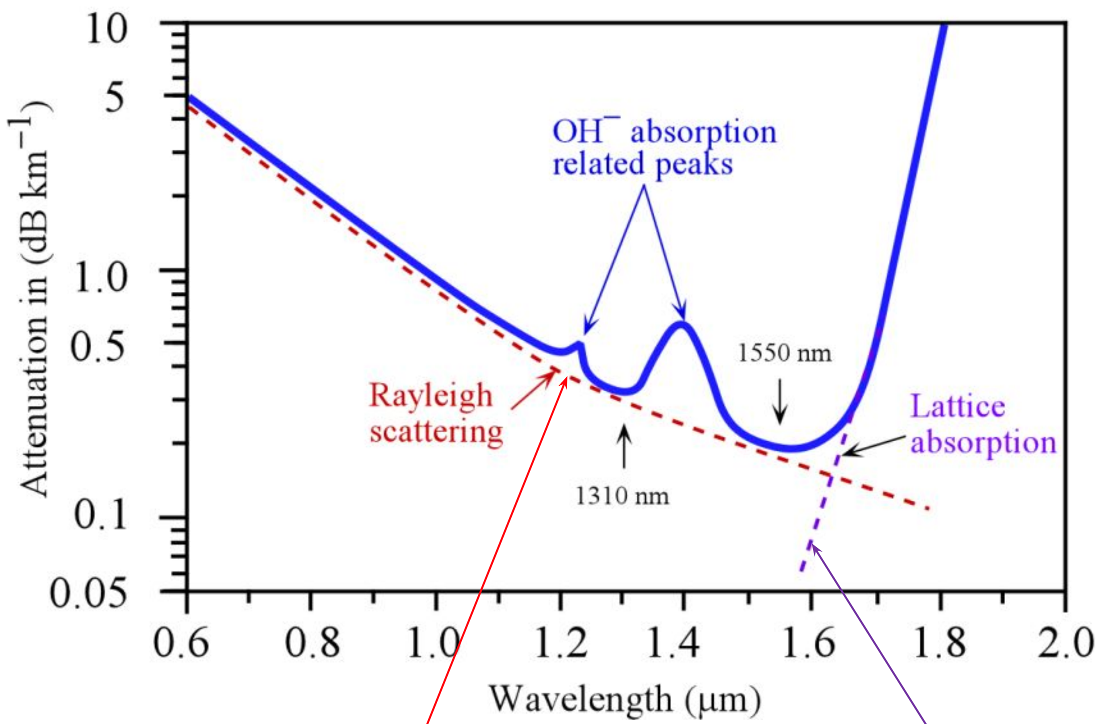
Attenuation vs. wavelength for a standard silica based fiber.

Low-water-peak fiber has no OH^- peak



E-band is available for communications with this fiber

Attenuation in Optical Fibers



$$\alpha_R = \frac{A_R}{\lambda^4}$$

$$\alpha_{\mathrm{FIR}} = A \exp\left(-B/\lambda\right)$$

Attenuation

$$\alpha_{\mathrm{FIR}} = A\exp(-B/\lambda)$$

$$\alpha_R = \frac{A_R}{\lambda^4}$$

A_R in dB km^{-1} µm^{4}

α_R in dB km^{-1}

λ in µm