Optics and Photonics

Lecture 05: Photodetectors and Image Sensors

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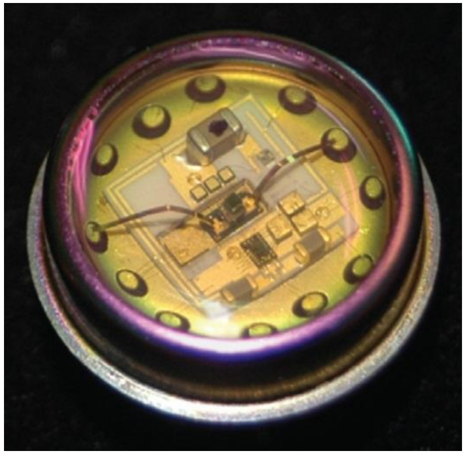
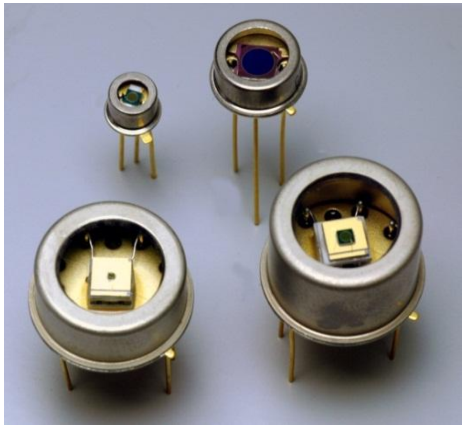
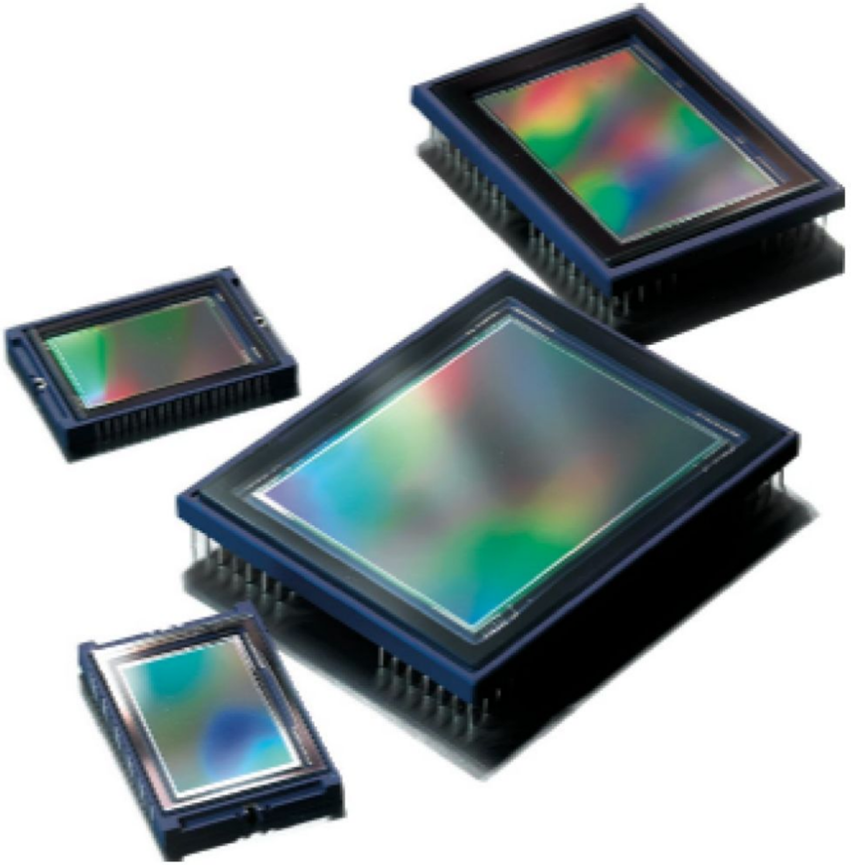
The inventors of the CCD (charge coupled device) image sensor at AT&T Bell Labs: Willard Boyle (left) and George Smith (right). The CCD was invented in 1969, the first CCD solid state camera was demonstrated in 1970, and a broadcast quality TV camera by 1975. (W. S. Boyle and G. E. Smith, “Charge Coupled Semiconductor Devices", *Bell Systems Technical Journal*, *49*, 587, 1970. (Courtesy of Alcatel-Lucent Bell Labs.)

Chapter 5
Photodetectors and Image Sensors

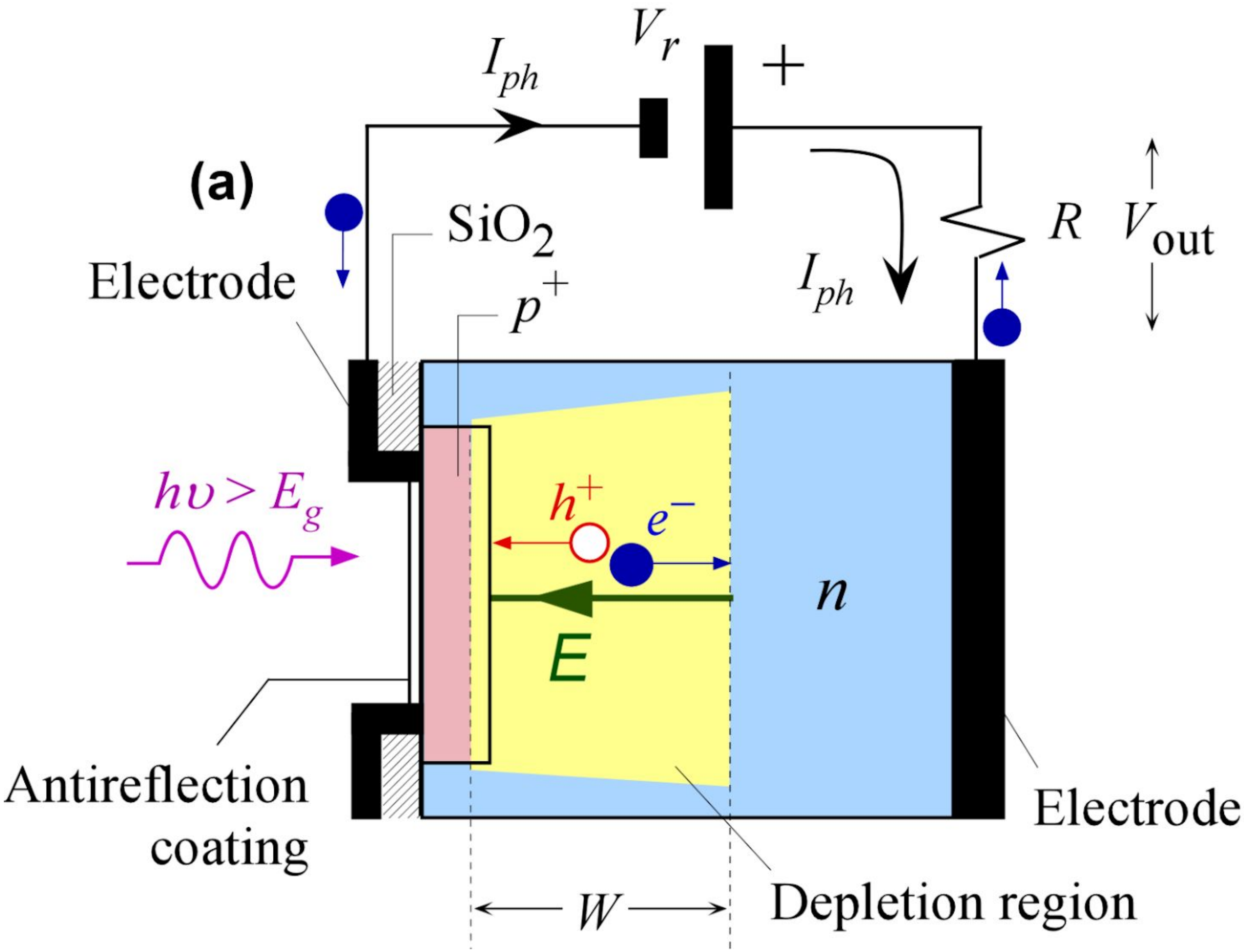


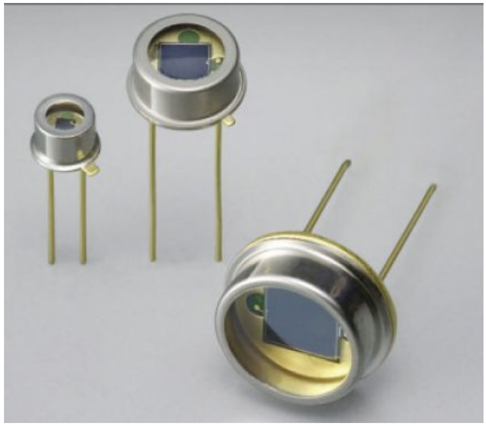




Top, courtesy of Voxtel Inc. Bottom, courtesy of Hamamatsu. Right, courtesy of Teledyne-DALSA

pn Junction Photodiode





Courtesy of Hamamatsu
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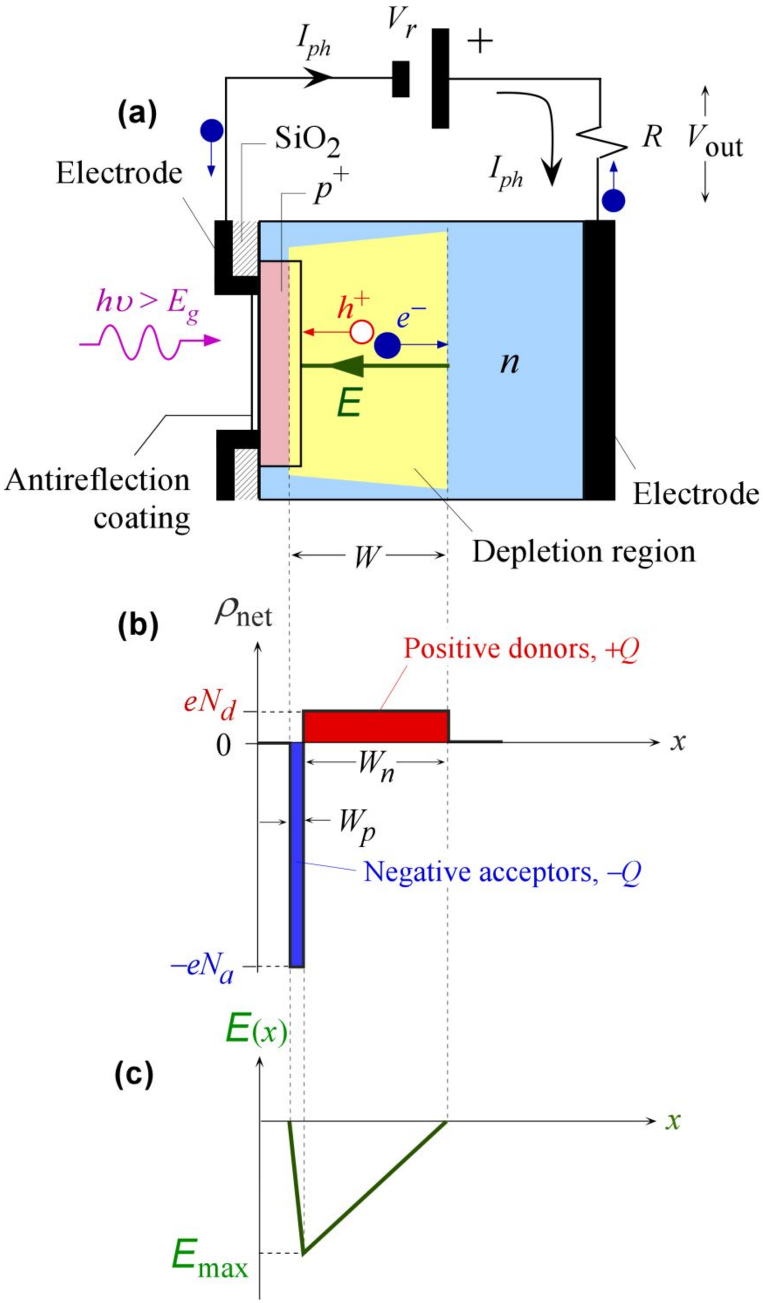
A schematic diagram of a reverse biased *pn* junction photodiode.

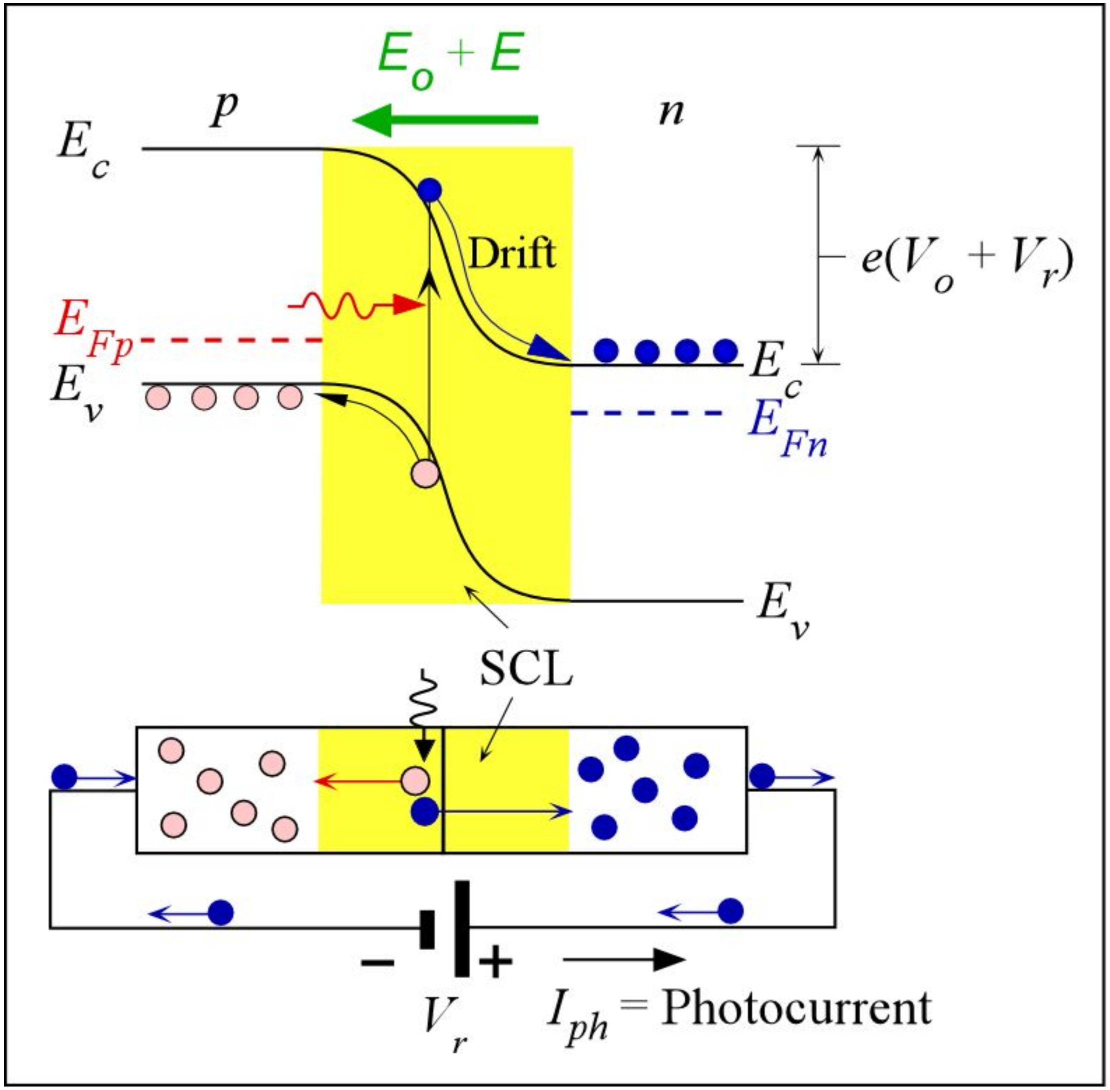


pn Junction Photodiode

(a) A schematic diagram of a reverse biased *pn* junction photodiode. (b) Net space charge across the diode in the depletion region. N_d and N_a are the donor and acceptor concentrations in the *p* and *n* sides. (c) The field in the depletion region.

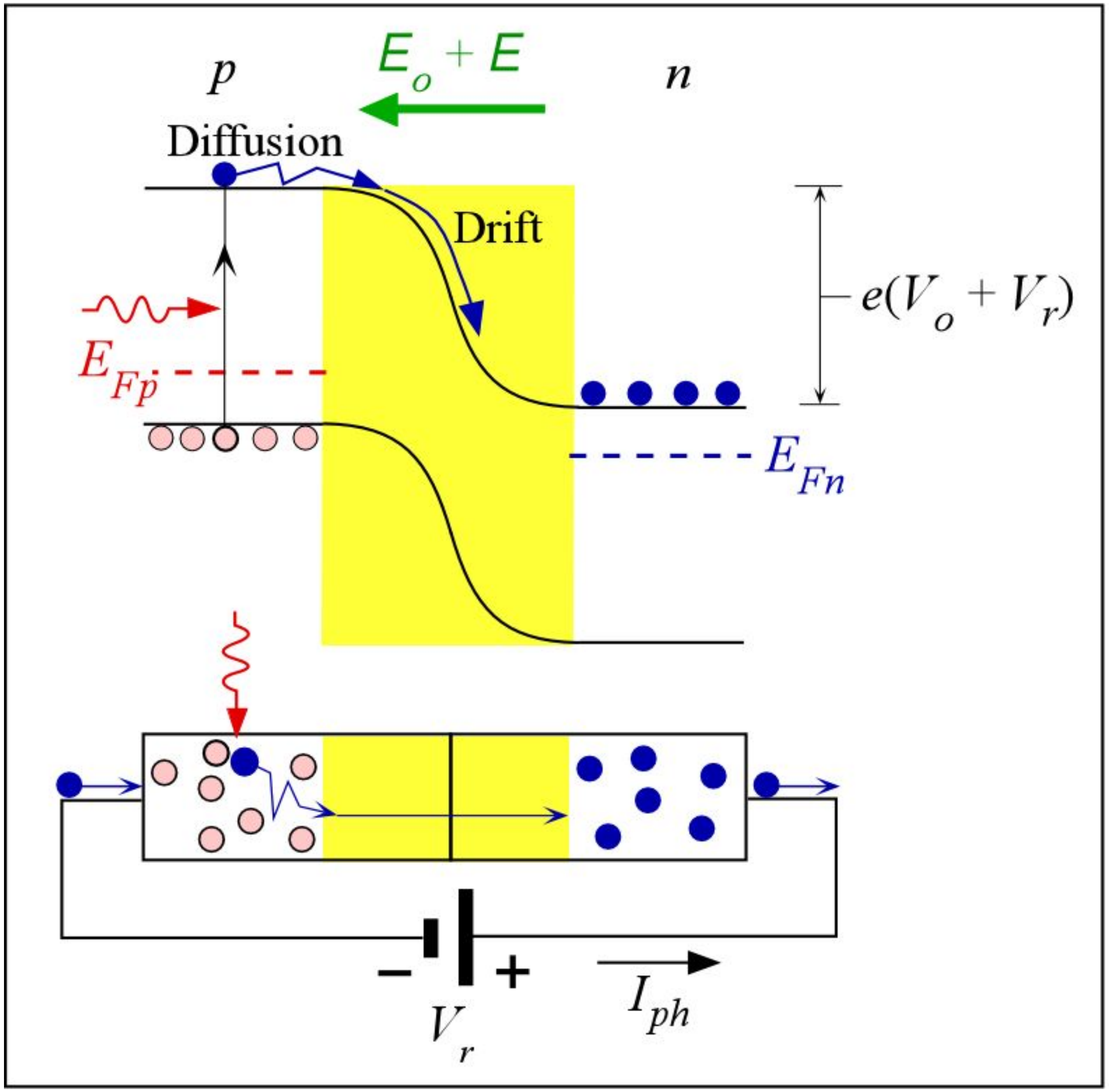
(Note: Depletion region shape in (a) is schematic only.)

pn Junction Photodiode



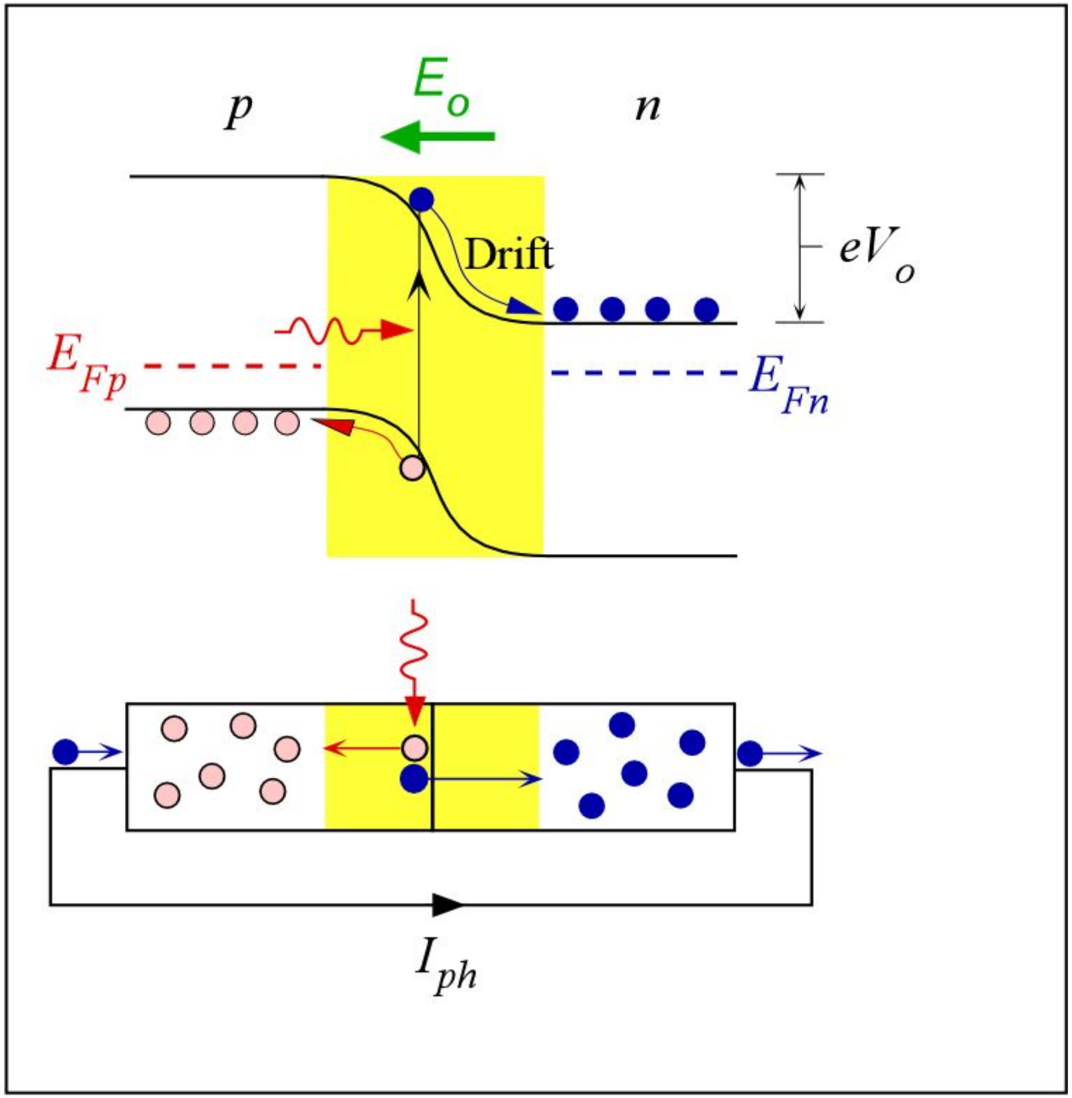
A reverse biased *pn* junction. Photogeneration inside the SCL generates an electron and a hole. Both fall their respective energy hills (electron along E_c and hole along E_v) *i.e.* they drift, and cause a photocurrent I_{ph} in the external circuit.

pn Junction Photodiode



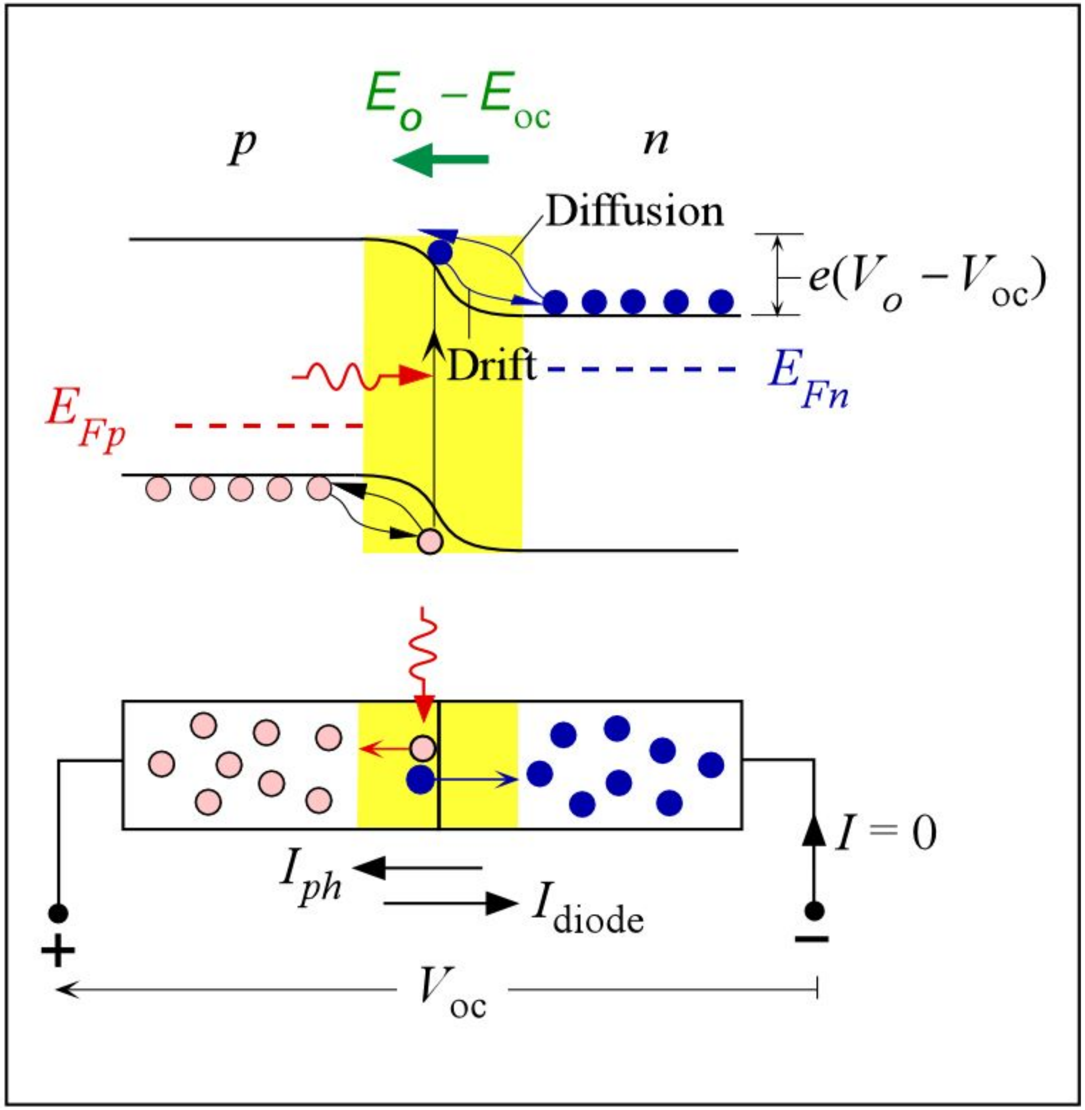
Photogeneration occurs in the neutral region. The electron has to diffuse to the depletion layer and then roll down the energy hill *i.e.* drift across the SCL.

pn Junction Photodiode



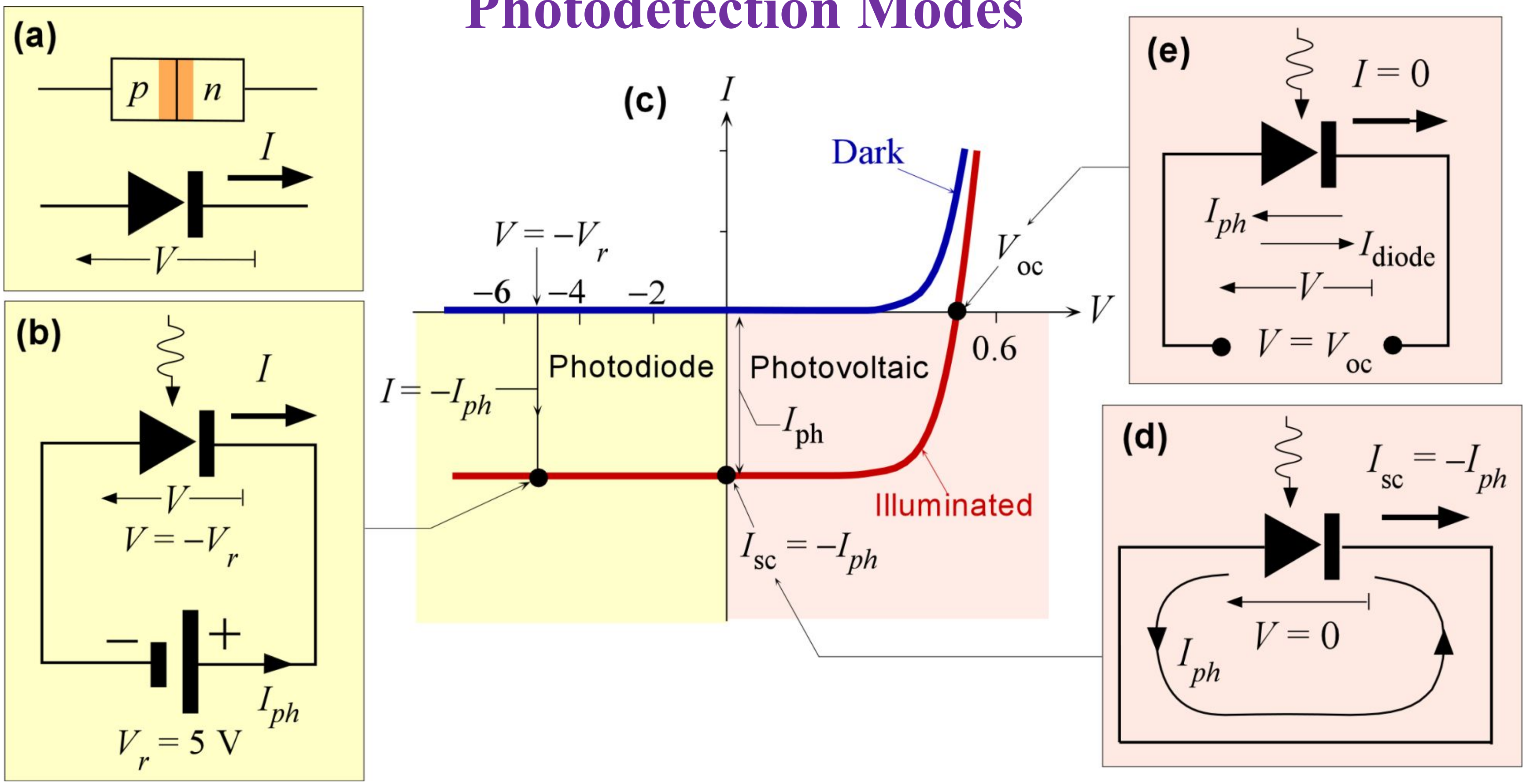
A shorted *pn* junction. The photogenerated electron and hole in the SCL roll down their energy hills, *i.e.* drift across the SCL, and cause a current I_{ph} in the external circuit.

pn Junction Photodiode



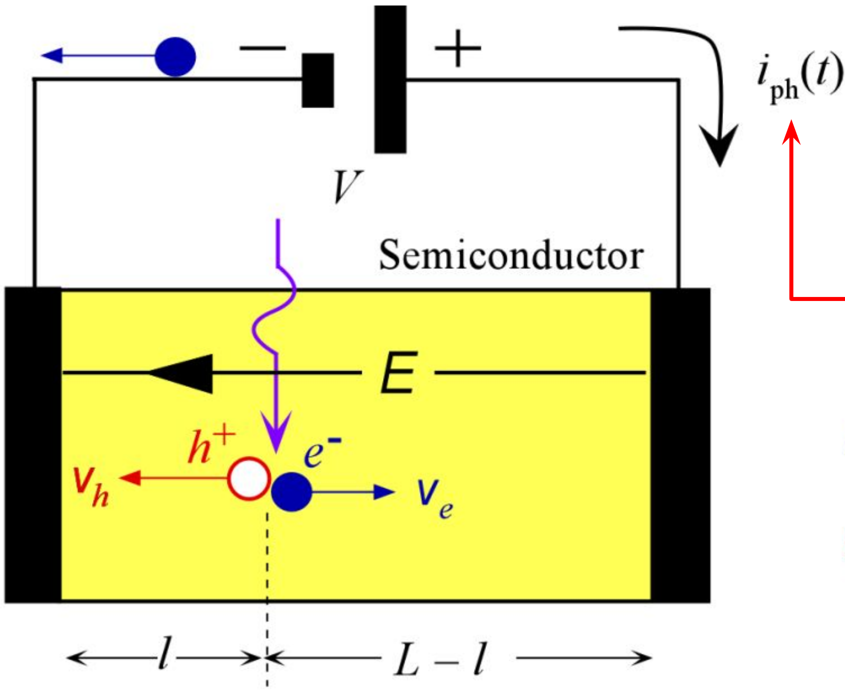
The *pn* junction in open circuit. The photogenerated electron and hole roll down their energy hills (drift) but there is a voltage V_{oc} across the diode that causes them the diffuse back so that the net current is zero.

Photodetection Modes



(a) The sign convention for the voltage V and current I for a *pn* junction. (b) If the *pn* junction is reverse biased by V_r = 5V, then $V = -V_r = -5$V. Under illumination, the *pn* junction current $I = -I_{ph}$ and is negative. (C) The I–V characteristics of a *pn* junction in the dark and under illumination. (d) A short circuit *pn* junction under illumination. The voltage $V = 0$ but there is a short circuit current so that $I = I_{sc} = -I_{ph}$. (e) An open circuit *pn* junction under illumination generates an open circuit voltage V_{oc}.

Shockley-Ramo Theorem



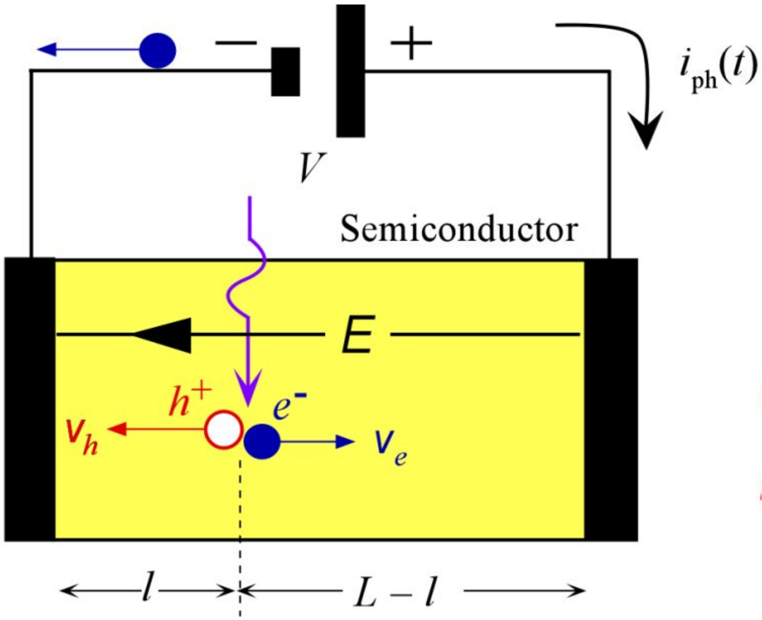
An electron and hole pair (EHP) is photogenerated at $x = l$. The electron and the hole drift in opposite directions with drift velocities v_h and v_e.

Question

What is the induced current $i_{ph}(t)$?

This is a simplified version of the more general treatment that examines the induced current on an electrode due to the motion of an electron. Its origins lie in tube-electronics in which engineers were interested in calculating how much current would flow into various electrodes of a vacuum tube as the electrons in the tube drifted. See W. Shockley, *J. Appl. Phys.*, **9**, 635, 1938 and S. Ramo, *Proc. IRE* **27**, 584, 1939.

Shockley-Ramo Theorem



External photocurrent due to the motion of this photogenerated electron is $i_e(t)$.

The electron is acted on by the force eE of the electric field.

When it moves a distance dx, work must be done by the external circuit. In time dt, the electron drifts a distance dx and does an amount of work $eEdx$

Work done $eEdx$ is provided by the battery in time dt

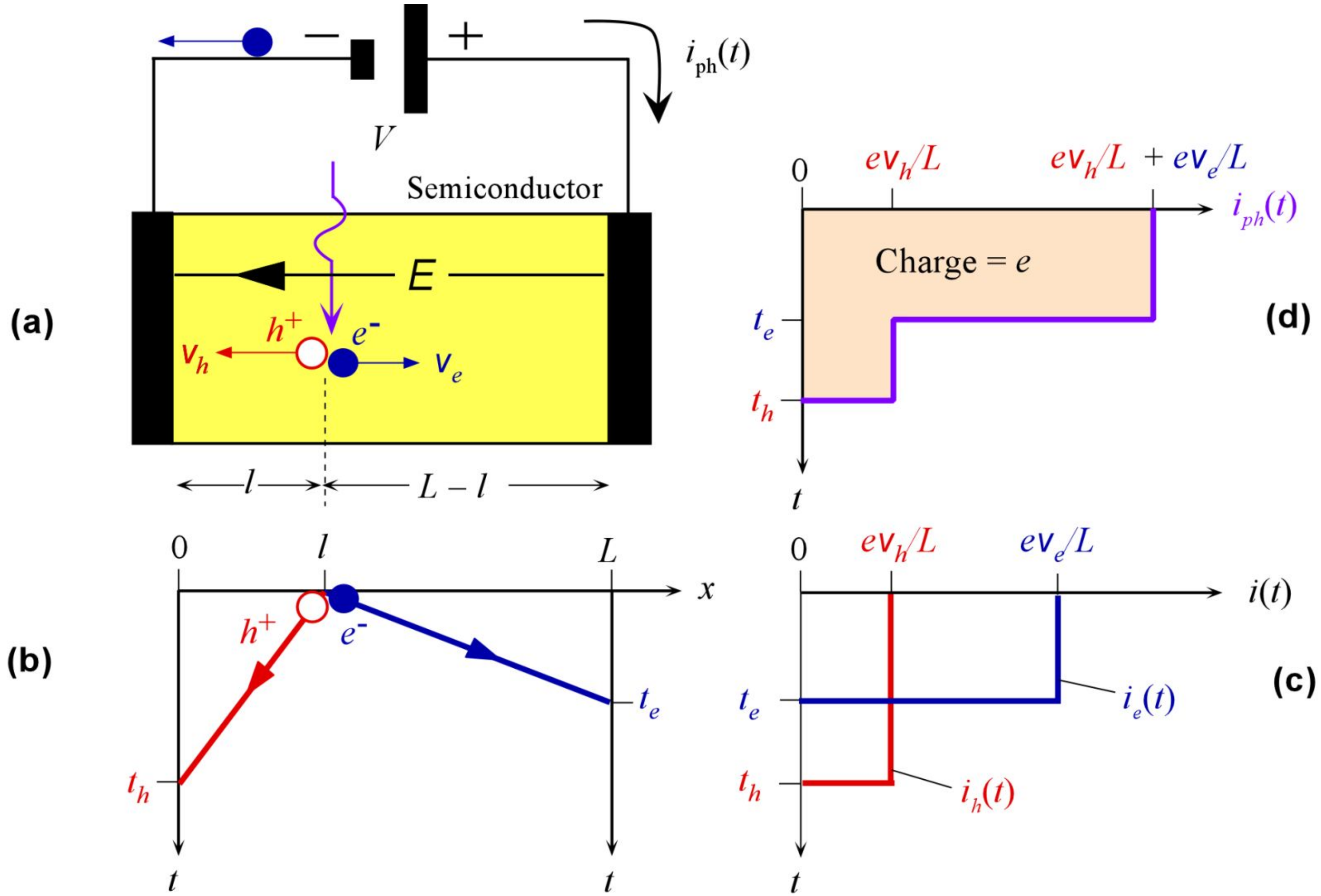
Electrical energy provided by the battery in time $dt = Vi_e(t)dt$

Thus, $eEdx = Vi_e(t)dt$. In time dt, the electron drift a distance $dx = v_e dt$

$$eEdx = Vi_e(t)dt \Rightarrow e(V/L)(v_e dt) = Vi_e(t)dt \Rightarrow \boxed{i_e(t) = \frac{ev_e}{L}}$$

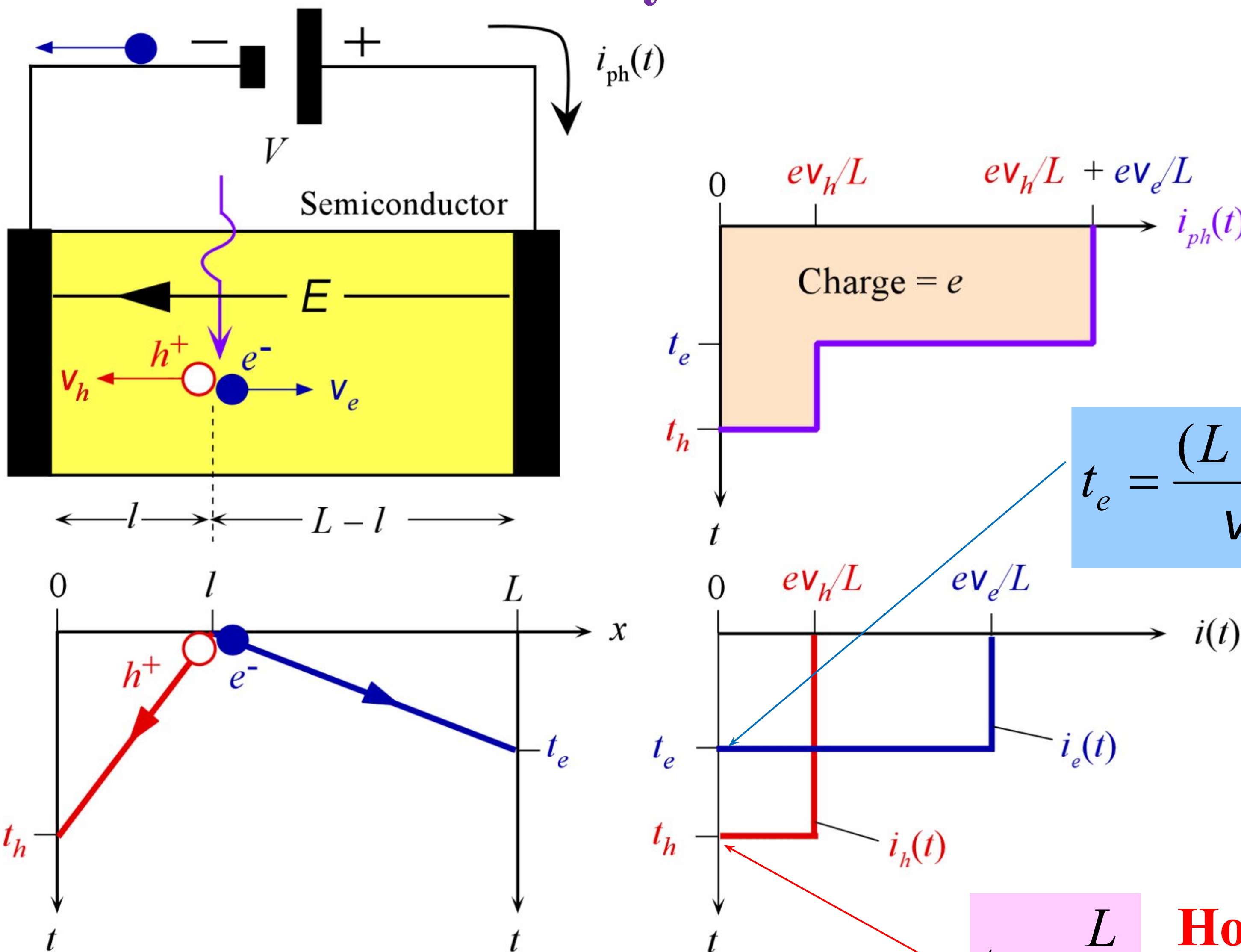
i_e flows **while the electron is drifting**, for a time $t_e = (L - l)/v_e$

Shockley-Ramo Theorem



(a) An electron and hole pair (EHP) is photogenerated at $x = l$. The electron and the hole drift in opposite directions with drift velocities v_h and v_e. (b) The electron arrives at time $t_e = (L - l)/v_e$ and the hole arrives at time $t_h = l/v_h$. (c) As the electron and hole drift, each generates an external photocurrent shown as $i_e(t)$ and $i_h(t)$. (d) The total photocurrent is the sum of hole and electron.

Shockley-Ramo Theorem



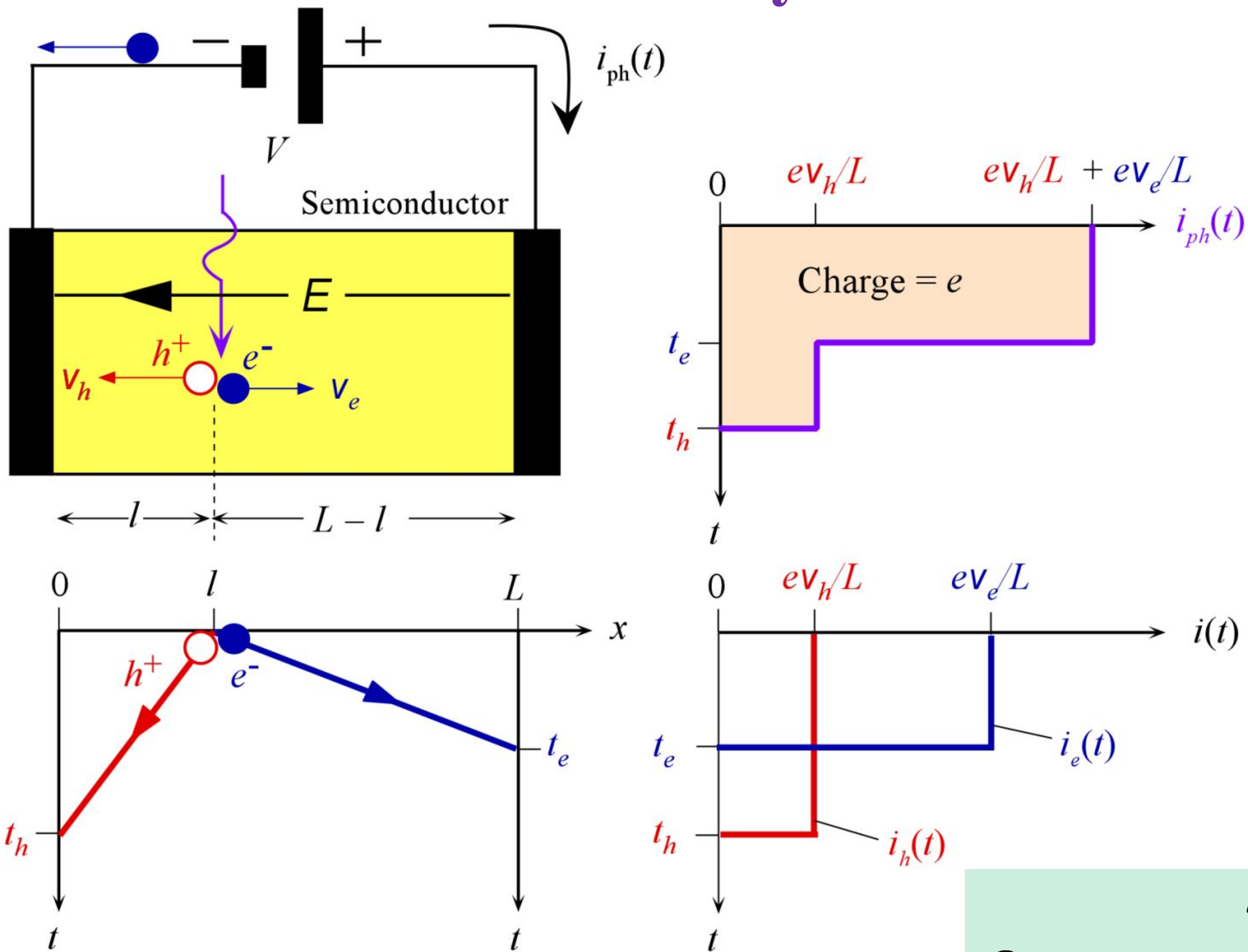
$$t_e = \frac{(L-l)}{v_e}$$

Electron transit times

$$t_h = \frac{L}{v_h}$$

Hole transit time

Shockley-Ramo Theorem



$$i_e(t) = \frac{ev_e}{L} \qquad t < t_e$$

$$i_h(t) = \frac{ev_h}{L} \qquad t < t_h$$

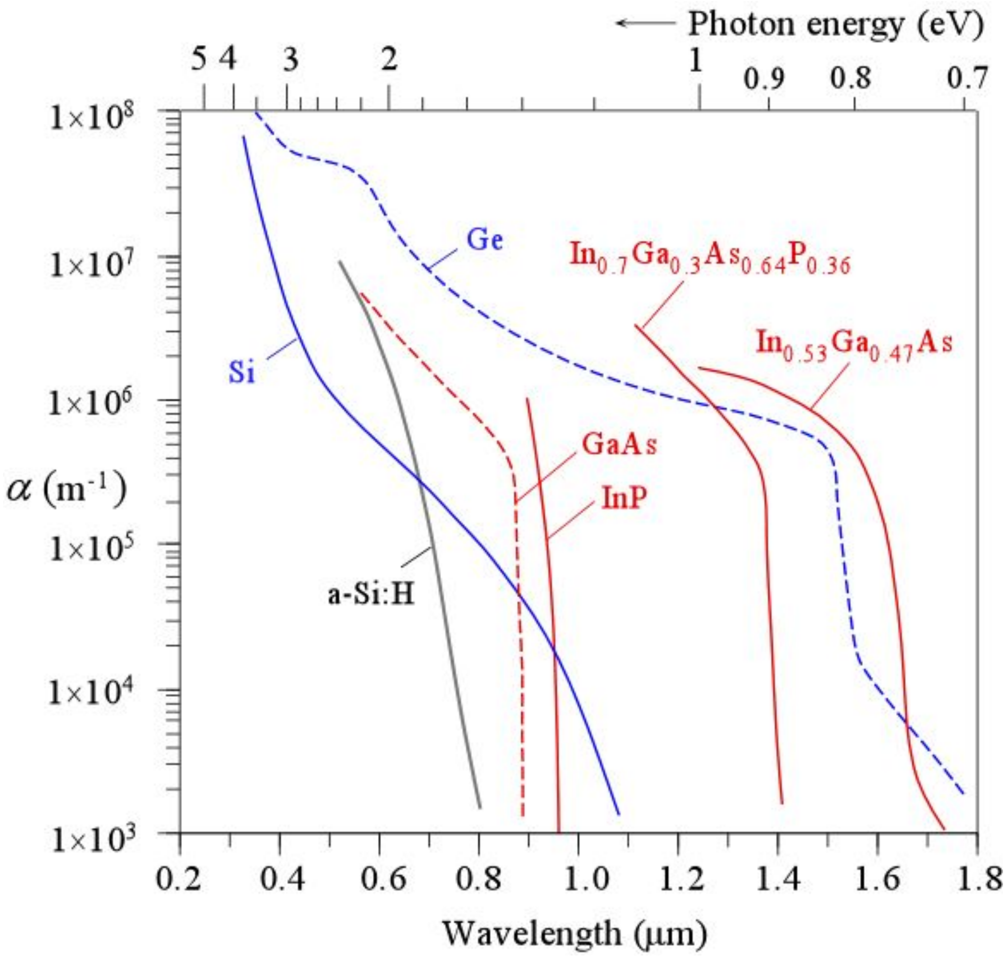
$$Q_{\text{collected}} = \int_0^{t_e} i_e(t)dt + \int_0^{t_h} i_h(t)dt = e$$

Total collected charge = *e*

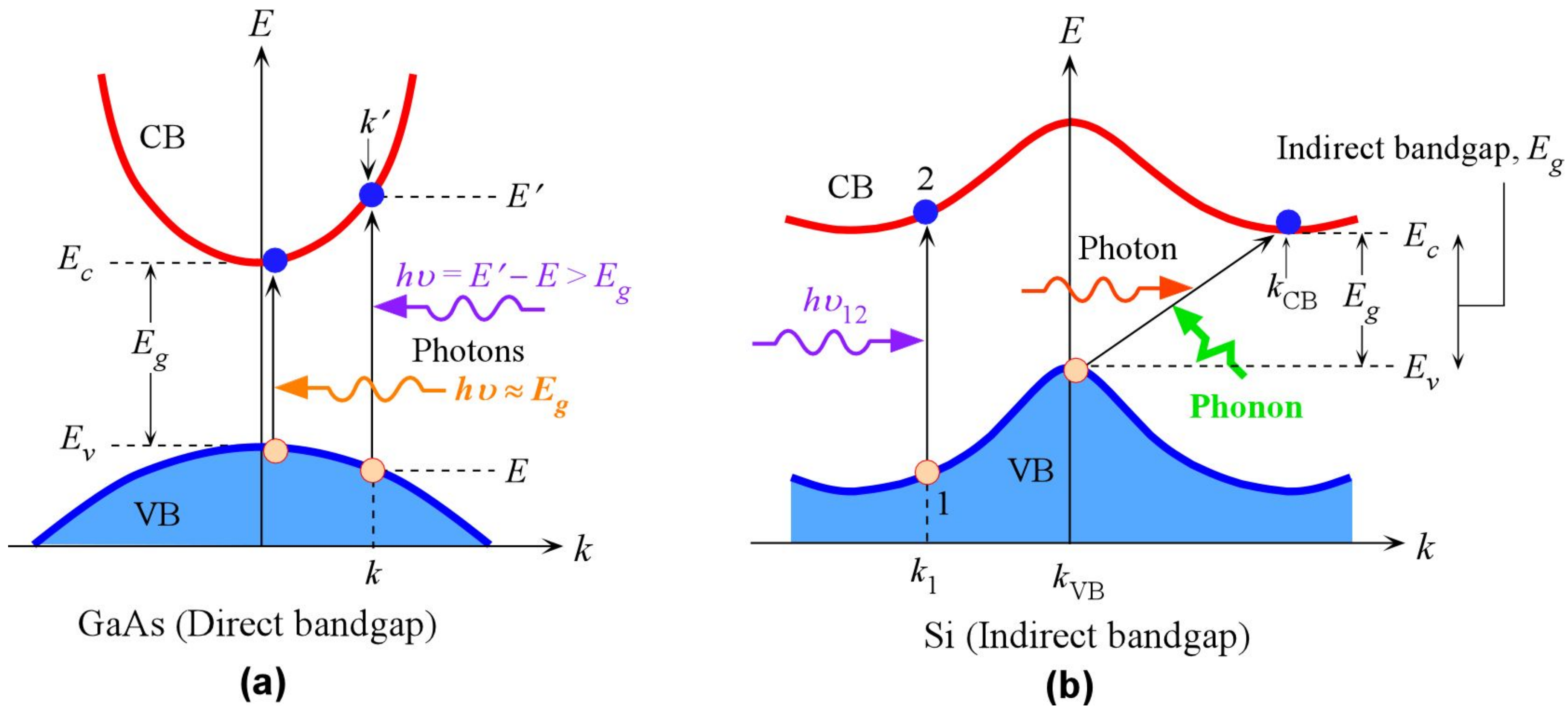
Absorption Coefficient α

$$I(x) = I_o \exp(-\alpha x)$$

$\delta = 1/\alpha$ = penetration or absorption depth



Absorption and Direct and Indirect Transitions



(a) Photon absorption in a direct bandgap semiconductor. (b) Photon absorption in an indirect bandgap semiconductor (VB, valence band; CB, conduction band)

Absorption and the Bandgap

Absorption cutoff wavelength

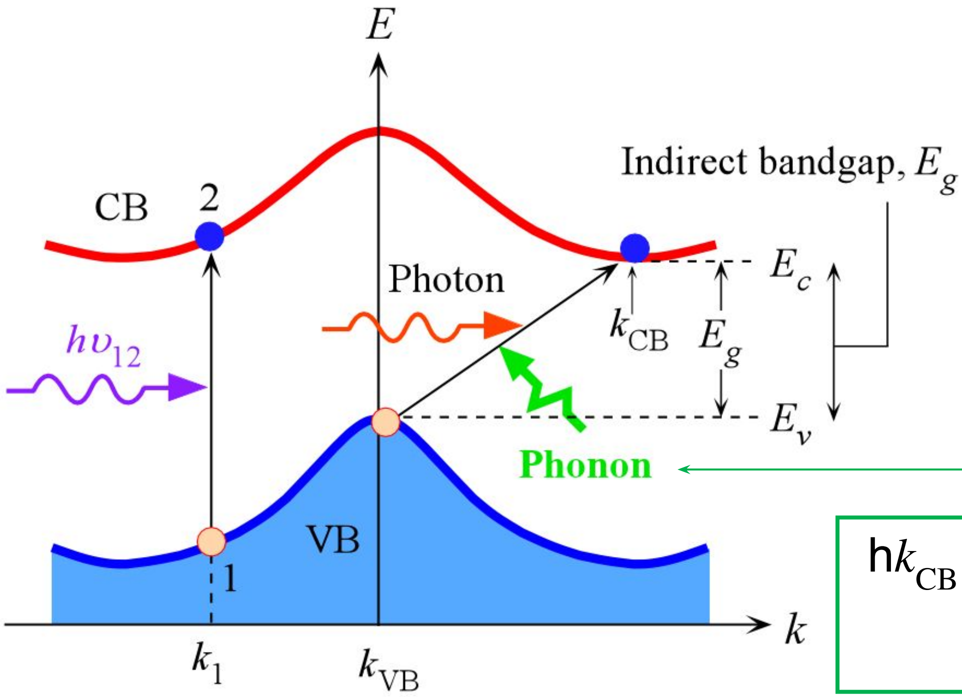
$$\lambda_g(\mu\mathrm{m}) \approx \frac{1.24}{E_g(\mathrm{eV})}$$

Wavelength in microns (micrometers)

Bandgap in eV

Wavelengths greater than roughly λ_g are not absorbed (by band-to band transitions)

Indirect Bandgap Semiconductors



Si (Indirect bandgap)

Photon energy absorbed

$h\upsilon = E_g \pm h\vartheta$

Phonon frequency

$hk_{CB} - hk_{VB}$ = Phonon momentum

$= hK$

Photon energy absorbed, $h\upsilon = E_g \pm h\vartheta$

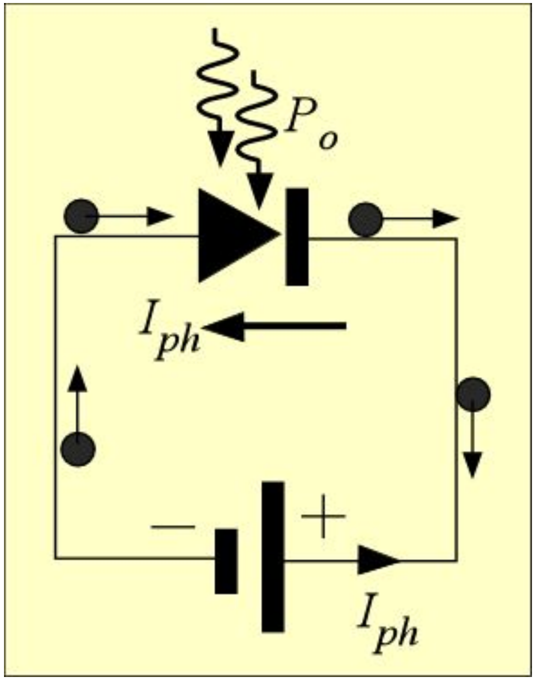
Phonon energy, small *e.g.* less than 0.1 eV

Semiconductors

Band gap energy E_g at 300 K, cut-off wavelength λ_g and type of bandgap (D = Direct and I = Indirect) for some photodetector materials

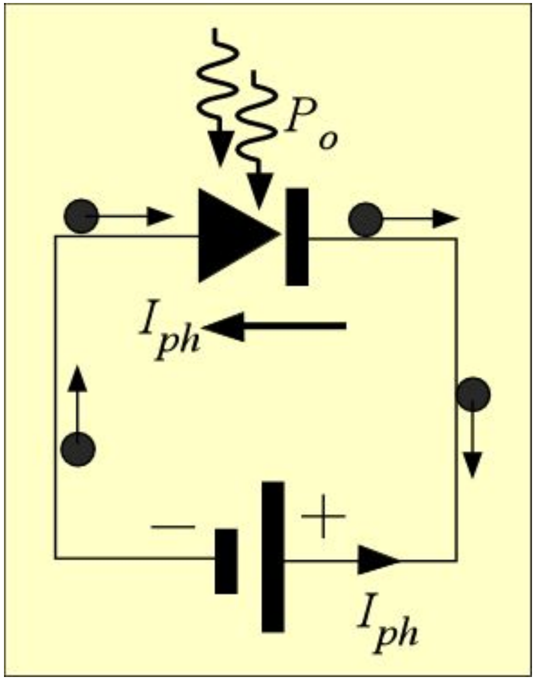
Semiconductor	E_g (eV)	λ_g (eV)	Type
InP	1.35	0.91	D
$GaAs_{0.88}Sb_{0.12}$	1.15	1.08	D
Si	1.12	1.11	I
$In_{0.7}Ga_{0.3}As_{0.64}P_{0.36}$	0.89	1.4	D
$In_{0.53}Ga_{0.47}As$	0.75	1.65	D
Ge	0.66	1.87	I
InAs	0.35	3.5	D
InSb	0.18	7	D

External quantum efficiency (QE) η_e of the detector



$$\eta_e = \frac{\text{Number of free EHP generated and collected}}{\text{Number of incident photons}}$$

$$\eta_e = \frac{I_{ph}/e}{P_o/h\upsilon}$$

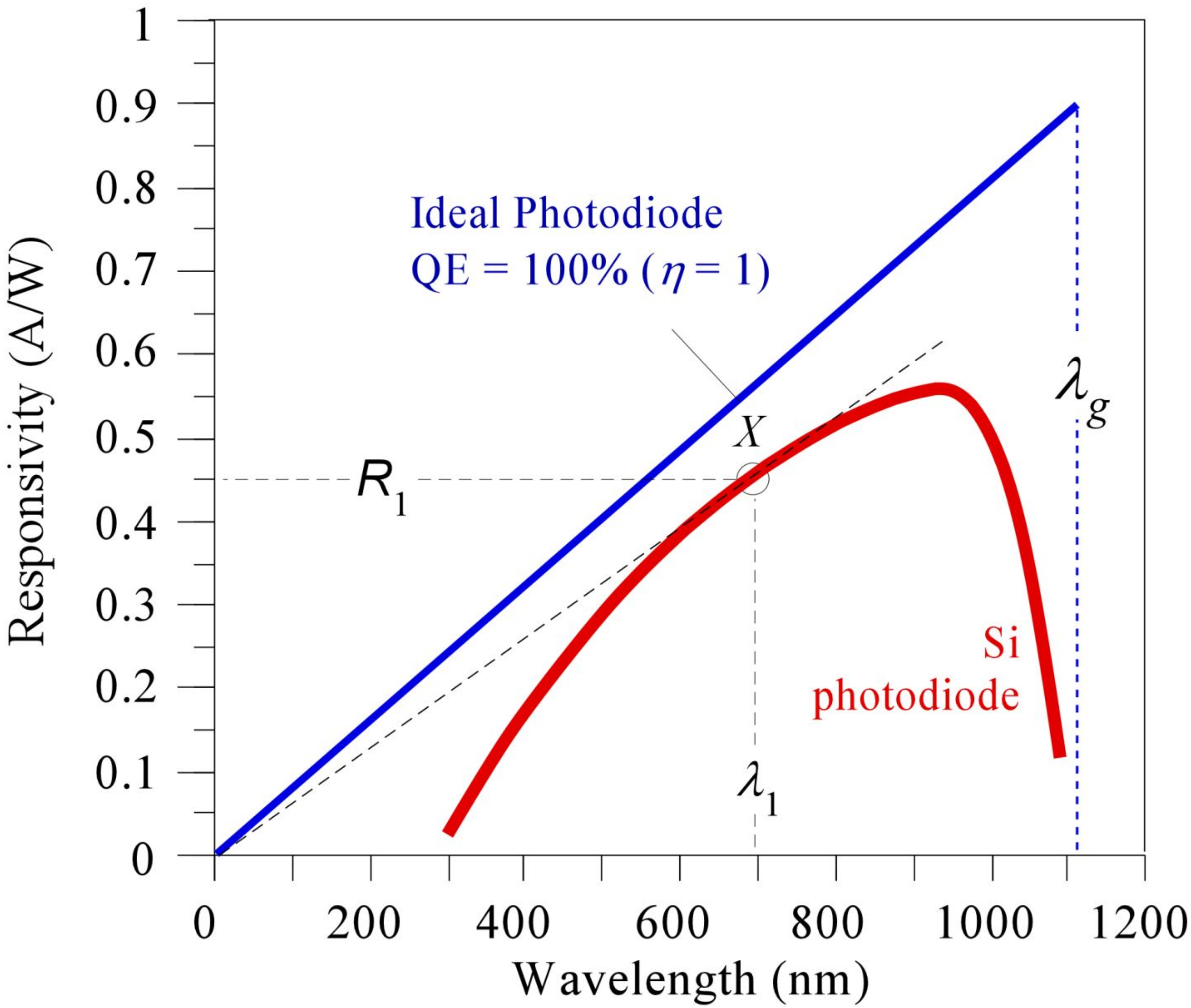


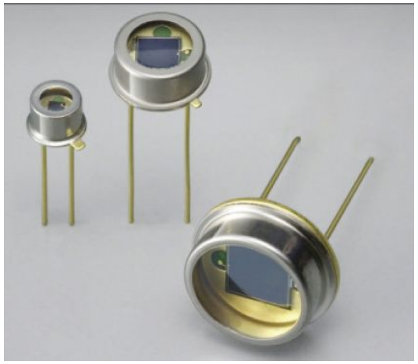
Responsivity *R*

$$R = \frac{\text{Photocurrent (A)}}{\text{Incident Optical Power (W)}} = \frac{I_{ph}}{P_o}$$

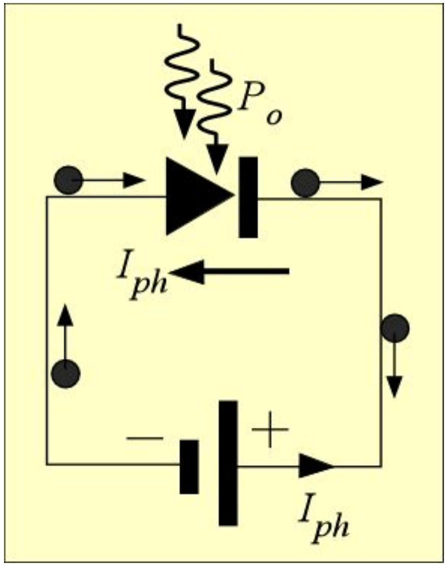
$$R = \eta_e \frac{e}{h\upsilon} = \eta_e \frac{e\lambda}{hc}$$

Responsivity *R*

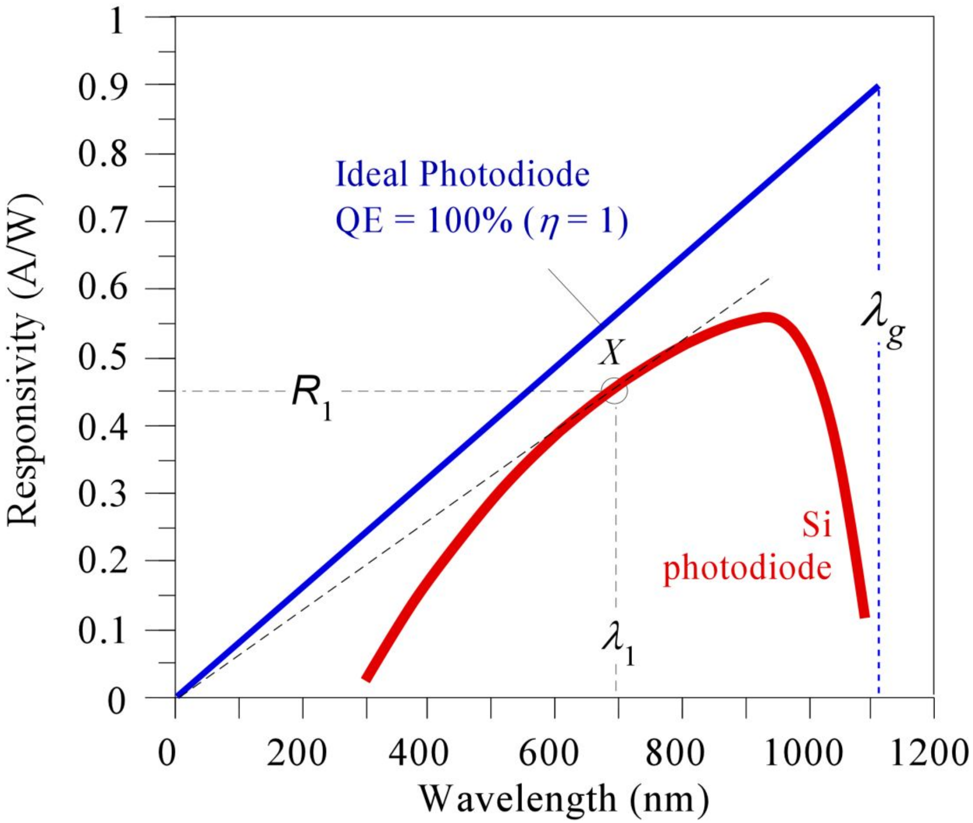




Si photodiodes of various sizes (S1336 series). (Courtesy of Hamamatsu)



Responsivity *R*



Responsivity (R) vs. wavelength (λ) for an ideal photodiode with QE = 100% (η_e = 1) and for a typical inexpensive commercial Si photodiode. The exact shape of the responsivity curve depends on the device structure.

The line through the origin that is a tangent to the responsivity curve at *X*, identifies operation at λ_1 with maximum QE

EXAMPLE: Quantum efficiency and responsivity

Consider the photodiode shown in Figure 5.7. What is the QE at peak responsivity? What is the QE at 450 nm (blue)? If the photosensitive device area is 1 mm^2, what would be the light intensity corresponding to a photocurrent of 10 nA at the peak responsivity?

Solution

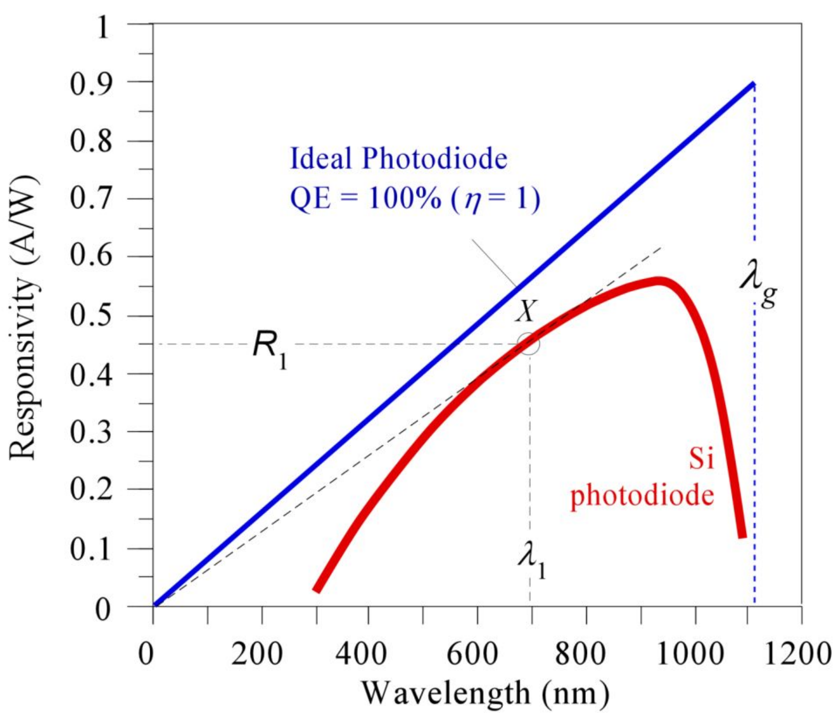
The peak responsibility in Figure 5.7 occurs at about $\lambda \approx 940$ nm where $R \approx 0.56$ A W^{-1}. Thus, from Eq. (5.4.4), that is $R = \eta_e e\lambda / hc$, we have

$$0.56\text{AW}^{-1} = \eta_e \frac{(1.6\times10^{-19}\ \text{C})(940\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})}$$

i.e. $\eta_e = 0.74$ or **74%**

We can repeat the calculation for λ = 450 nm, where $R \approx 0.24$ AW^{-1}, which gives η_e = 0.66 or 66%.

From the definition of responsivity, $R = I_{ph}/P_o$, we have 0.56 AW^{-1} = $(10\times10^{-9}$ A$)/P_o$, *i.e.* $P_o = 1.8\times10^{-8}$ W or 18 nW. Since the area is 1 mm^2 the intensity must be 18 nW mm^{-2}.



EXAMPLE: Maximum quantum efficiency

Show that a photodiode has maximum QE when

$$\boxed{\frac{dR}{d\lambda} = \frac{R}{\lambda}} \qquad (5.4.5)$$

that is, when the tangent X at λ_1 in Figure 5.7 passes through the origin (R = 0, λ = 0). Hence determine the wavelengths where the QE is maximum for the Si photodiode in Figure 5.7

Solution

From Eq. (5.4.4) the QE is given by

$$\eta_e = \frac{hcR(\lambda)}{e\lambda} \qquad (5.4.6)$$

where $R(\lambda)$ depends on λ and there is also λ in the denominator. We can differentiate Eq. (5.4.6) with respect to λ and then set to zero to find the maximum point X. Thus

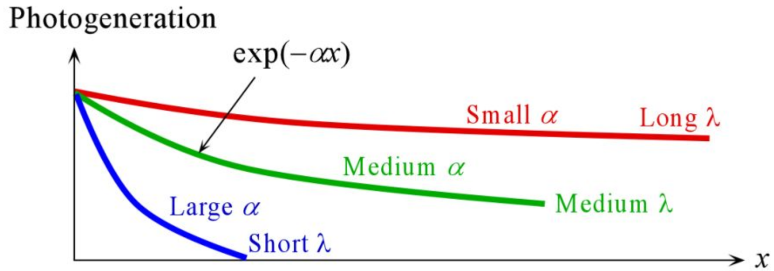
$$\frac{d\eta_e}{d\lambda} = \frac{hc}{e\lambda}\frac{dR}{d\lambda} - \frac{hcR}{e}\left(\frac{1}{\lambda^2}\right) = 0$$

which leads to Eq. (5.4.5). Equation (5.4.5) represents a line through the origin that is a tangent to the R vs λ curve. This tangential point is X in Figure 5.7 , where λ_1 = 700 nm and R_1 = 0.45 AW^{-1}. Then, using Eq. (5.4.6), the maximum QE is

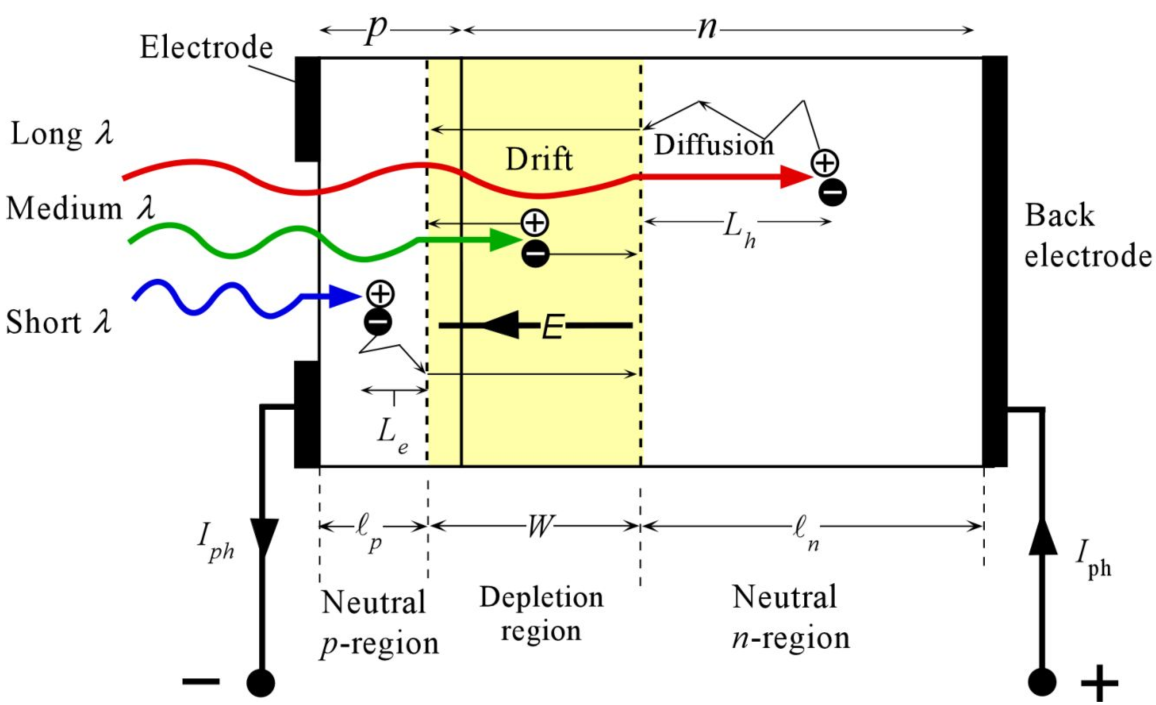
η_e = (6.626×10^{-34} J s) (3×10^{8} m s^{-1}) (0.45 A W^{-1}) / (1.6×10^{-19} C) (700×10^{-9} m)

= 0.80 or 80%

External Quantum Efficiency and Responsivity



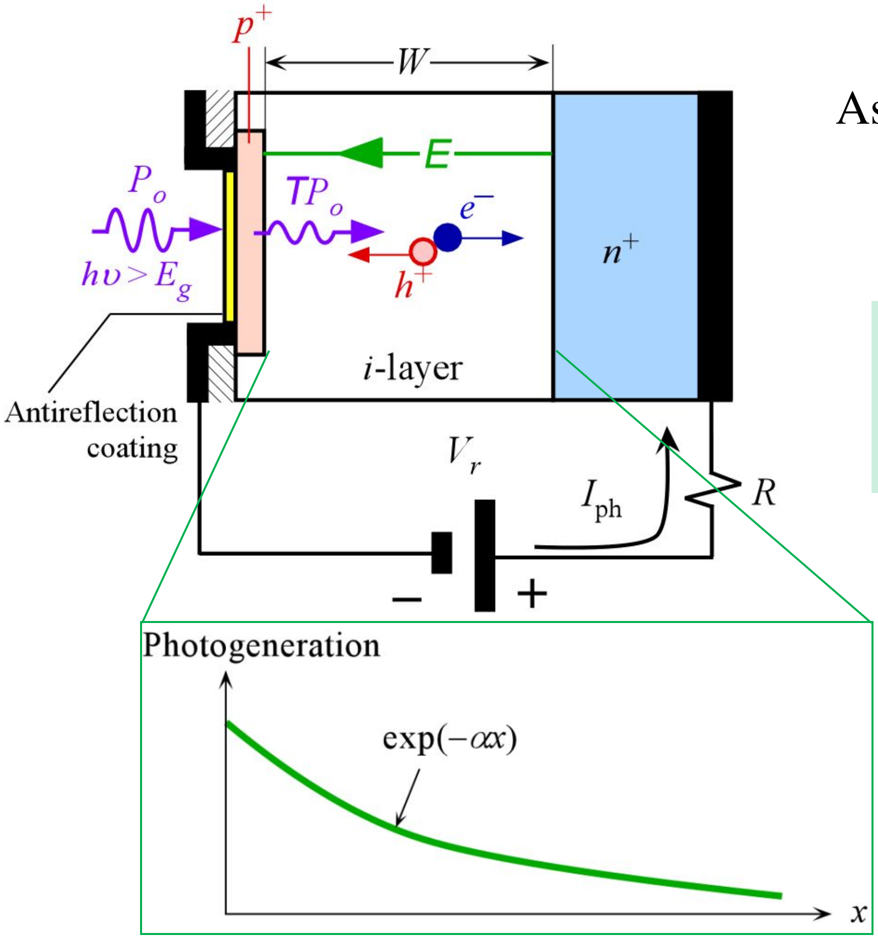
Schematic photogeneration profiles



Different contributions to the photocurrent I_{ph}. Photogeneration profiles corresponding to short, medium and long wavelengths are also shown.

Internal Quantum Efficiency η_i

$$\eta_i = \text{Internal Quantum Efficiency} = \frac{\text{Number of EHP photogenerated}}{\text{Number of absorbed photons}}$$



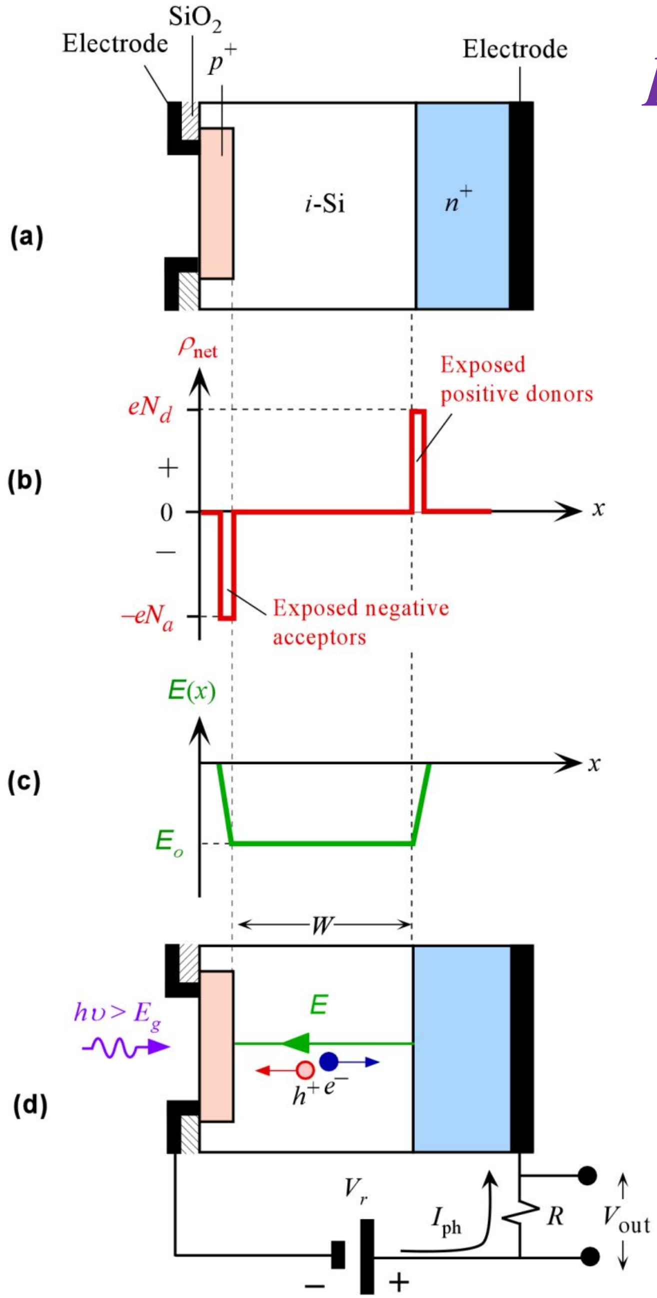
Assuming l_p is very thin, and assuming $W >> L_h$

$$I_{ph} \approx \frac{e\eta_i TP_o(0)}{h\upsilon}[1 - \exp(-\alpha W)]$$

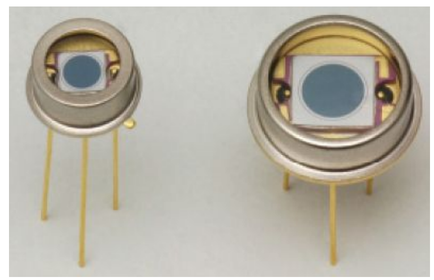
T = Transmission coefficient of AR coating

α = Absorption coefficient

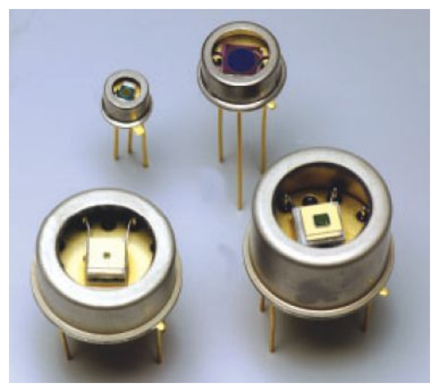
pin Photodiode



The schematic structure of an idealized pin photodiode (b) The net space charge density across the photodiode. (c) The built-in field across the diode. (d) The pin photodiode reverse biased for photodetection.



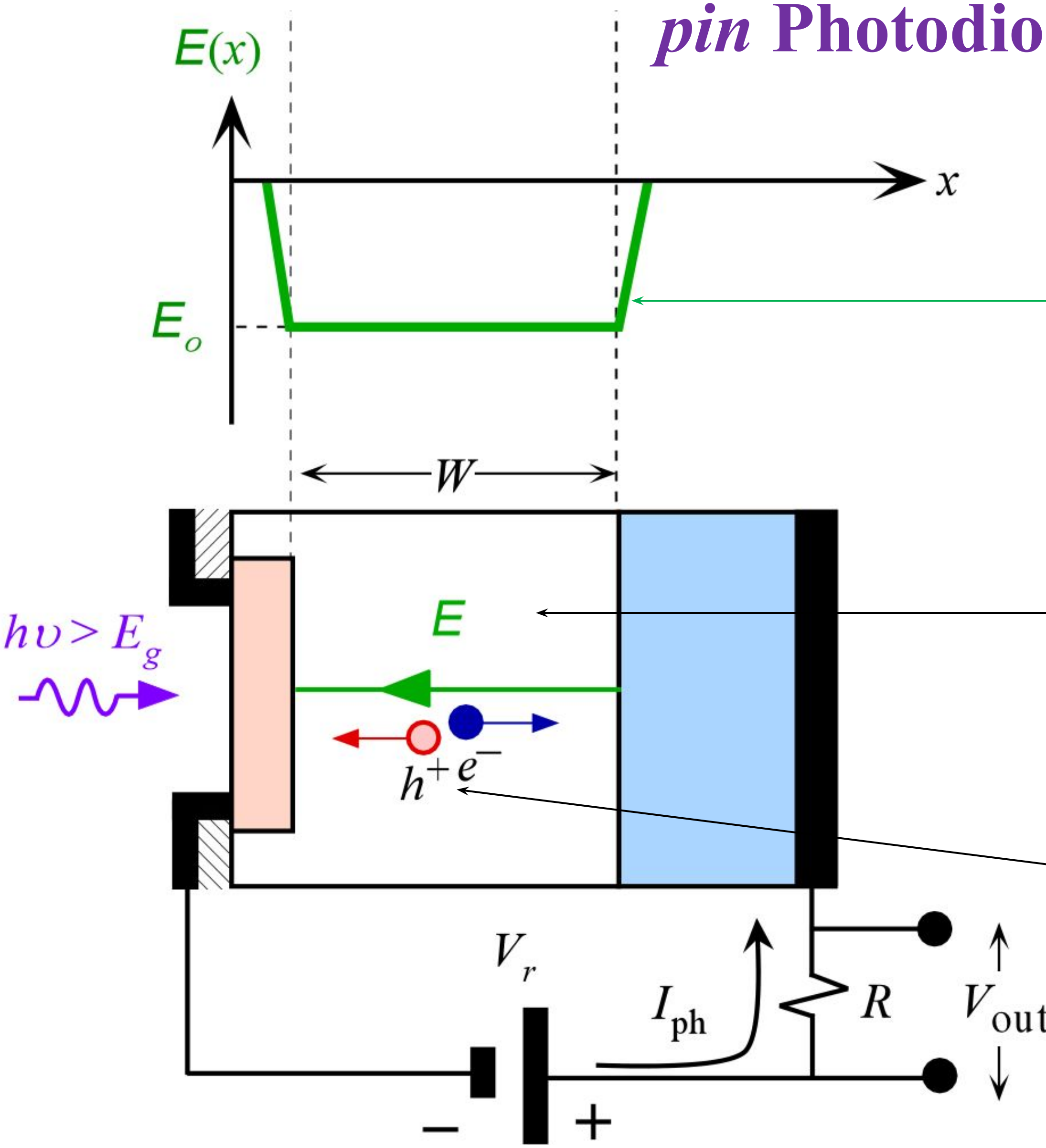
Si pin



InGaAs pin

Courtesy of Hamamatsu

pin Photodiode

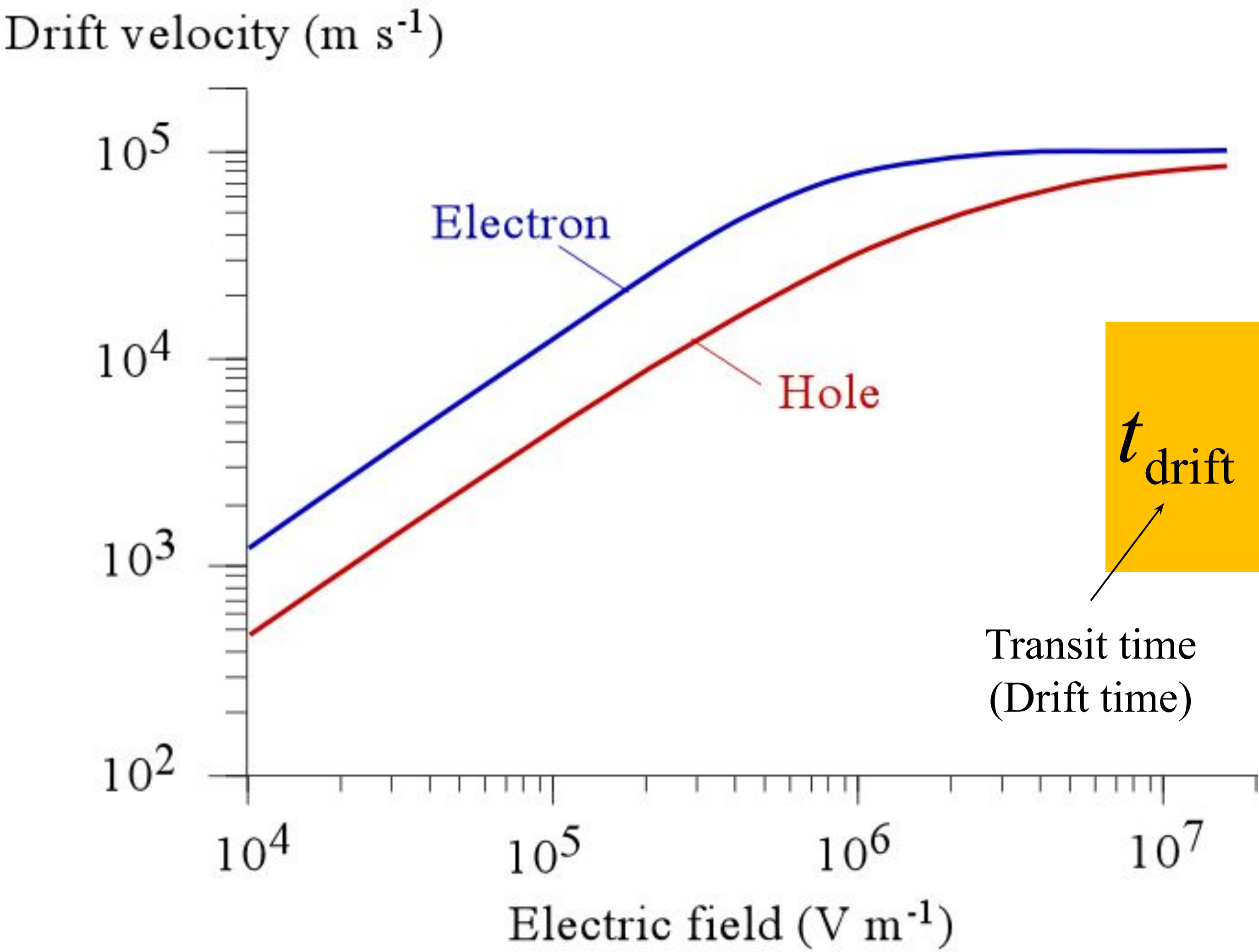


$$E = E_o + \frac{V_r}{W} \approx \frac{V_r}{W}$$

$$C_{\text{dep}} = \frac{\varepsilon_o \varepsilon_r A}{W}$$

$$t_{\text{drift}} = \frac{W}{v_d}$$

pin Photodiode



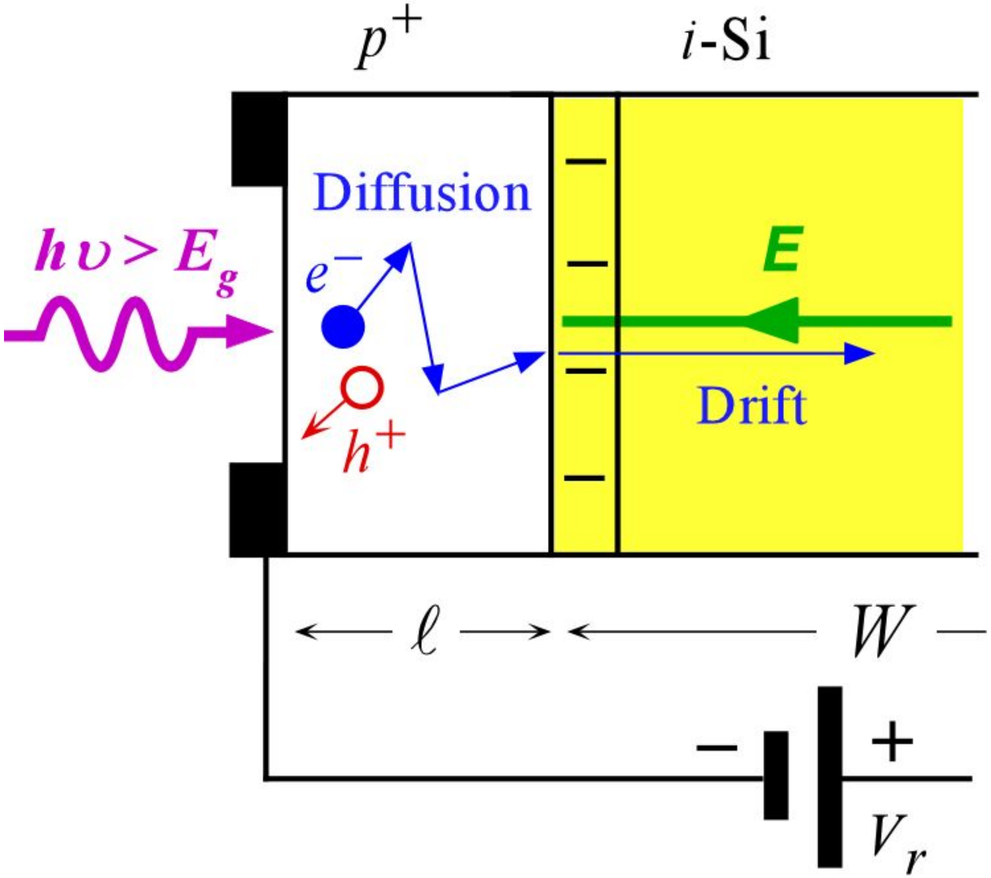
Drift velocity vs. electric field for holes and electrons in Si.

Width of *i*-region
(Depletion region)

$$t_{\mathrm{drift}} = \frac{W}{v_d}$$

Transit time
(Drift time)

Drift velocity



pin Photodiode Speed

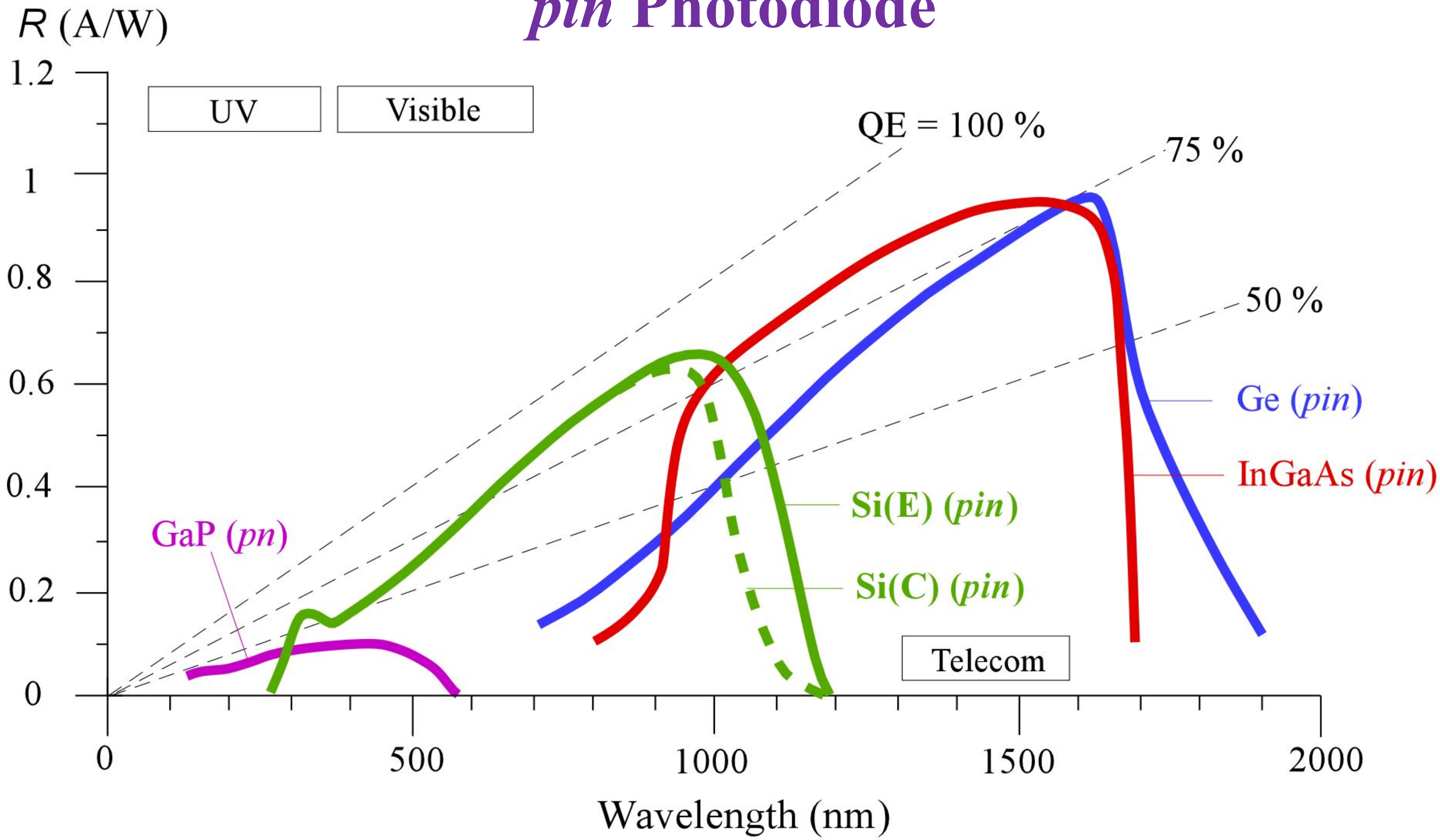
A reverse biased *pin* photodiode is illuminated with a short wavelength light pulse that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across.

In time *t*, an electron, on average, diffuses a distance l given by

$$\boxed{l = (2D_e t)^{1/2}}$$

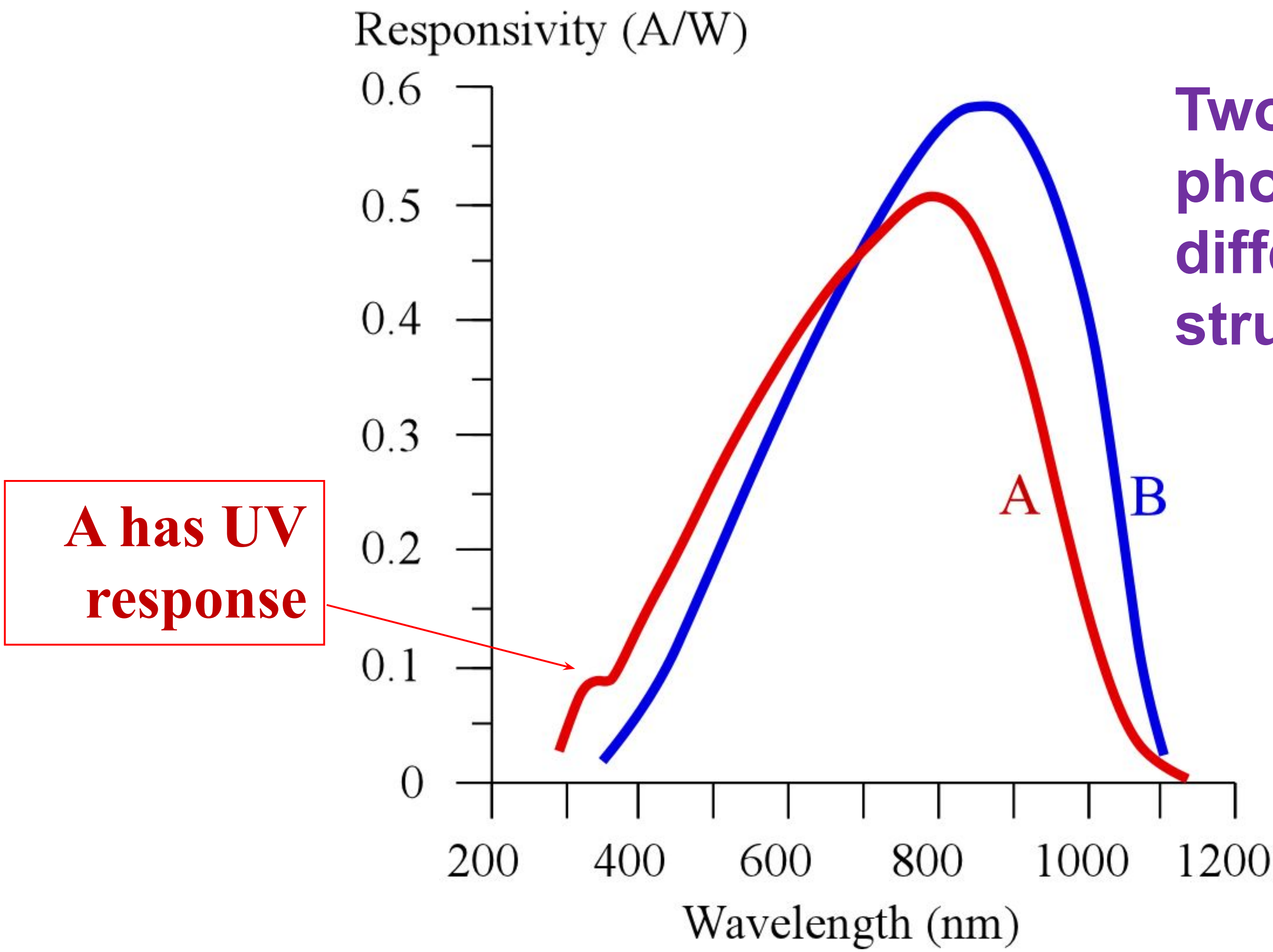
Electron diffusion coefficient

pin Photodiode



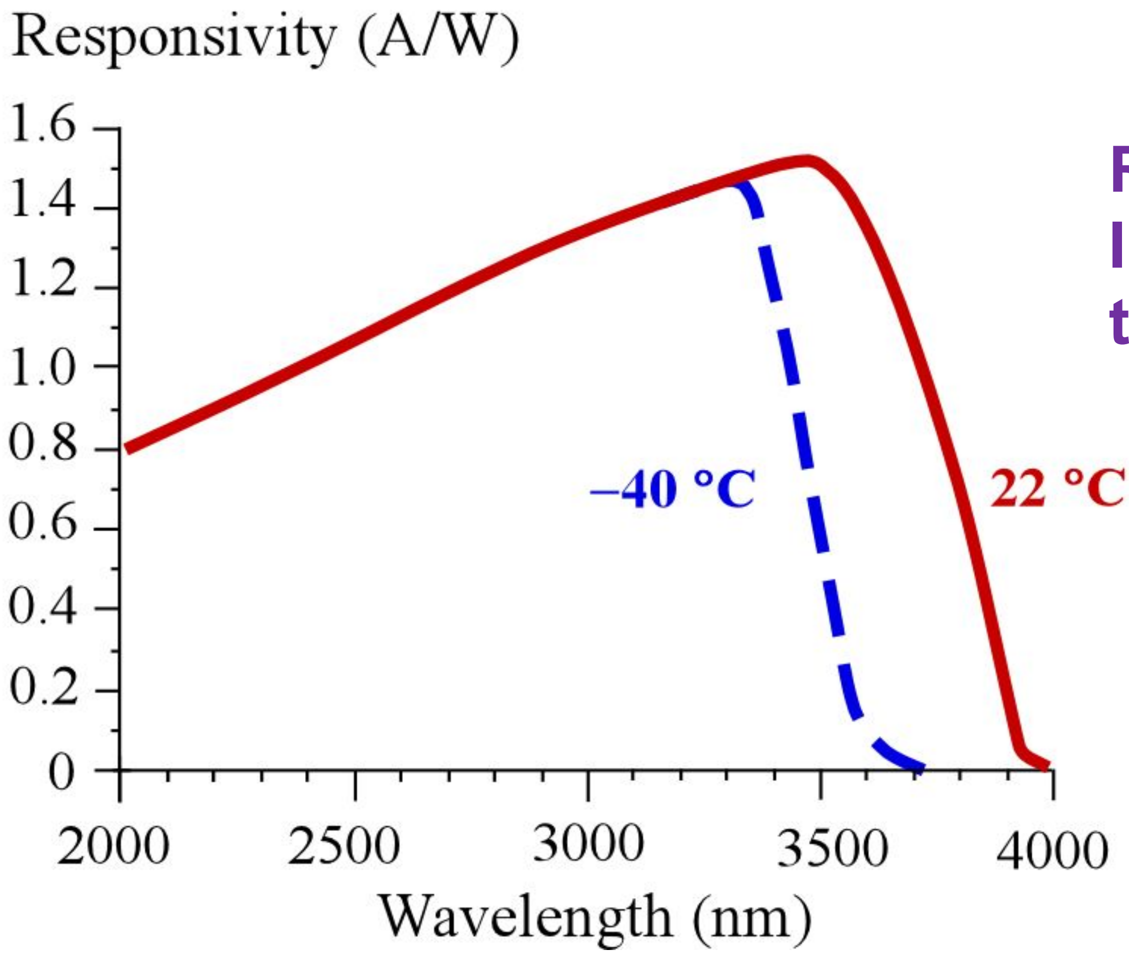
The responsivity of Si, InGaAs and Ge *pin* type photodiodes. The *pn* junction GaP detector is used for UV detection. GaP (Thorlabs, FGAP71), Si(E), IR enhanced Si (Hamamatsu S11499), Si(C), conventional Si with UV enhancement, InGaAs (Hamamatsu, G8376), and Ge (Thorlabs, FDG03). The dashed lines represent the responsivity due to QE = 100 %, 75% and 50 %.

Responsivity *R* depends on the device structure



Two Si *pin* photodiodes with different device structures.

Responsivity *R* depends on the temperature



Responsivity of an InAs photodiode at two temperatures

EXAMPLE: Responsivity of a *pin* photodiode

A Si *pin* photodiode has an active light receiving area of diameter 0.4 mm. When radiation of wavelength 700 nm (red light) and intensity 0.1 mW cm^{-2} is incident, it generates a photocurrent of 56.6 nA. What is the responsivity and external QE of the photodiode at 700 nm?

Solution

The incident light intensity I = 0.1 mW cm^{-2} means that the incident power for conversion is

$P_o = AI = [\pi(0.02 \text{ cm})^2](0.1\times10^{-3} \text{ W cm}^{-2}) = 1.26\times10^{-7}$ W or **0.126 μW**.

The responsivity is

$R = I_{ph}/P_o = (56.6 \times 10^{-9} \text{ A})/(1.26\times10^{-7} \text{ W}) = \mathbf{0.45\ A\ W^{-1}}$

The QE can be found from

$$\eta = R\frac{hc}{e\lambda} = (0.45\ \text{A W}^{-1})\frac{(6.62\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})}{(1.6\times10^{-19}\ \text{C})(700\times10^{-9}\ \text{m})} = 0.80 = \mathbf{80\,\%}$$

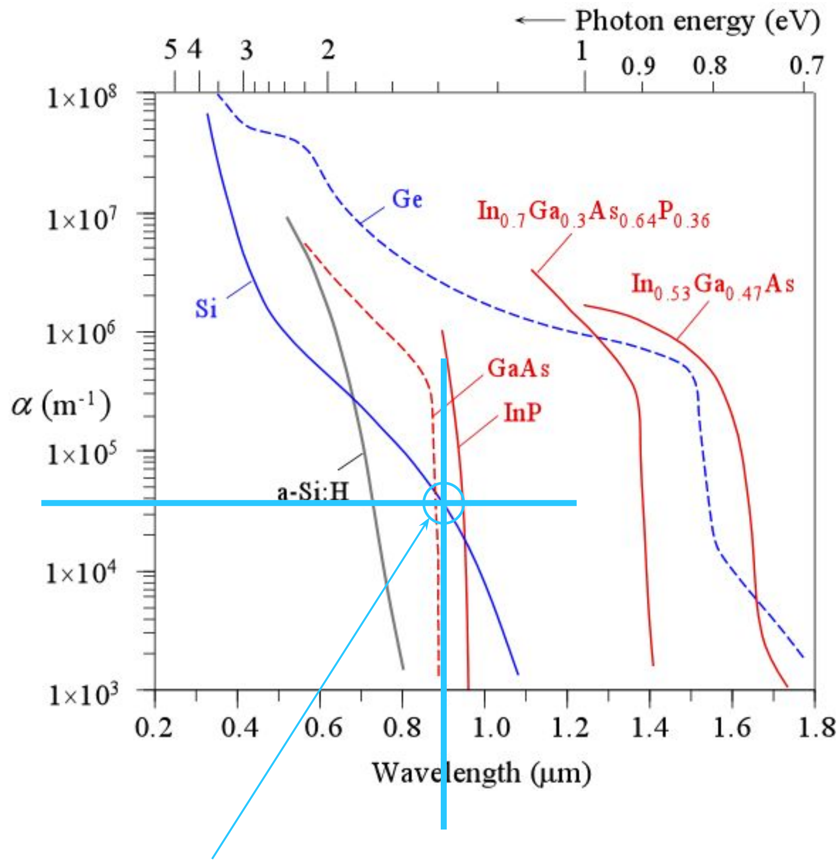
EXAMPLE: Operation and speed of a *pin* photodiode

A Si *pin* photodiode has an *i*-Si layer of width 20 μm. The p^+-layer on the illumination side is very thin (0.1 μm). The *pin* is reverse biased by a voltage of 100 V and then illuminated with a very short optical pulse of wavelength 900 nm. What is the duration of the photocurrent if absorption occurs over the whole *i*-Si layer?

Solution

From Figure 5.5 , the absorption coefficient at 900 nm is ~ 3×10^4 m^{-1} so that the absorption depth is ~33 μm. We can assume that absorption and hence photogeneration occurs over the entire width *W* of the *i*-Si layer. The field in the *i*-Si layer is

$$E \approx V_r / W$$
$$= (100\text{ V})/(20\times10^{-6}\text{ m})$$
$$= 5\times10^6\text{ V m}^{-1}$$



Note: The absorption coefficient is between 3×10^4 m^{-1} and 4×10^4 m^{-1}

EXAMPLE: Operation and speed of a *pin* photodiode
Solution (continued)

At this field the electron drift velocity v_e is very near its saturation at 10^5 m s^{-1}, whereas the hole drift velocity v_h, ≈ 7×10^4 m s^{-1} as shown in Figure 5.10. Holes are slightly slower than the electrons. The transit time t_h of holes across the *i*-Si layer is

$$t_h = W/v_h = (20\times10^{-6}\text{ m})/(7\times10^4\text{ m s}^{-1})$$
$$= 2.86\times10^{-10}\text{ s or } \mathbf{0.29\ ns}$$

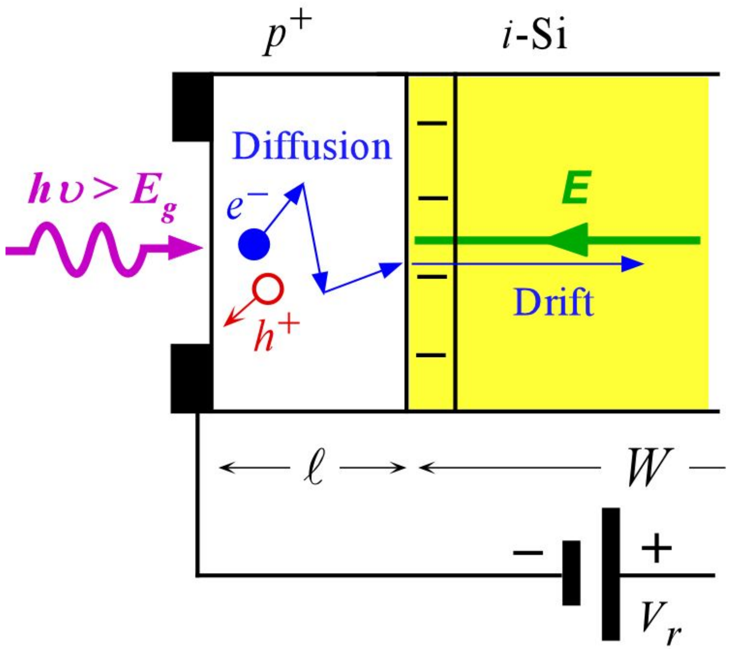
This is the response time of the *pin* as determined by the transit time of the slowest carriers, holes, across the *i*-Si layer. To improve the response time, the width of the *i*-Si layer has to be narrowed but this decreases the quantity of photons absorbed and hence reduces the responsivity. There is therefore a trade off between speed and responsivity.

EXAMPLE : Photocarrier Diffusion in a *pin* photodiode

A reverse biased *pin* photodiode is illuminated with a short wavelength light pulse that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across by the field in this region. What is the speed of response of this photodiode if the *i*-Si layer is 20 μm and the p^+-layer is 1 μm and the applied voltage is 60 V? The diffusion coefficient (D_e) of electrons in the heavily doped p^+-region is approximately 3×10^{-4} m^2 s^{-1}.

Solution

There is no electric field in the p^+-side outside the depletion region as shown in Figure 5.12 . The photogenerated electrons have to make it across to the n^+-side to give rise to a photocurrent. In the p^+-side, the electrons move by diffusion. In time *t,* an electron, on average, diffuses a distance l given by



$$l = [2D_e t]^{1/2}$$

The *diffusion time* t_{diff} is the time it takes for an electron to diffuse across the p^+-side (of length l) to reach the depletion layer and is given by

EXAMPLE: Photocarrier Diffusion in a *pin* photodiode
Solution (continued)

$t_{\text{diff}} = l^2/(2D_e) = (1\times10^{-6}\text{ m})^2 / [2(3\times10^{-4}\text{ m}^2\text{ s}^{-1})] = 1.67\times10^{-9}$ s or **1.67 ns**.

On the other hand, once the electron reaches the depletion region, it becomes drifted across the width W of the *i*-Si layer at the saturation drift velocity since the electric field here is $E = V_r / W$ = 60 V / 20 μm = 3×10^6 V m^{-1}; and at this field the electron drift velocity v_e saturates at 10^5 m s^{-1}. The drift time across the *i*-Si layer is

$$t_{\text{drift}} = W / v_e = (20\times10^{-6}\text{ m}) / (1\times10^5\text{ m s}^{-1}) = 2.0\times10^{-10}\text{ s or } \mathbf{0.2\ ns}.$$

Thus, the response time of the *pin to* a pulse of short wavelength radiation that is absorbed near the surface is very roughly $t_{\text{diff}} + t_{\text{drift}}$ or 1.87 ns. Notice that the diffusion of the electron is much slower than its drift. In a proper analysis, we have to consider the **diffusion and drift of many carriers, and we have to average ($t_{\text{diff}} + t_{\text{drift}}$) for all the electrons**.

EXAMPLE: Steady state photocurrent in the *pin* photodiode

Consider a pin photodiode that is reverse biased and illuminated, as in Figure 5.9, and operating under steady state conditions.

Assume that the photogeneration takes place inside the depletion layer of width W, and the neutral p-side is very narrow.

If the incident optical power on the semiconductor is $P_o(0)$, then $TP_o(0)$ will be transmitted, where T is the transmission coefficient.

At a distance x from the surface, the optical power $P_o(x) = TP_o(0)\exp(-\alpha x)$.

In a small volume δx at x, the absorbed radiation power (by the definition of α) is $\alpha P_o(x)\delta x$, and the number of photons absorbed per second is $\alpha P_o(x)\delta x / h\upsilon$.

Of these absorbed photons, only a fraction η_i will photogenerate EHPs, where η_i is the **internal quantum efficiency** IQE.

Thus, $\eta_i \alpha P_o(x)\delta x / h\upsilon$ number of EHPs will be generated per second.

EXAMPLE: Steady state photocurrent in the *pin* photodiode

We assume these will drift through the depletion region and thereby contribute to the photocurrent. The current contribution δI_{ph} from absorption and photogeneration at x within the SCL will thus be

$$\delta I_{ph} = \frac{e\eta_i \alpha P_o(x)\delta x}{h\upsilon} = \frac{e\eta_i \alpha T P_o(0)}{h\upsilon}\exp(-\alpha x)\delta x$$

We can integrate this from x = 0 (assuming l_p is very thin) to the end of $x = W$, and assuming $W >> L_h$ to find

$$\boxed{I_{ph} \approx \frac{e\eta_i T P_o(0)}{h\upsilon}[1-\exp(-\alpha W)]}$$

Steady state photocurrent pin photodiode (5.5.4)

where the approximate sign embeds the many assumptions we made in deriving Eq. (5.5.4). Consider a *pin* photodiode without an AR coating so that $T = 0.68$. Assume $\eta_i = 1$. The *SCL* width is 20 μm. If the device is to be used at 900 nm, what would be the photocurrent if the incident radiation power is 100 nW? What is the responsivity? Find the photocurrent and the responsivity if a perfect AR coating is used. What is the primary limiting factor? What is the responsivity if $W = 40$ μm?

EXAMPLE: Steady state photocurrent in the *pin* photodiode

Solution (continued)

From Figure 5.5, at $\lambda = 900$ nm, $\alpha \approx 3 \times 10^4$ m^{-1}. Further for $\lambda = 0.90$ μm, the photon energy $h\upsilon = 1.24 / 0.90 = 1.38$ eV. Given $P_o(0) = 100$ nW, we have

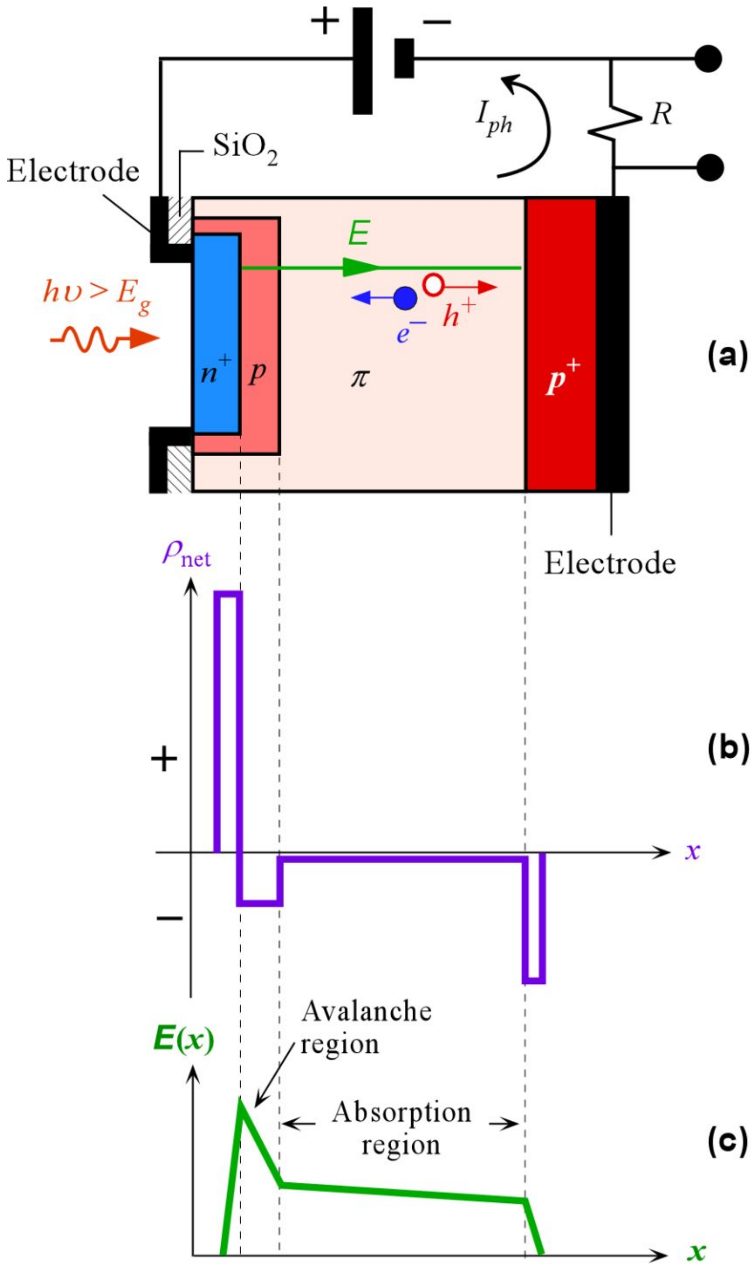
$$I_{ph} \approx \frac{(1.6\times10^{-19})(1)(0.68)(100\times10^{-9})}{(1.38\times1.6\times10^{-19})}[1-\exp(-3\times10^4\times20\times10^{-6})] = \mathbf{22\ nA}$$

and the responsivity R = 22 nA / 100 nW = **0.22 A W^{-1}**, which is on the low-side.

Consider next, a perfect AR coating so that $T = 1$, and using Eq. (5.5.4) again, we find $I_{ph} = 32.7$ nA and ***R* = 0.33 A W^{-1}**, a significant improvement.

The factor $[1-\exp(-\alpha W)]$ is only 0.451, and can be significantly improved by making the SCL thicker. Setting $W = 40$ μm, gives $[1-\exp(-\alpha W)] = 0.70$ and R = 0.51, which is close to values for commercial devices.

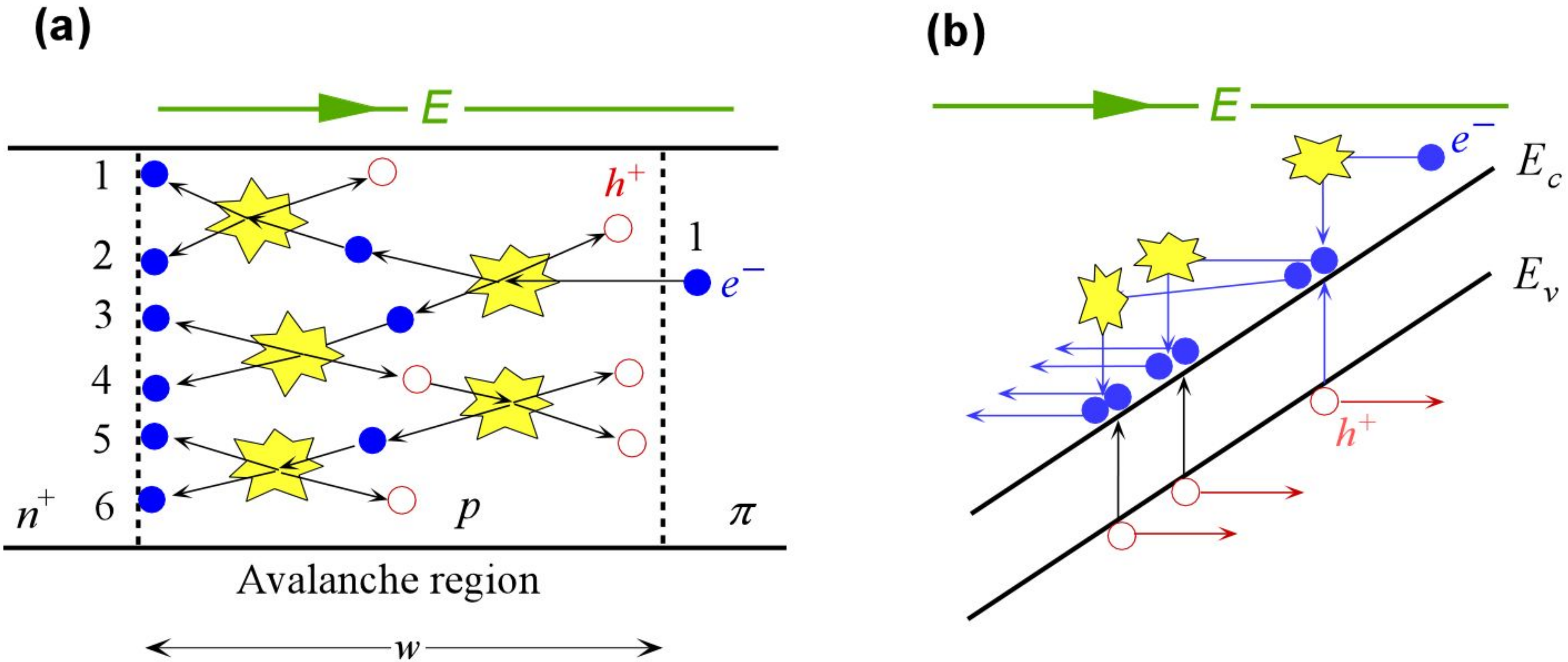
The maximum theoretical photocurrent would be obtained by setting $\exp(-\alpha W) \approx 0$, $T = 1$, $\eta_i = 1$, which gives $I_{ph} = 73$ nA and R = **0.73 A W^{-1}**.



Avalanche Photodiode

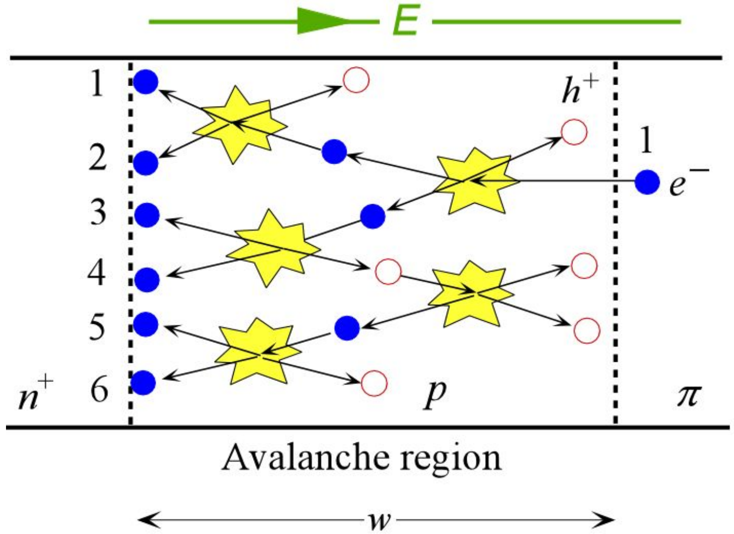
(a) A schematic illustration of the structure of an avalanche photodiode (APD) biased for avalanche gain. (b) The net space charge density across the photodiode. (c) The field across the diode and the identification of absorption and multiplication regions.

Avalanche Photodiode



(a) A pictorial view of impact ionization processes releasing EHPs and the resulting avalanche multiplication. (b) Impact of an energetic conduction electron with crystal vibrations transfers the electron's kinetic energy to a valence electron and thereby excites it to the conduction band

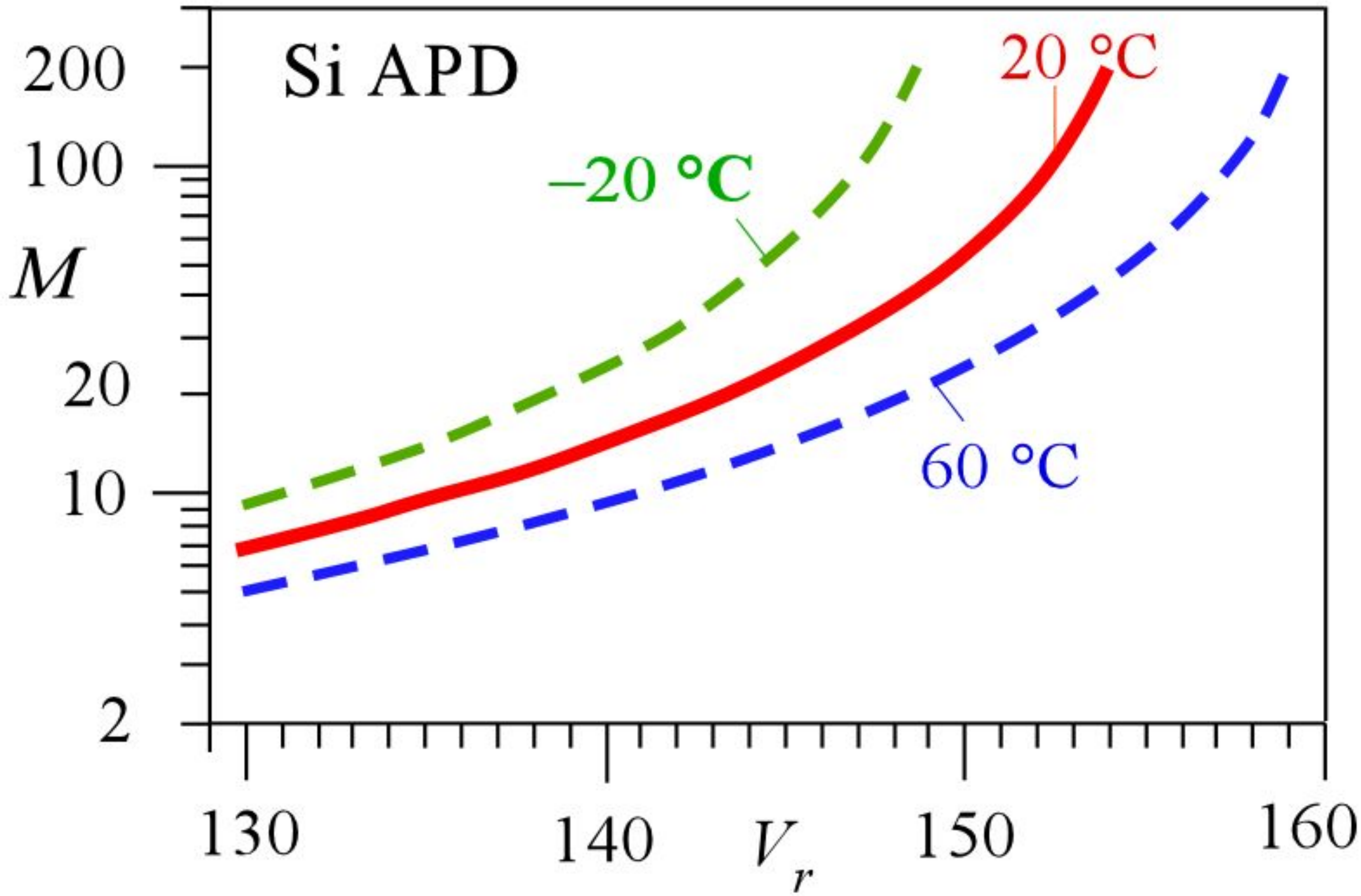
Avalanche Photodiode Gain or Multiplication M



$$M = \frac{\text{Multiplied photocurrent}}{\text{Primary unmultiplied photocurrent}} = \frac{I_{ph}}{I_{pho}}$$

$$M = \frac{1}{1 - \left(\dfrac{V_r}{V_{\text{br}}}\right)^m}$$

Avalanche Photodiode

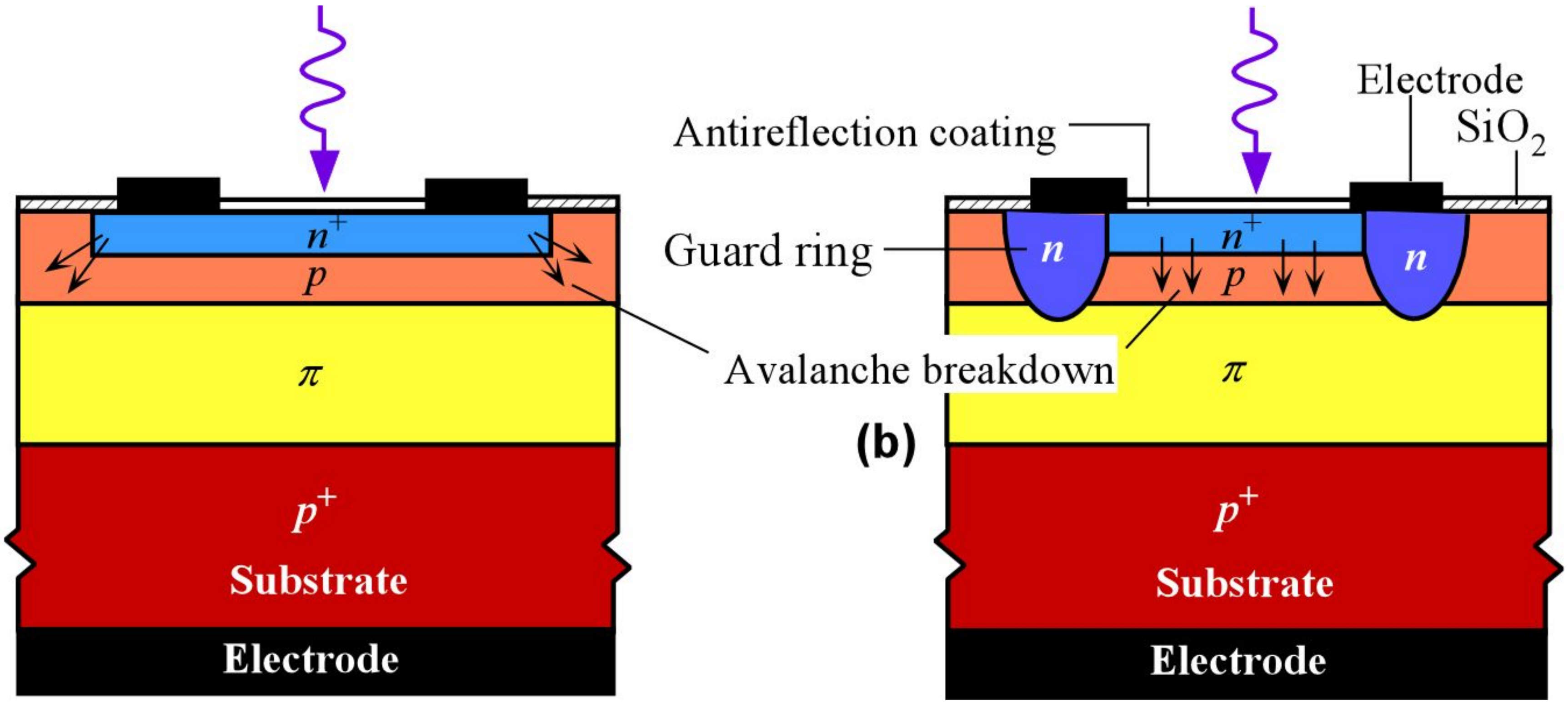


Typical multiplication (gain) M vs. reverse bias characteristics for a typical commercial Si APD, and the effect of temperature. (M measured for a photocurrent generated at 650 nm of illumination)

Avalanche Photodiode

(a)

(b)



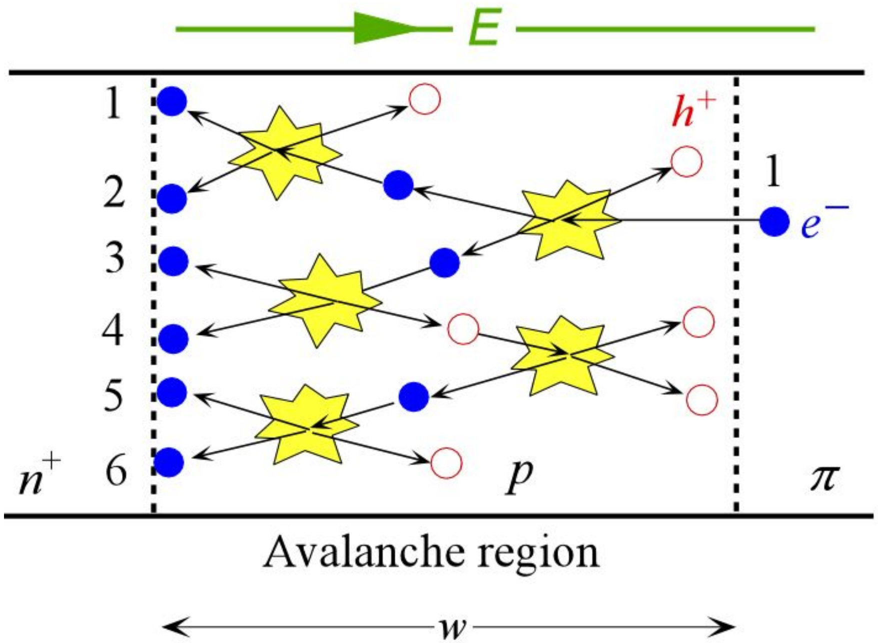
(a) A Si APD structure without a guard ring. (b) A schematic illustration of the structure of a more practical Si APD. Note: SiO_2 is silicon dioxide and serves as an insulating passivation layer.

Photodiode Comparison

Photodiode	λ_{range}	λ_{peak}	R at λ_{peak}	Gain	I_d For 1 mm^2	Features
	nm	nm	A/W			
GaP *pin*	150–550	450	0.1	<1	1 nm	UV detection[a]
GaAsP *pn*	150–750	500–720	0.2–0.4	<1	0.005–0.1 nA	UV to visible, covering the human eye, low I_d.
GaAs *pin*	570–870	850	0.5–0.5	<1	0.1–1 nA	High speed and low I_d
Si *pn*	200–1100	600–900	0.5–0.6	<1	0.005–0.1 nA	Inexpensive, general purpose, low I_d
Si *pin*	300–1100	800–1000	0.5–0.6	<1	0.1–1 nA	Faster than *pn*
Si APD	400–1100	800–900	0.4–0.6[b]	10–10^3	1–10 nA[c]	High gains and fast
Ge *pin*	700–1800	1500–1580	0.4–0.7	<1	0.1–1 μA	IR detection, fast.
Ge APD	700–1700	1500–1580	0.4–0.8[b]	10–20	1–10 μA[c]	IR detection, fast
InGaAs *pin*	800–1700	1500–1600	0.7–1	<1	1–50 nA	Telecom, high speed, low I_d
InGaAs APD	800–1700	1500–1600	0.7–0.95[b]	10–20	0.05–10 μA[c]	Telecom, high speed and gain.
InAs *pn*	2–3.6 μm	3.0–3.5 μm	1–1.5	<1	>100 μA	Photovoltaic mode. Normally cooled
InSb *pn*	4–5.5 μm	5 μm	3	<1	Large	Photovoltaic mode. Normally cooled

NOTE: [c]FGAP71 (Thorlabs); [a]At M = 1; [c]At operating multiplication.

Avalanche Photodiode Gain or Multiplication *M*



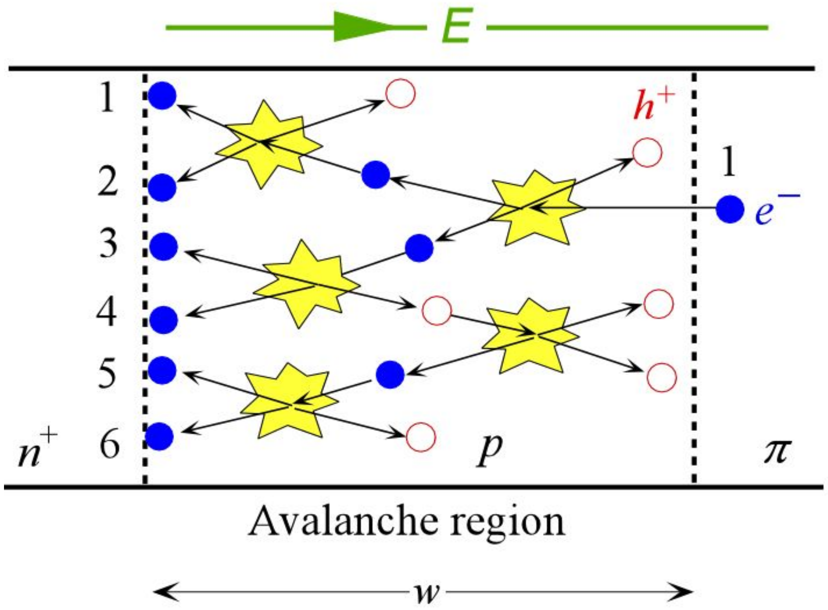
Ionization coefficient ratio

$$k = \frac{\text{Ionization coefficient for holes}}{\text{Ionization coefficient for electrons}} = \frac{\alpha_h}{\alpha_e}$$

$$\alpha_e = A\exp(-B/E)$$

Chyoweth's law

Avalanche Photodiode Gain or Multiplication M



Electrons only

$$M = \exp(\alpha_e w)$$

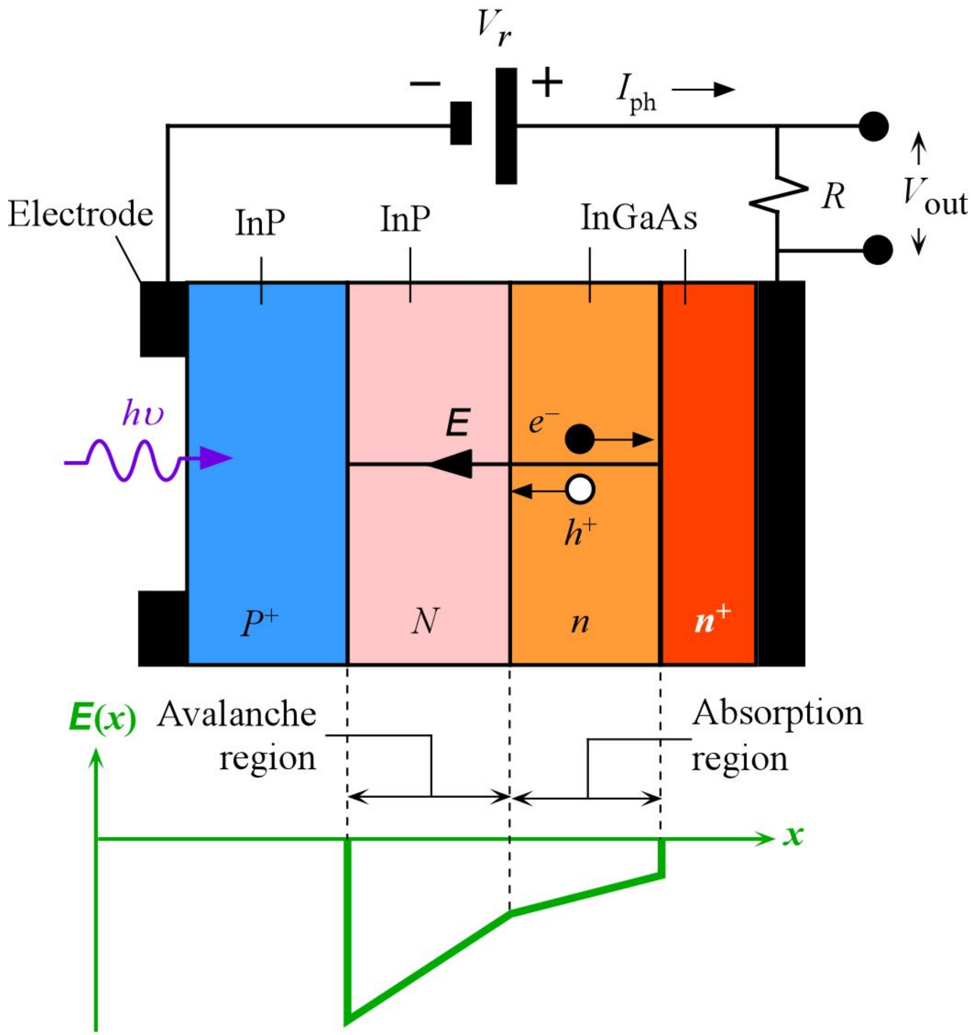
Ionization coefficient

Electrons and holes

$$M = \frac{1-k}{\exp[-(1-k)\alpha_e w] - k}$$

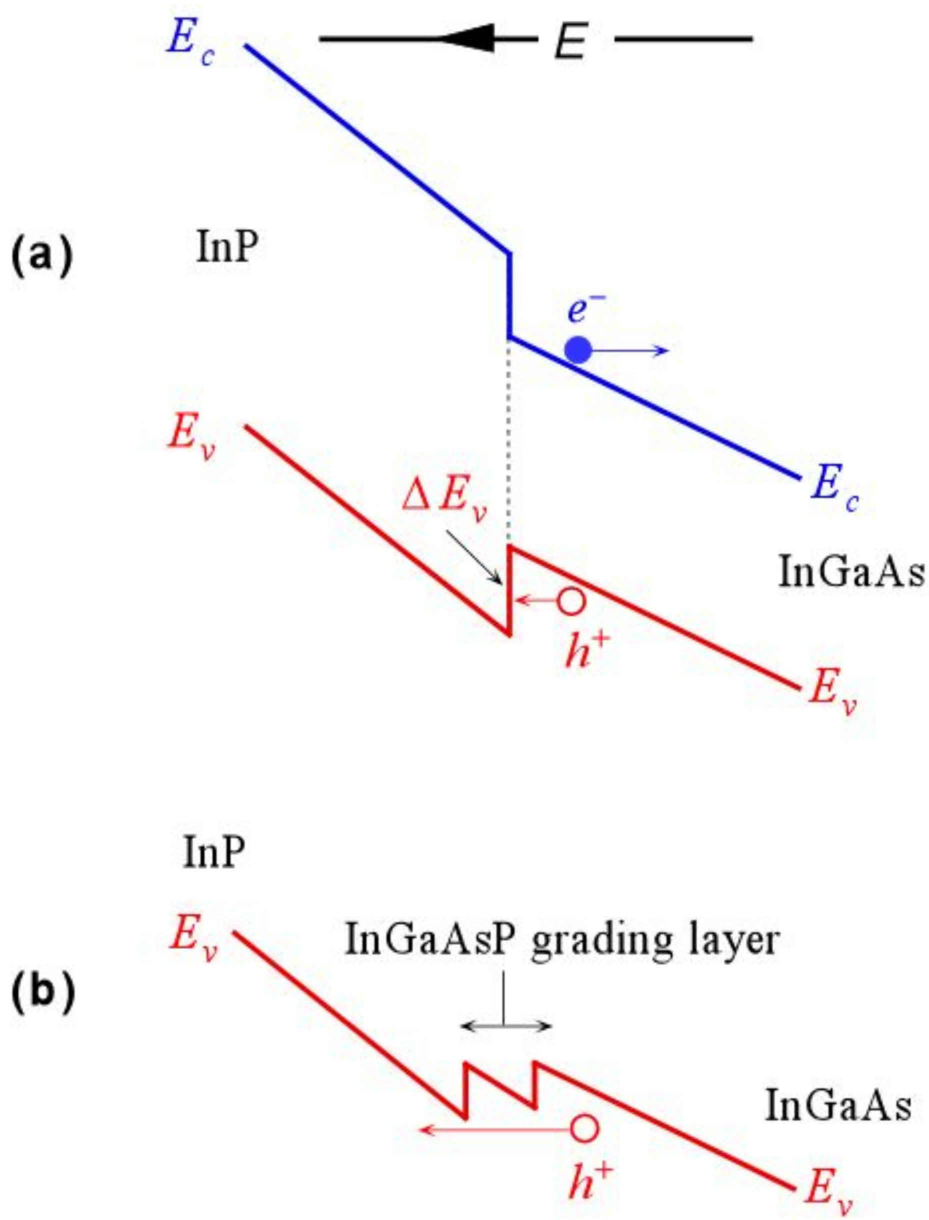
$$k = \alpha_h / \alpha_e$$

Heterojunction Photodiodes: SAM



Simplified schematic diagram of a **separate absorption and multiplication** (SAM) APD using a heterostructure based on InGaAs-InP. *P* and *N* refer to *p* and *n* -type wider-bandgap semiconductor.

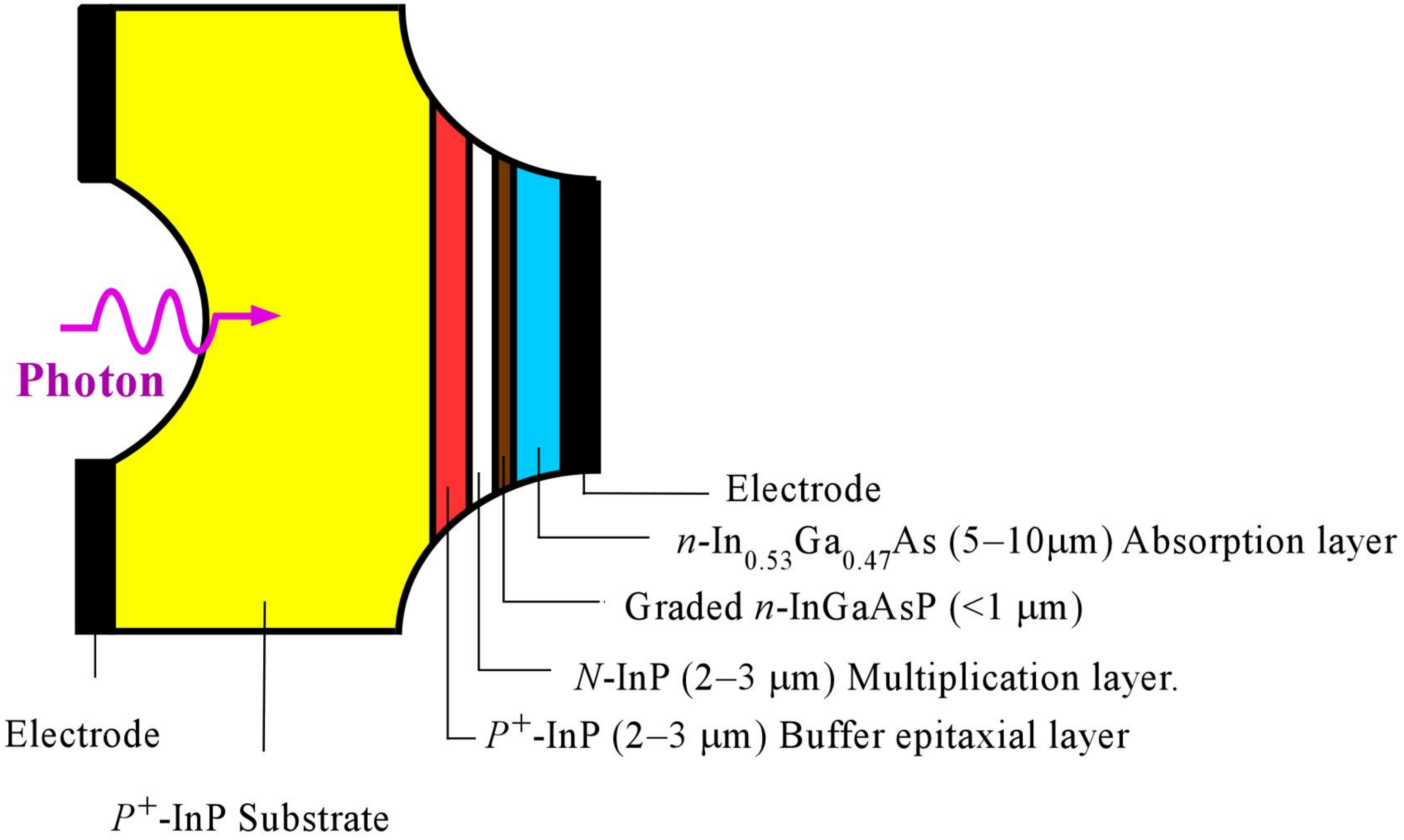
Heterojunction Photodiodes: SAM



(a) Energy band diagrams for a SAM detector with a step junction between InP and InGaAs. There is a valence band step ΔE_v from InGaAs to InP that slows hole entry into the InP layer.

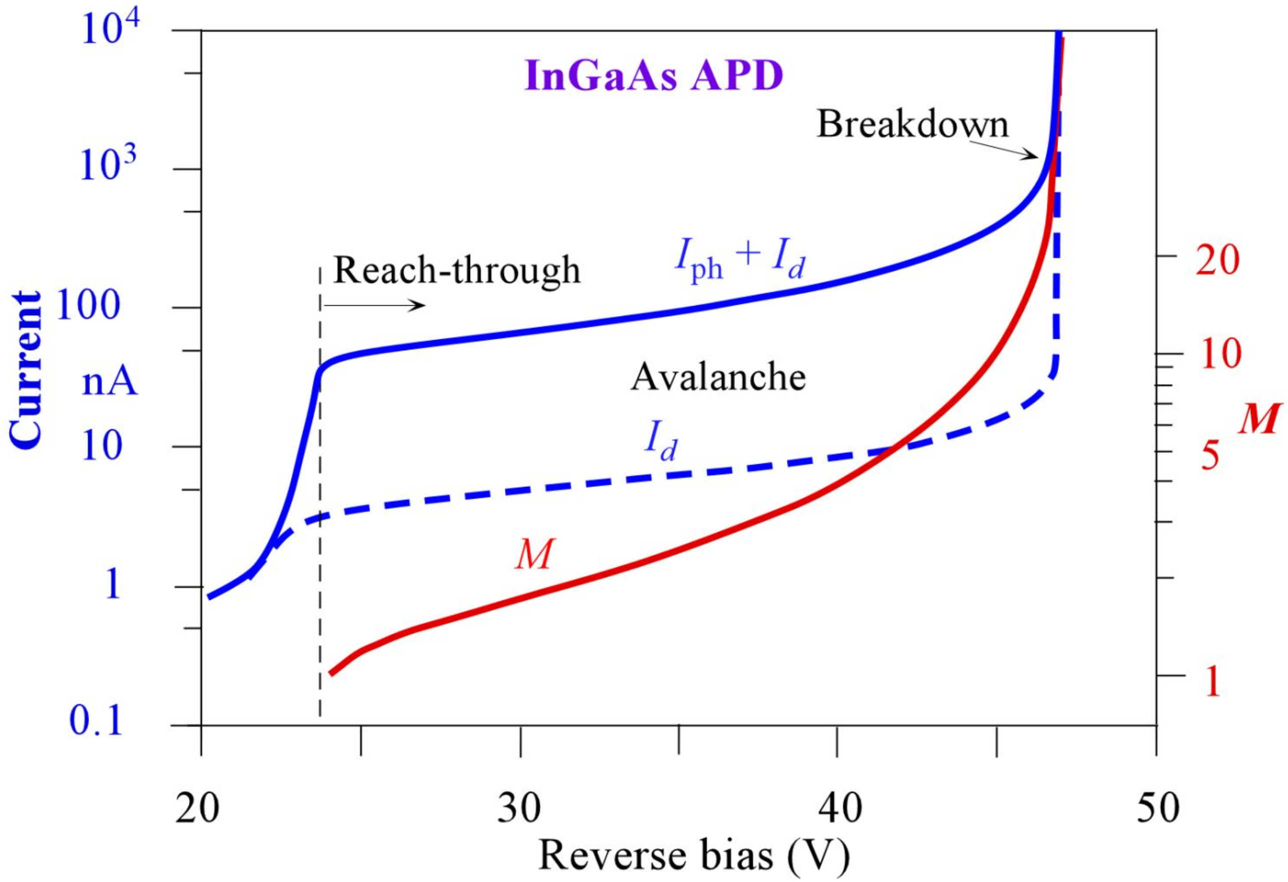
(b) An interposing grading layer (InGaAsP) with an intermediate bandgap breaks ΔE_v and makes it easier for the hole to pass to the InP layer for a detector with a graded junction between InP and InGaAs. This is the SAGM structure.

Heterojunction Photodiodes: SAM



Simplified schematic diagram of a more practical mesa-etched SAGM layered APD

APD Characteristics



Typical current and gain (M) vs. reverse bias voltage for a commercial InGaAs reach-through APD. I_d and I_{ph} are the dark current and photocurrent respectively. The input optical power is ~100 nW. The gain M is 1 when the diode has attained reach-through and then increases with the applied voltage. (The data extracted selectively from Voxtel Catalog, Voxtel, Beaverton, OR 97006)

EXAMPLE: InGaAs APD Responsivity

An InGaAs APD has a quantum efficiency (QE, η_e) of 60 % at 1.55 μm in the absence of multiplication (M = 1). It is biased to operate with a multiplication of 12. Calculate the photocurrent if the incident optical power is 20 nW. What is the responsivity when the multiplication is 12?

Solution

The responsivity at M = 1 in terms of the quantum efficiency is

$$R = \eta_e \frac{e\lambda}{hc} = (0.6)\frac{(1.6\times10^{-19}\ \mathrm{C})(1550\times10^{-9}\ \mathrm{m})}{(6.626\times10^{-34}\ \mathrm{J\ s})(3\times10^{8}\ \mathrm{m\ s^{-1}})} = \mathbf{0.75\ A\ W^{-1}}$$

If I_{pho} is the primary photocurrent (unmultiplied) and P_o is the incident optical power then by definition, $R = I_{pho}/P_o$ so that

$$I_{pho} = RP_o$$
$$= (0.75\ \mathrm{A\ W^{-1}})(20\times10^{-9}\ \mathrm{W})$$
$$= 1.5\times10^{-8}\ \mathrm{A\ or\ 15\ nA.}$$

The photocurrent I_{ph} in the APD will be I_{pho} multiplied by M,

$$I_{ph} = MI_{pho}$$
$$= (12)(1.5\times10^{-8}\ \mathrm{A})$$
$$= 1.80\times10^{-7}\ \mathrm{A\ or\ 180\ nA.}$$

The responsivity at M = 12 is

$$R' = I_{ph}/P_o = MR = (12) / (0.75) = \mathbf{9.0\ A\ W^{-1}}$$

EXAMPLE: Silicon APD

A Si APD has a QE of 70 % at 830 nm in the absence of multiplication, that is $M = 1$. The APD is biased to operate with a multiplication of 100. If the incident optical power is 10 nW what is the photocurrent?

Solution

The unmultiplied responsivity is given by,

$$R = \eta_e \frac{e\lambda}{hc} = (0.70)\frac{(1.6\times10^{-19}\ \text{C})(830\times10^{-9}\ \text{m})}{(6.626\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})} = 0.47\ \text{A W}^{-1}$$

The unmultiplied primary photocurrent from the definition of R *is*

$$I_{pho} = RP_o = (0.47\ \text{A W}^{-1})(10\times10^{-9}\ \text{W}) = 4.7\ \text{nA}$$

The multiplied photocurrent is

$$I_{ph} = MI_{pho} = (100)(4.67\ \text{nA}) = \mathbf{470\ nA} \text{ or } \mathbf{0.47\ \mu A}$$

EXAMPLE: Avalanche multiplication in Si APDs

The electron and hole ionization coefficients α_e and α_h in silicon are approximately given by Eq. (5.6.4) with $A \approx 0.740\times10^6$ cm^{-1}, $B \approx 1.16\times10^6$ V cm^{-1} for electrons (α_e) and $A \approx 0.725\times10^6$ cm^{-1} and $B \approx 2.2\times10^6$ V cm^{-1} for holes (α_h). Suppose that the width w of the avalanche region is 0.5 μm. Find the multiplication gain M when the applied field in this region reaches 4.00×10^5 V cm^{-1}, 4.30×10^5 V cm^{-1} and 4.38×10^5 V cm^{-1}. What is your conclusion?

Solution

At the field of $E = 4.00\times10^5$ V cm^{-1}, from Eq. (5.6.4)

$$\alpha_e = A\exp(-B/E)$$

$$= (0.74\times10^6 \text{ cm}^{-1})\exp[-(1.16\times10^6 \text{ V cm}^{-1})/(4.00\times10^5 \text{ V cm}^{-1})] = 4.07\times10^4 \text{ cm}^{-1}.$$

Similarly using Eq. (5.6.4) for holes, $\alpha_h = 2.96\times10^3$ cm^{-1}. Thus $k = \alpha_h/\alpha_e = 0.073$. Using this k and α_e above in Eq. (5.6.6) with $w = 0.5\times10^{-4}$ cm,

$$M = \frac{1-0.073}{\exp[-(1-0.073)(4.07\times10^4 \text{ cm})(0.5\times10^{-4} \text{ cm}^{-1})]-0.073} = 11.8$$

Note that if we had only electron avalanche without holes ionizing, then the multiplication would be

$$M_e = \exp(\alpha_e w) = \exp[(4.07\times10^4 \text{ cm}^{-1})(0.5\times10^{-4} \text{ cm})] = 7.65$$

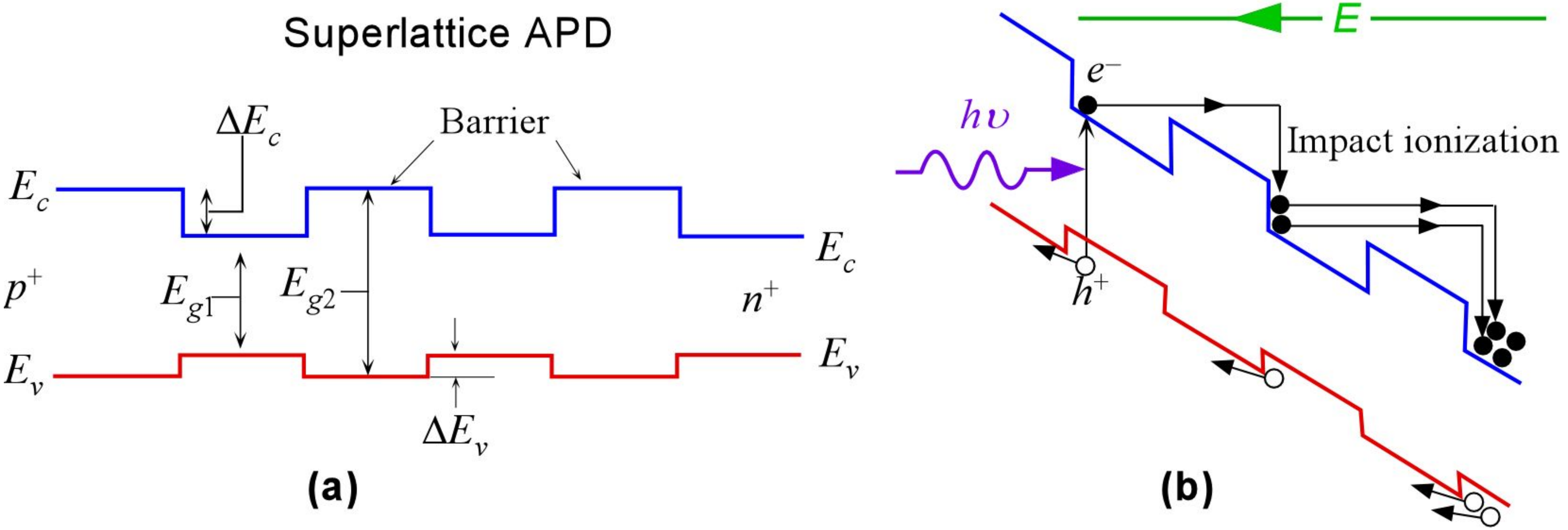
EXAMPLE: Avalanche multiplication in Si APDs

Solution (contiued)

We can now repeat the calculations for $E = 4.30\times10^5$ V cm^{-1} and again for $E = 4.38\times10^5$ V cm^{-1}. The results are summarized in Table 5.3 for both M and M_e. **Notice how quickly *M* builds up with the field and how a very small change at high fields causes an enormous change in *M* that eventually leads to a breakdown**. (M running away to infinity as V_r increases.) Notice also that in the presence of only electron-initiated ionization, M_e simply increases without a sharp run-away to breakdown.

E (V cm^{-1})	a_e (cm^{-1})	a_h (cm^{-1})	k	M	M_e	Comment
4.00×10^5	4.07×10^4	2.96×10^3	0.073	11.8	7.65	M and Me not too different at low E
4.30×10^5	4.98×10^4	4.35×10^3	0.087	57.2	12.1	7.5% increase in E, large difference between M and M_e
4.38×10^5	5.24×10^4	4.77×10^3	0.091	647	13.7	1.9% increase in E

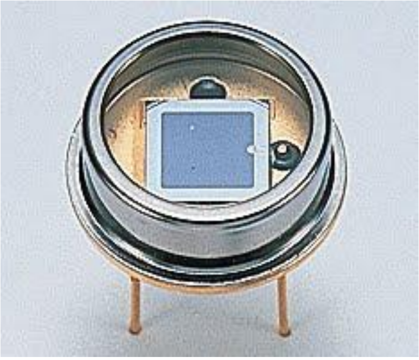
Superlattice APD
Multiple Quantum Well Detectors



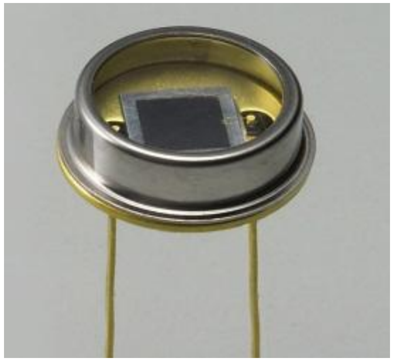
(a) Energy band diagram of a MQW superlattice APD.
(b) Energy band diagram with an applied field and impact ionization.

Schottky Junction Photodiodes

Schottky kunction type metal-semiconductor-metal (MSM) type photodetectors. (Courtesy of Hamamatsu)



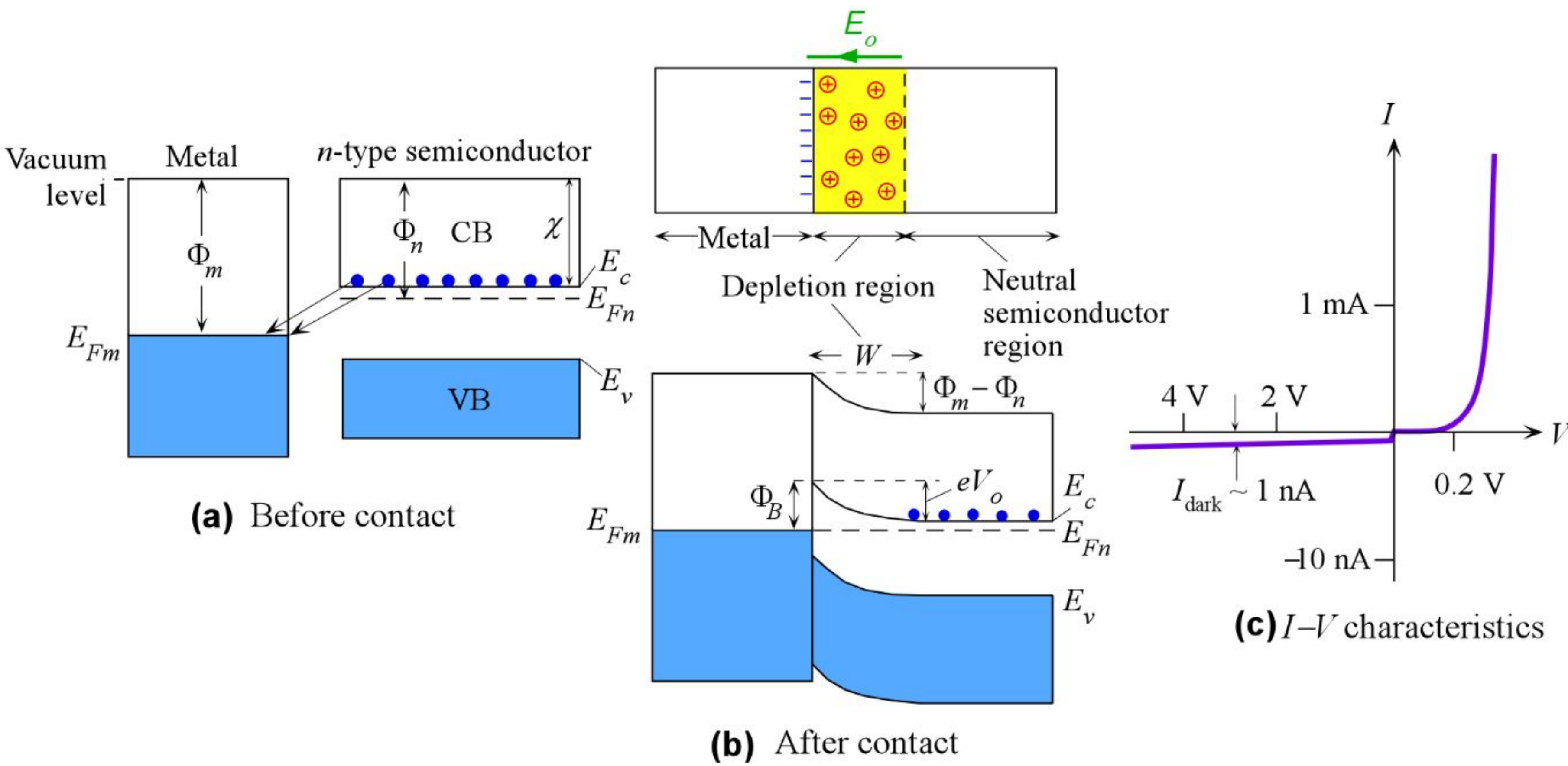
GaAsP Schottky junction photodiode for 190-680 nm detection, from UV to red (Courtesy of Hamamatsu)



GaP Schottky junction photodiode for 190 nm to 550 nm detection. (Courtesy of Hamamatsu)

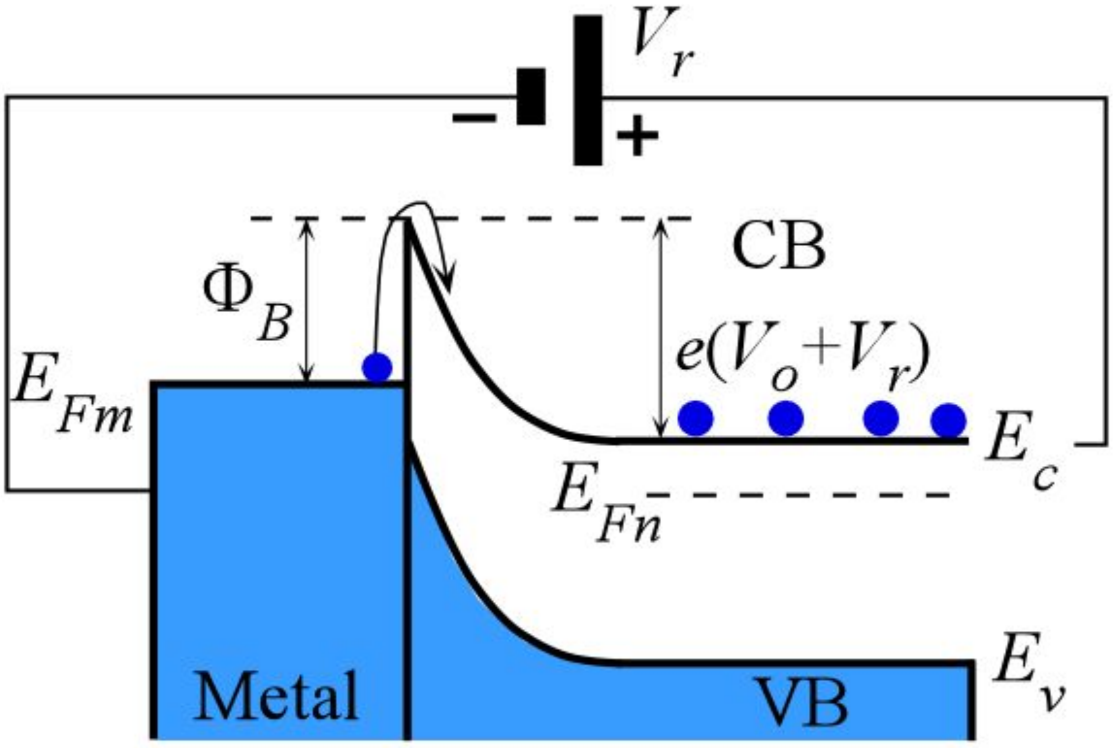
AlGaN Scottky junction photodiode for UV detection (Courtesy of sglux, Germany)

Schottky Junction



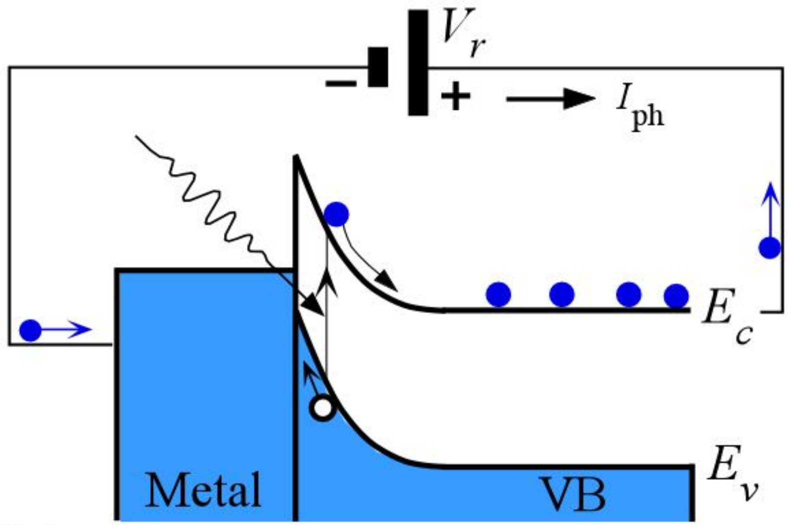
(a) Metal and an *n*-type semiconductor before contact. The metal work function Φ_m is greater than that of the *n*-type semiconductor (b) A Schottky junction forms between the metal and the semiconductor. There is a depletion region in the semiconductor next to the metal and a built-in field E_o (c) Typical *I* vs. *V* characteristics of a Schottky contact device.

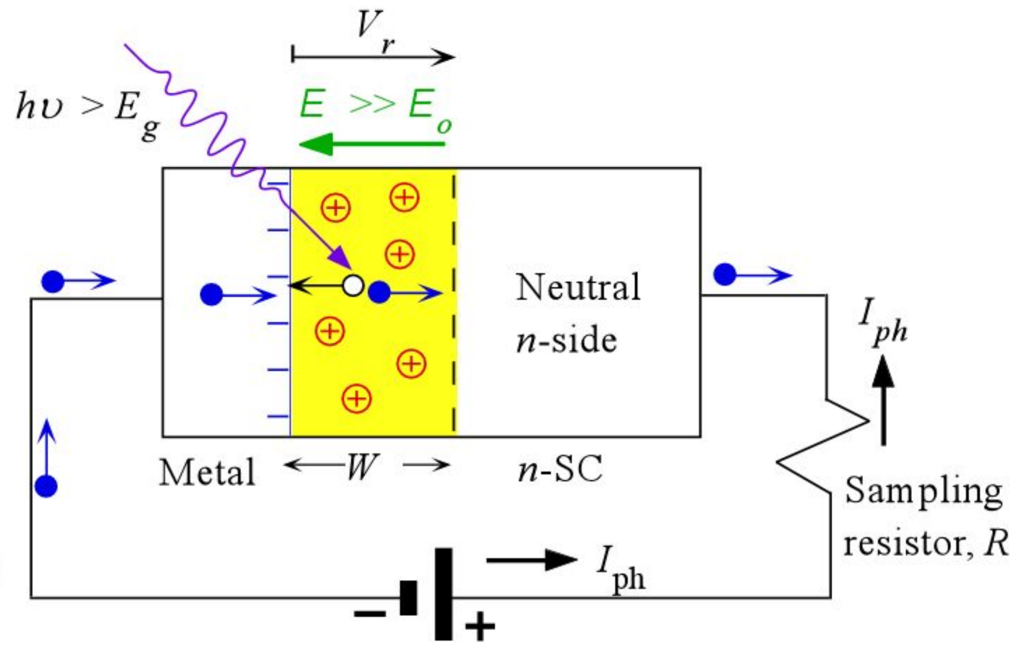
Schottky Junction



Reverse biased Schottky junction and the dark current due to the injection of electrons from the metal into the semiconductor over the barrier Φ_B.

Schottky Junction





LEFT: Photogeneration in the depletion region and the resulting photocurrent. RIGHT: The Schottky junction photodetector

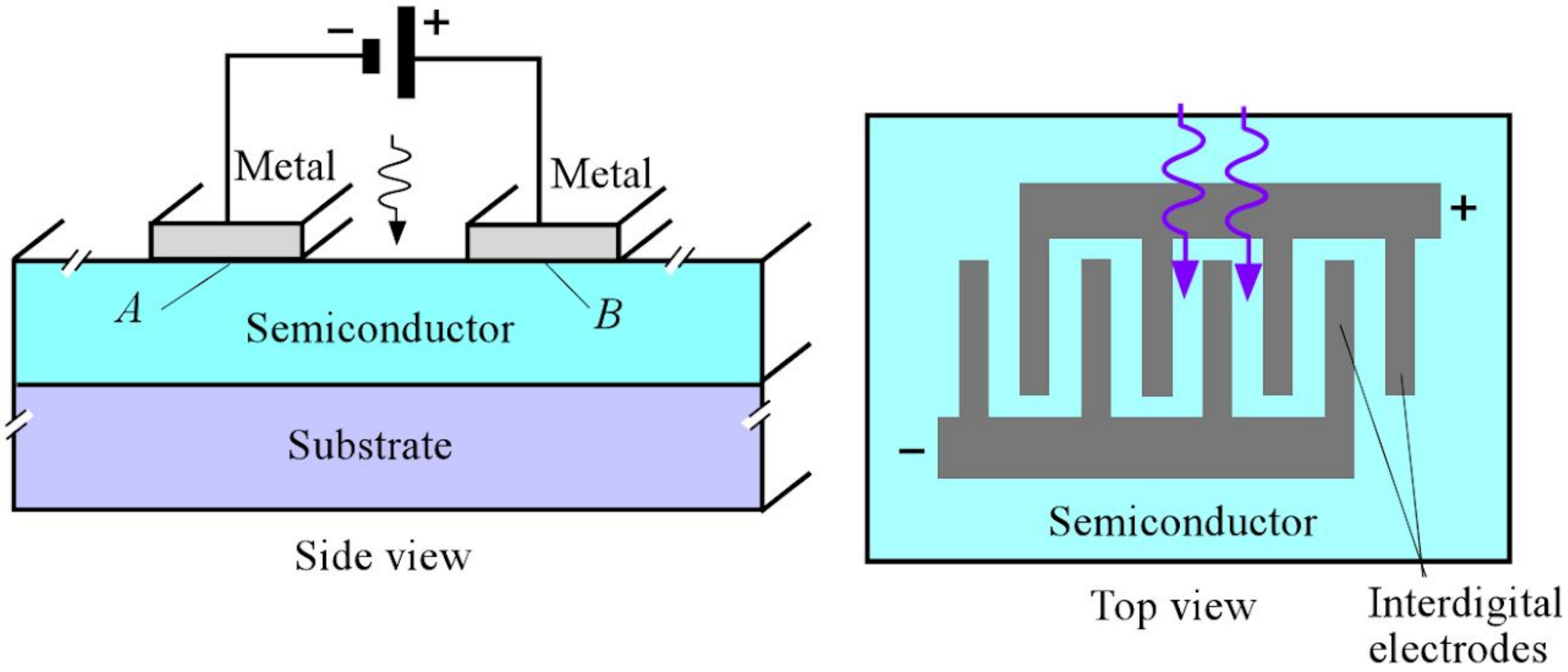
Schottky Junction Photodiodes

Schottky junction based photodetectors and some of their features. τ_R and τ_F are the rise and fall times of the output of the photodetector for an optical pulse input. The rise and fall times represent the times required for the output to rise from 10% to 90% of its final steady state value and to fall from 90% to 10% of its value before the optical pulse is turned off.

Schottky junction	λ range nm	R_{peak} (at peak) (A/W)	J_{dark} per mm^2	Features with typical values
GaAsP	190−680	0.18 (610 nm)	5 pA	UV to red, τ_R = 3.5 μs. (G1126 series[a])
GaP	190−550	0.12 (440 nm)	5 pA	UV to green, τ_R = 5 μs. (G1961[a])
AlGaN	220−375	0.13 (350 nm)	1 pA	Measurement of UV; blind to visible light. (AG38S[b])
GaAs	320−900	0.2 (830 nm)	~ 1 nA	Wide bandwidth > 10 GHz, τ_R < 30 ps. (UPD-30-VSG-P[c])
InGaAs MSM	850−1650	0.4 (1300 nm)	5 μA	Optical high speed measurements, τ_R = 80 ps, τ_F = 160 ps. (G7096[a])
GaAs MSM	450−870	0.3 (850 nm)	0.1 nA	Optical high speed measurements, τ_R = 30 ps, τ_F = 30 ps. (G4176[a])

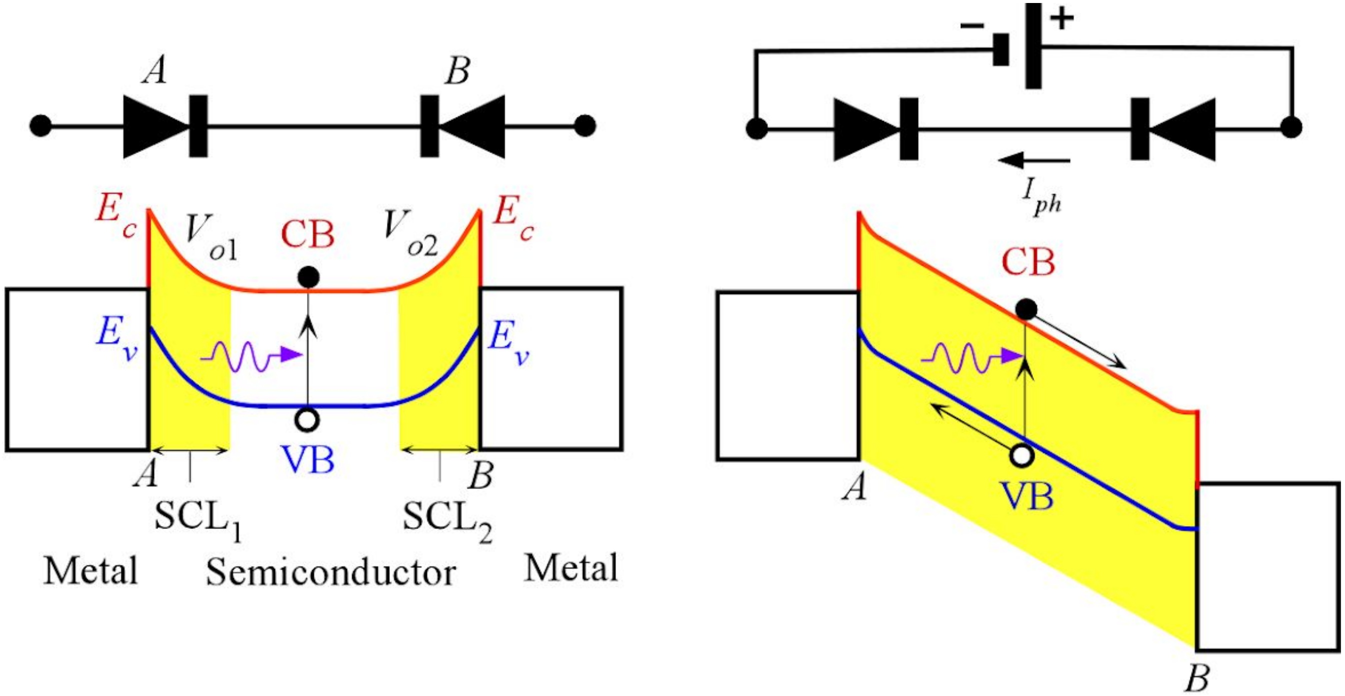
[a]Hamamatsu (Japan); [b]sglux (Germany); [c]Alphalas

Schottky Junction Photodiodes



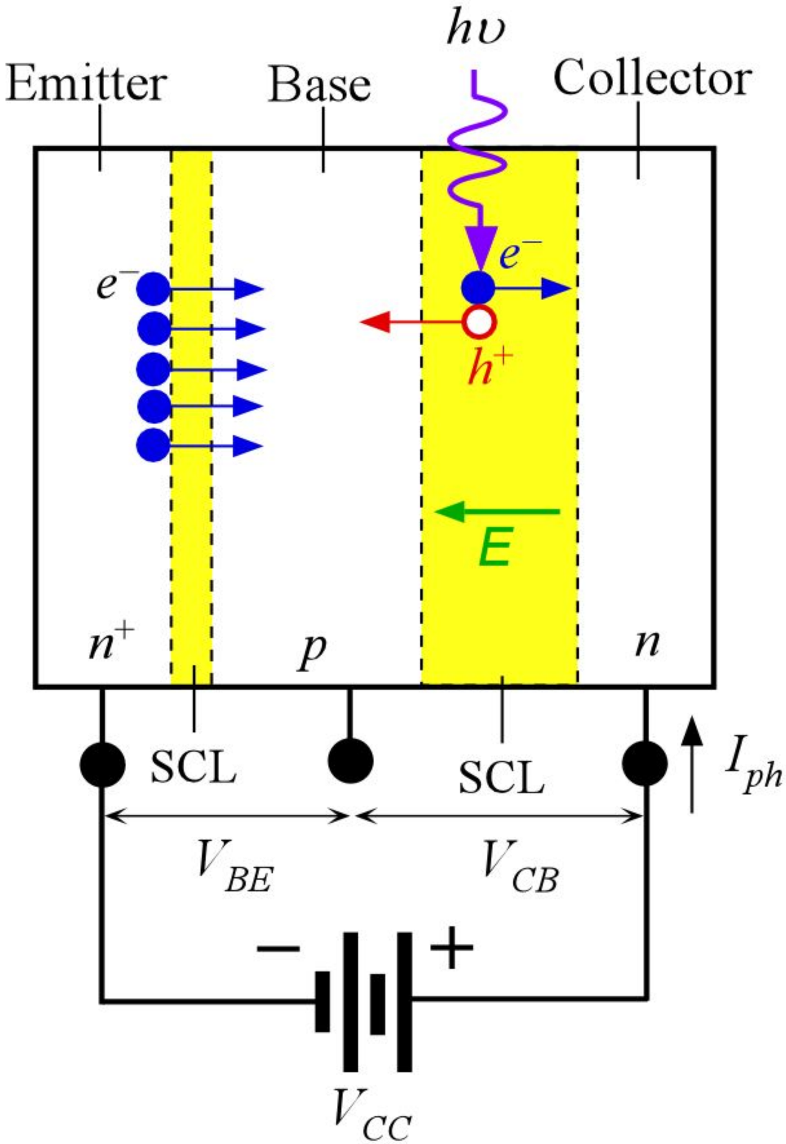
LEFT: The metal electrodes are on the surface of the semiconductor crystal (which is grown on a suitable substrate). RIGHT: The electrodes are configured to be interdigital and on the surface of the crystal.

Schottky Junction Photodiodes



LEFT: Two neighboring Schottky junctions are connected end-to-end, but in opposite directions as shown for *A* and *B*. The energy band diagram without any bias is symmetrical. The grey areas represent the SCL_1 and SCL_2 at *A* and *B*. RIGHT: Under a sufficiently large bias, the SCL_1 from *A* extends and meets that from *B* so that the whole semiconductor between the electrodes is depleted. There is a large field in this region, and the photogenerated EHPs become separated and then drifted, which results in a photocurrent.

Phototransistor



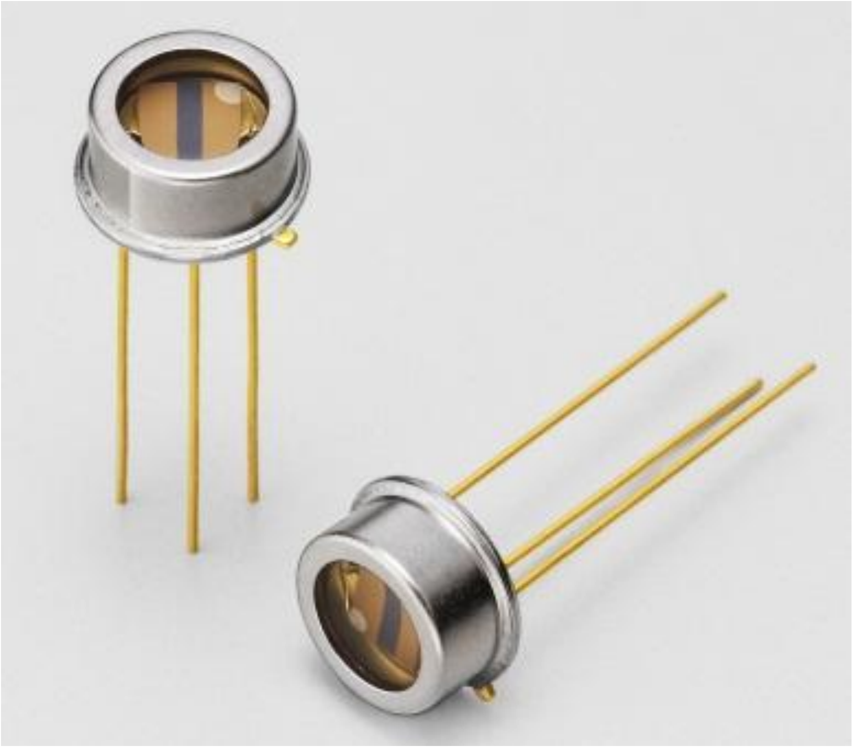
Transistor action

$$I_E \propto \exp(eV_{BE}/k_BT)$$

Gain

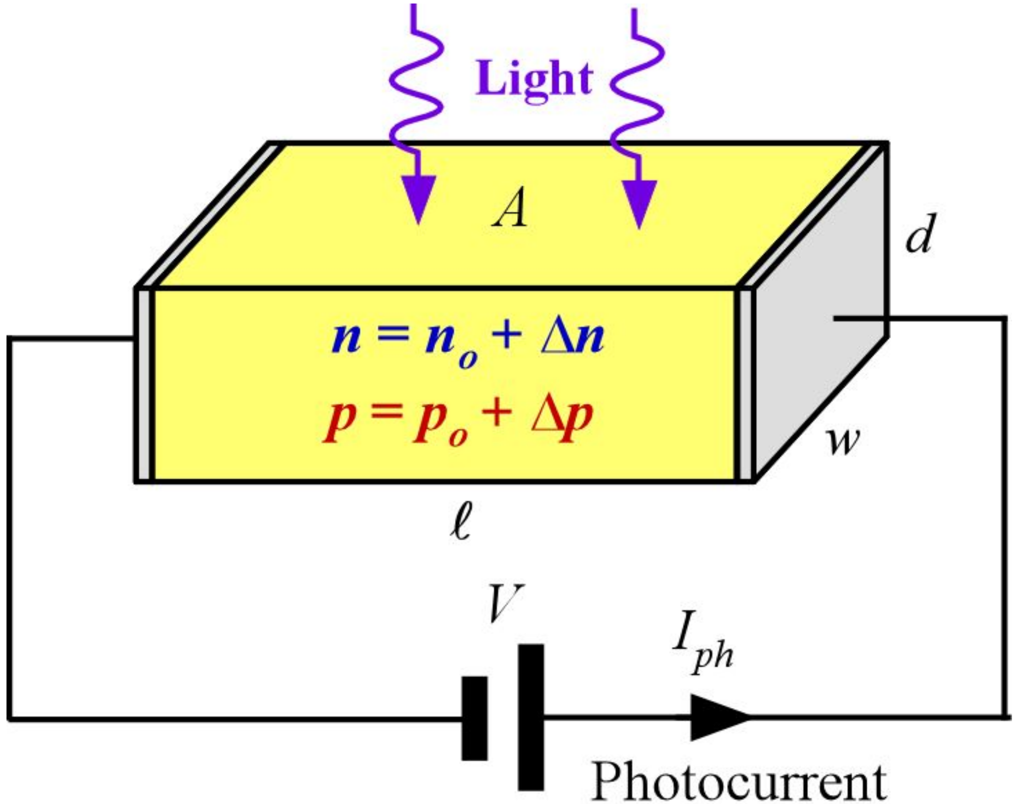
$$I_{ph} \approx \beta I_{pho}$$

Photoconductive Detectors



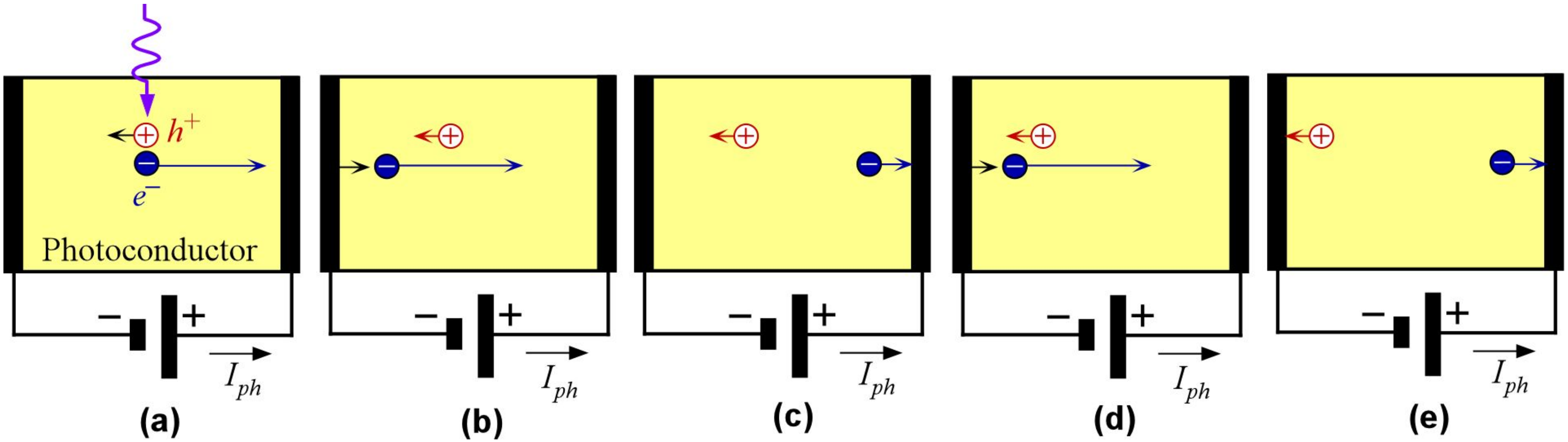
PbS (lead sulfide) photoconductive detectors for the detection of IR radiation up to 2.9 μm. They are typically used in such applications as radiation thermometers, flame monitors, water content and food ingredient analyzers, spectrophotometers *etc.*. (P9217 series) (Courtesy of Hamamatsu.)

Photoconductive Detectors



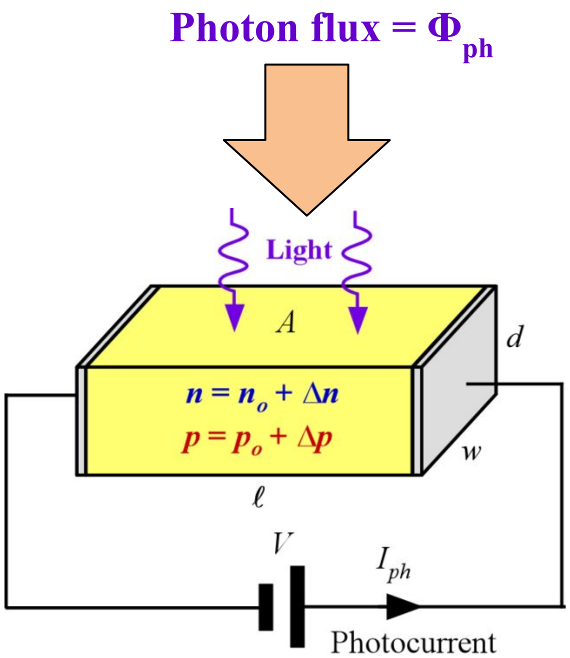
A semiconductor slab of length l, width w and depth d is illuminated with light of wavelength λ

Photoconductive Detectors



A photoconductor with ohmic contacts (contacts not limiting carrier entry) can exhibit gain. As the slow hole drifts through the photoconductors, many fast electrons enter and drift through the photoconductor because, at any instant, the photoconductor must be neutral. Electrons drift faster which means as one leaves, another must enter.

Photoconductivity $\Delta\sigma$ and Photocurrent Density J_{ph}



Photogeneration rate

$$g_{\mathrm{ph}} = \frac{\eta_i A \Phi_{\mathrm{ph}}}{Ad} = \frac{\eta_i \left(\frac{I}{h\nu}\right)}{d} = \frac{\eta_i I \lambda}{hcd}$$

η_i = Internal quantum efficiency

Steady state illumination

$$\frac{d\Delta n}{dt} = g_{\mathrm{ph}} - \frac{\Delta n}{\tau} = 0$$

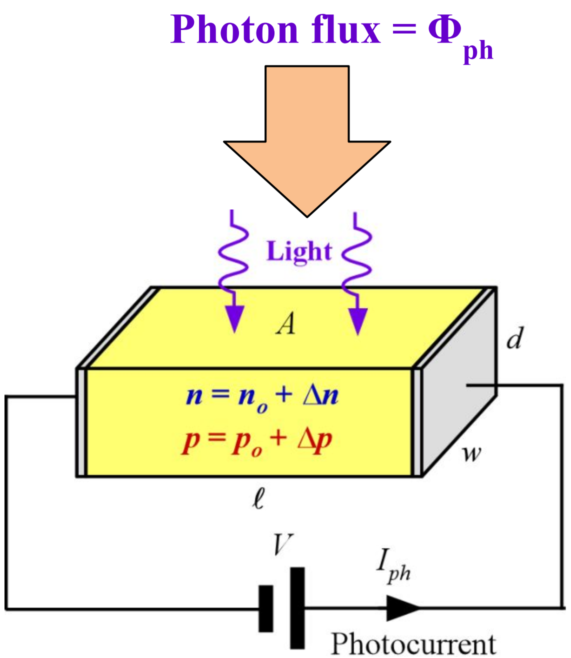
Photoconductivity

$$\Delta\sigma = e\mu_e \Delta n + e\mu_h \Delta p = e\Delta n(\mu_e + \mu_h)$$

$$\Delta\sigma = \frac{e\eta_i I \lambda \tau(\mu_e + \mu_h)}{hcd}$$

$$J_{\mathrm{ph}} = \Delta\sigma \frac{V}{\ell} = \Delta\sigma E$$

Photoconductive Gain



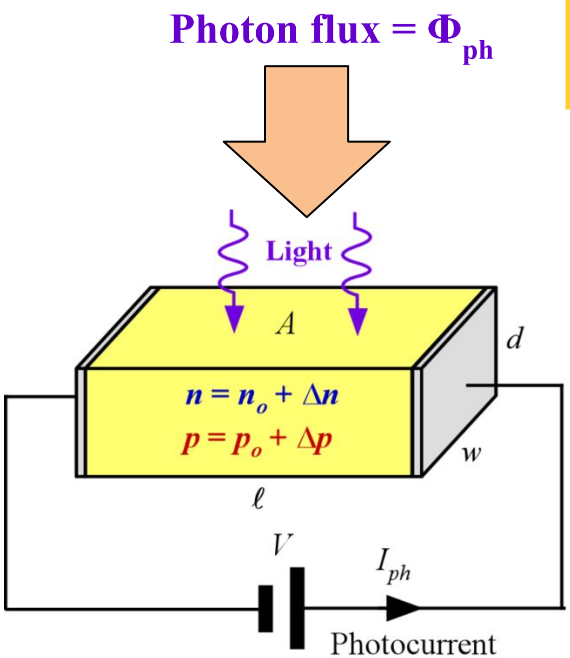
$$\text{Rate of electron flow} = \frac{I_{\text{ph}}}{e} = \frac{wdJ_{\text{ph}}}{e} = \frac{\eta_i I w \lambda \tau (\mu_e + \mu_h) E}{hc}$$

$$\text{Rate of electron generation} = (\text{Volume}) g_{\text{ph}} = (wd\square) g_{\text{ph}} = w\square \frac{\eta_i I \lambda}{hc}$$

Photoconductive gain *G*

$$G = \frac{\text{Rate of electron flow in external circuit}}{\text{Rate of electron generation by light absorption}} = \frac{\tau(\mu_e + \mu_h)E}{\square}$$

Photoconductive Gain



$$G = \frac{\text{Rate of electron flow in external circuit}}{\text{Rate of electron generation by light absorption}} = \frac{\tau(\mu_e + \mu_h)E}{\ell}$$

Electron and hole transit times (time to cross the semiconductor) are

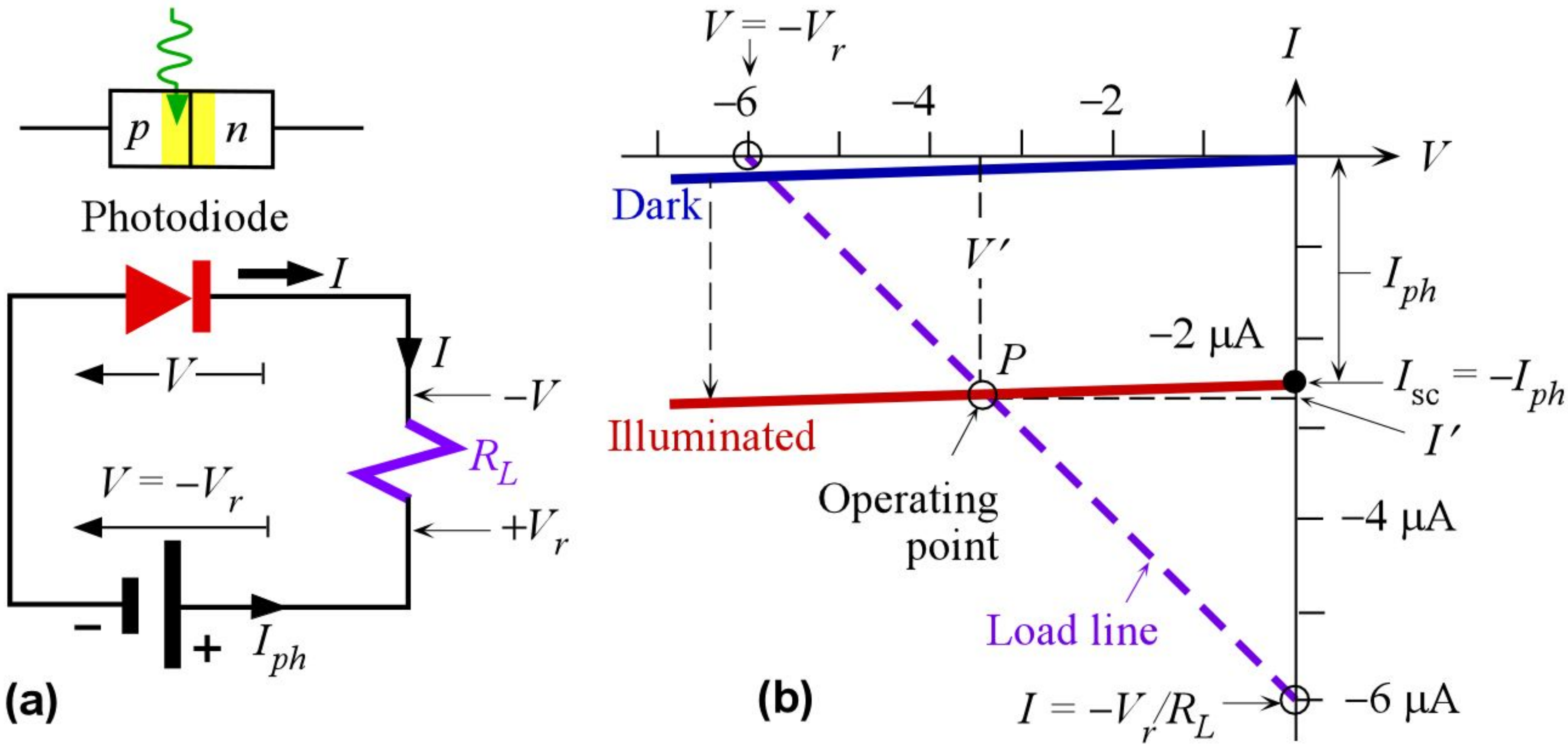
$$t_e = l/(\mu_e E)$$ Electron

$$t_h = l/(\mu_h E)$$ Hole

Photoconductive gain *G*

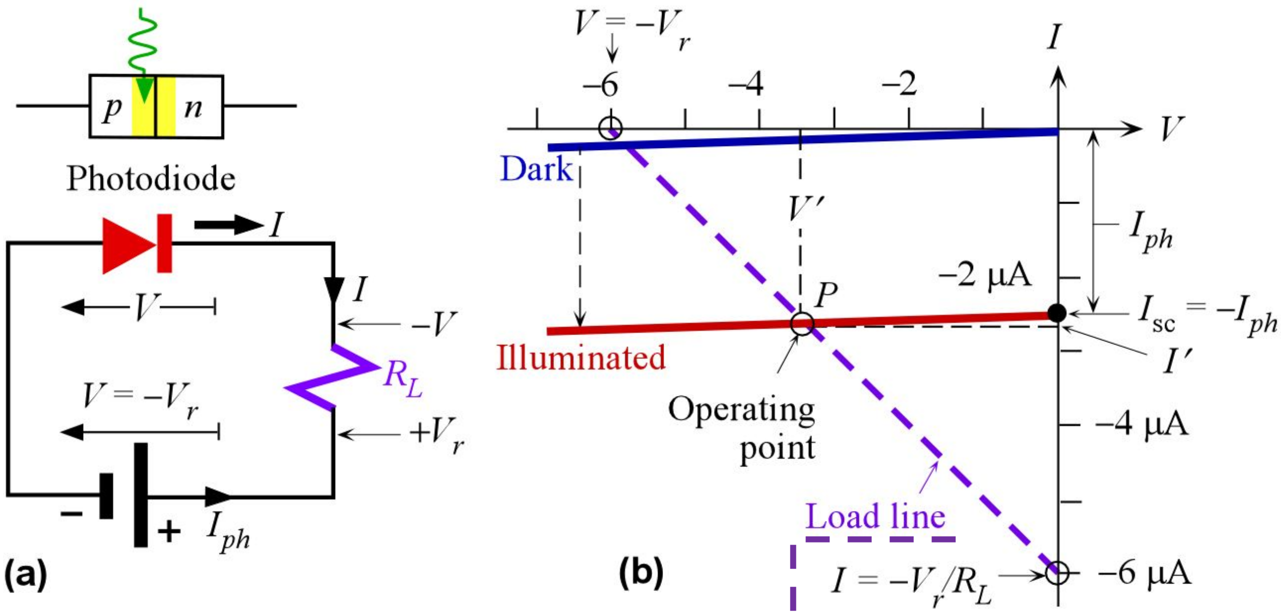
$$G = \frac{\tau}{t_e} + \frac{\tau}{t_h} = \frac{\tau}{t_e}\left(1 + \frac{\mu_h}{\mu_e}\right)$$

Basic Photodiode Circuits



(a) The photodiode is reverse biased through R_L and illuminated. Definitions of positive I and V are shown as if the photodiode were forward biased. (b) $I-V$ characteristics of the photodiode with positive I and V definitions in (a). The load line represents the behavior of the load R. The operating point is P where the current and voltage are I' and V'.

Basic Photodiode Circuits: The Load Line



P is the operating point

$V' = -3.5$ V

$I' = -2.5$ μA

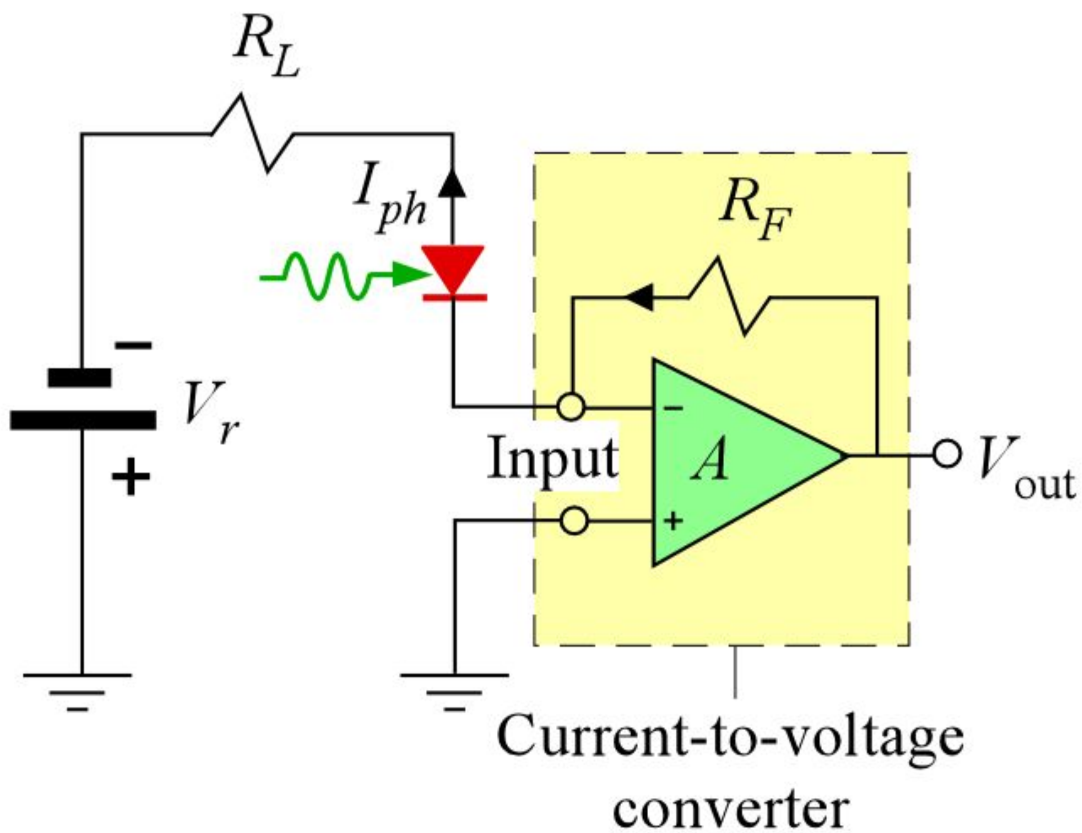
$I' \approx I_{ph}$

The current through R_L is

$$I = -(V + V_r)/R_L$$

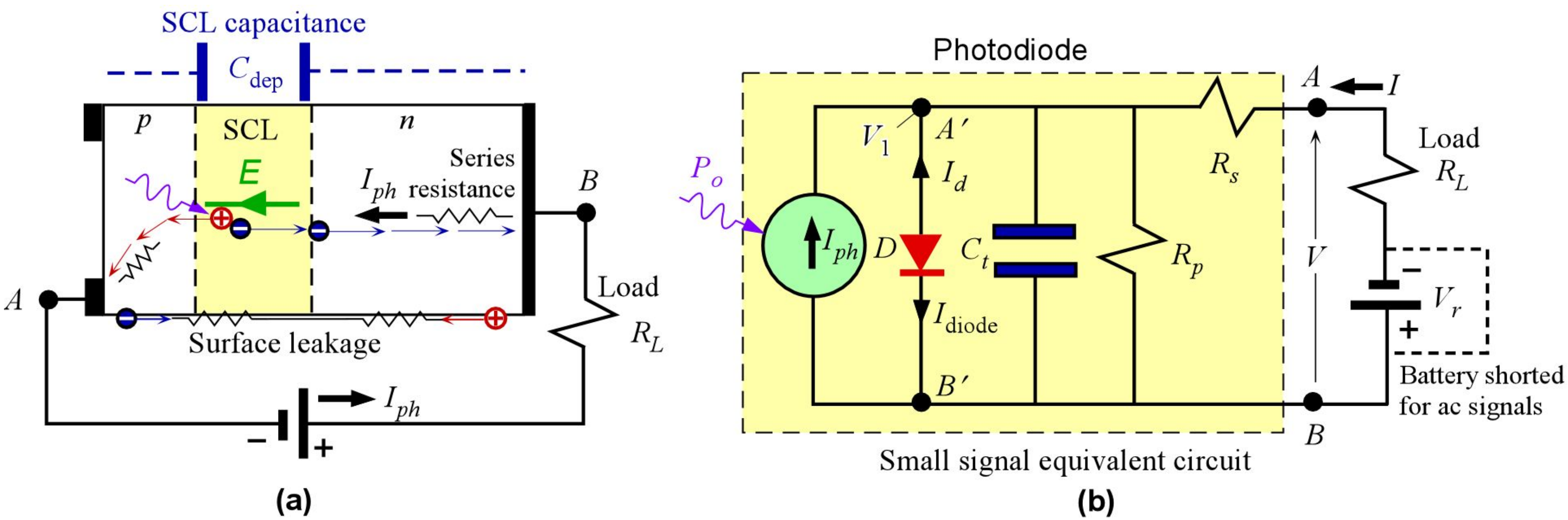
This is the **load line** shown in the figure. *P* is the intersection of the load line with the photodiode *I* vs. *V* curve and is the operating point.

Basic Photodiode Circuits



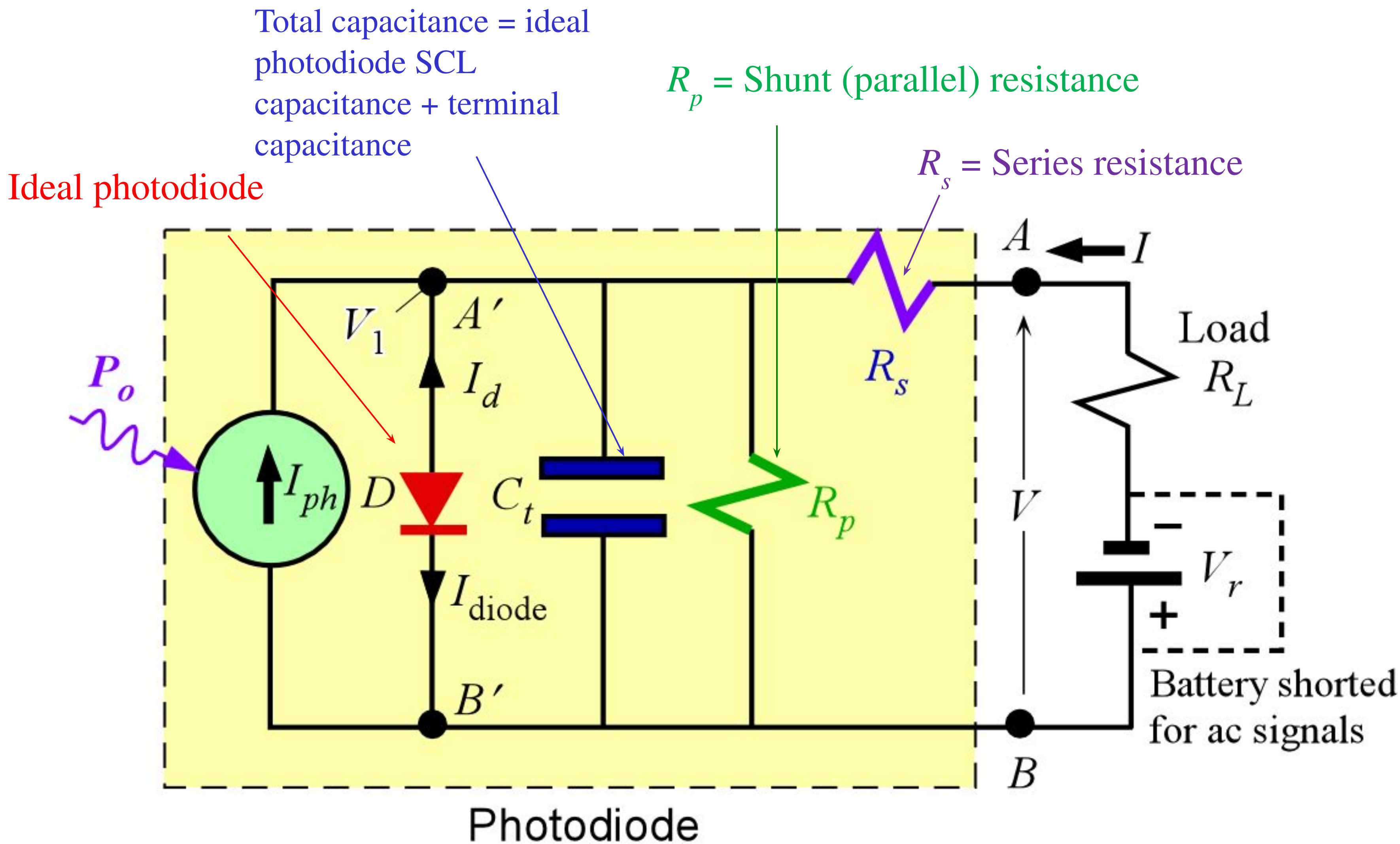
A simple circuit for the measurement of the photocurrent I_{ph} by using a current-voltage converter or a transimpedance amplifier. The reverse bias V_r is a positive number. Note that biasing circuit for the op amp is not shown.

Photodiode Equivalent Circuit

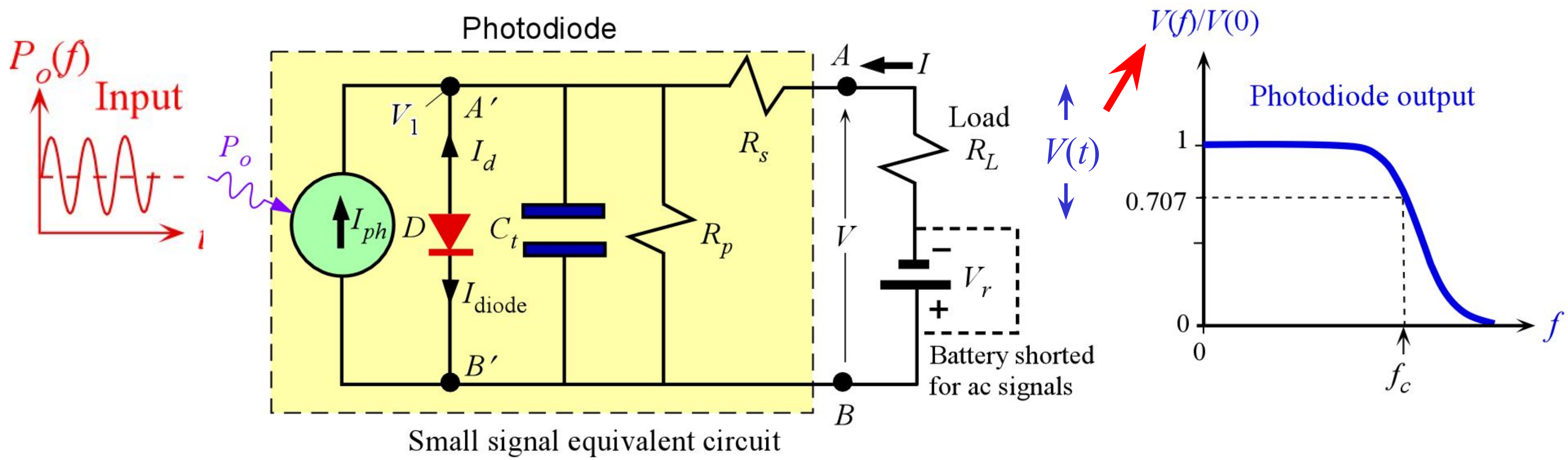


(a) A real photodiode has series and parallel resistances R_s and R_p and a SCL capacitance C_{dep}. *A* and *C* represent anode and cathode terminals. (b) The equivalent circuit of a photodiodes. For ac (or transient) signals, the battery can be shorted since ac signals will simply pass through the battery.

Reverse Biased Photodiode Equivalent Circuit



Cutoff Frequency f_c



The cutoff frequency or the bandwidth of the PD

$$f_c = \frac{1}{2\pi R_{\text{eq}} C_t} \approx \frac{1}{2\pi (R_s + R_L) C_t} \approx \frac{1}{2\pi R_L C_t}$$

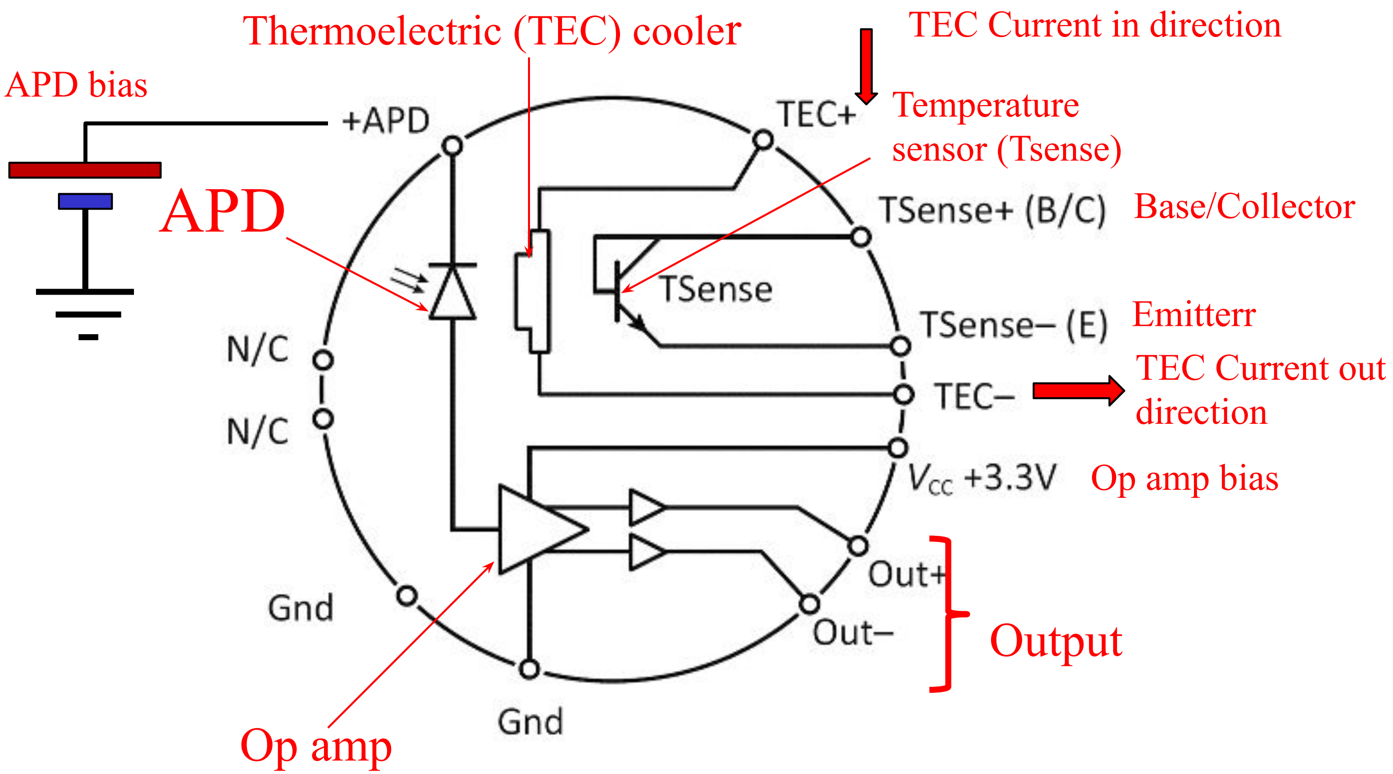
R_{eq} is equivalent resistance and represents (R_s + R_L) in parallel with R_p

Assumption

Drift time of carriers is much less than $1/f_c$.

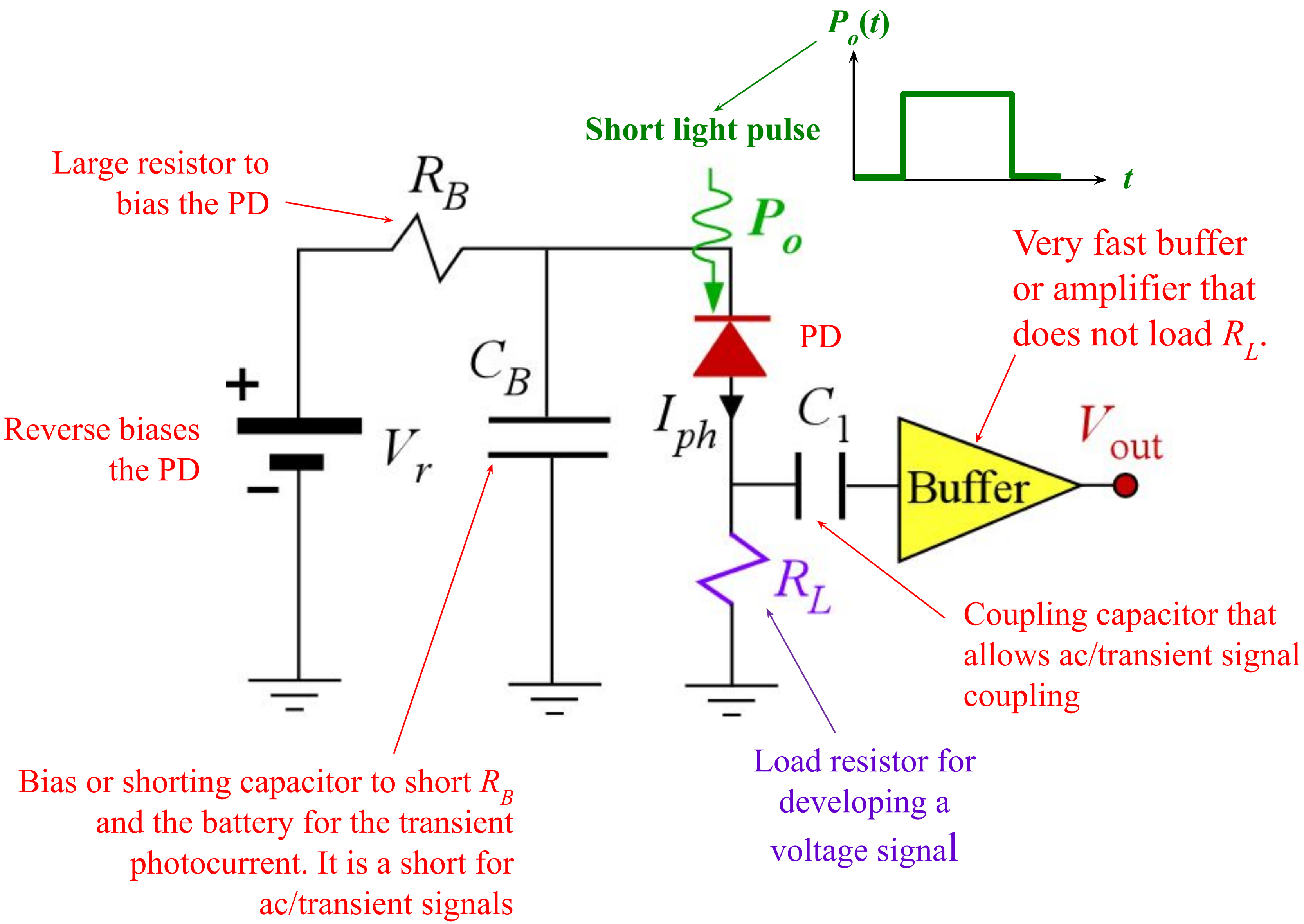
Response is not limited by drift and diffusion times of caries within the device.

A Commercial Photoreceiver



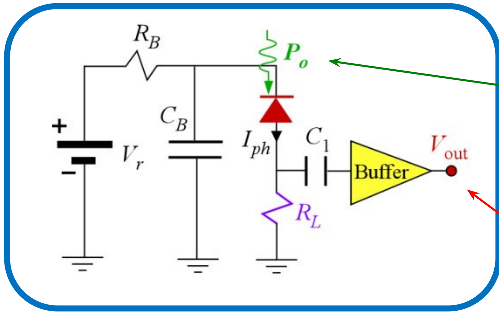
A photoreceiver that has an InGaAs APD and peripheral electronics (ICs) to achieve high gain and high sensitivity. There is also a thermoelectric cooler (TEC) and a temperature sensor (TSense). Courtesy of Voxtel Inc (www.voxtel-inc.com)

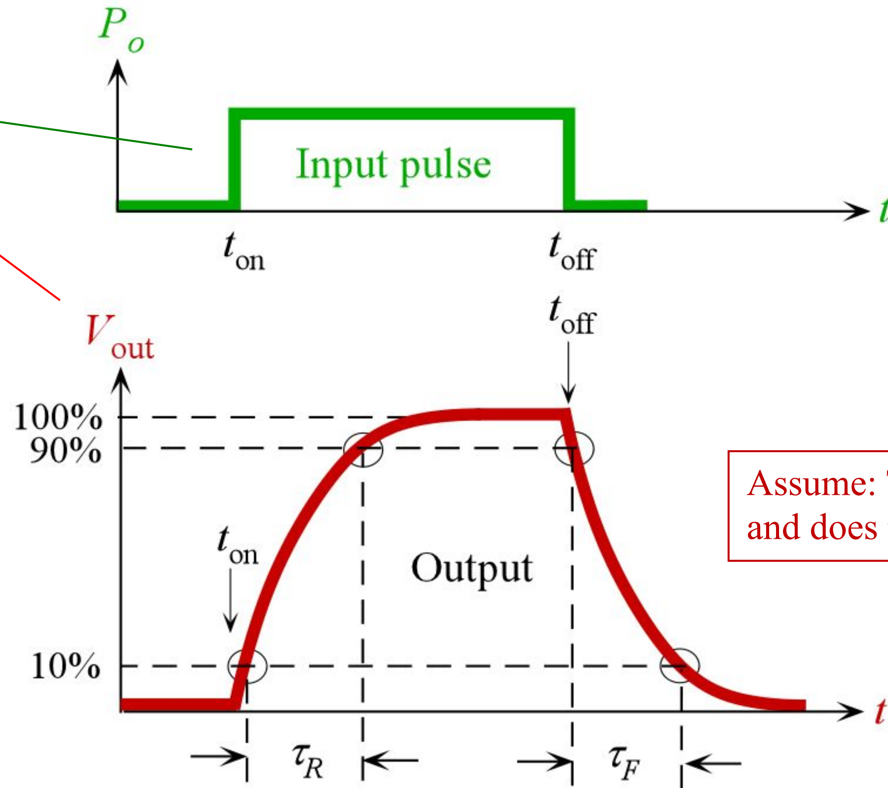
Pulsed Excitation



Pulsed Excitation

The Experiment





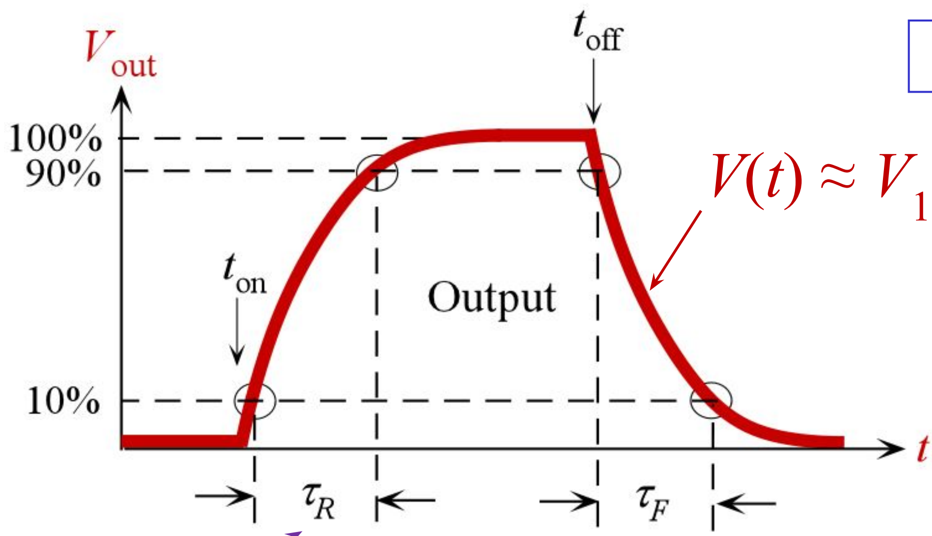
Assume: The buffer is extremely fast and does not limit the response

Rise time

Fall time

Are these related to f_c?

Rise and Fall Times, and Bandwidth



$\tau = (R_s + R_L)C_t \approx R_L C_t$

$V(t) \approx V_{100}\exp(-t'/\tau)$

Measured from t_{off}

Rise time

Very roughly, $\tau_R \approx \tau_F$

Fall time

$$\tau_F = 2.2\tau$$

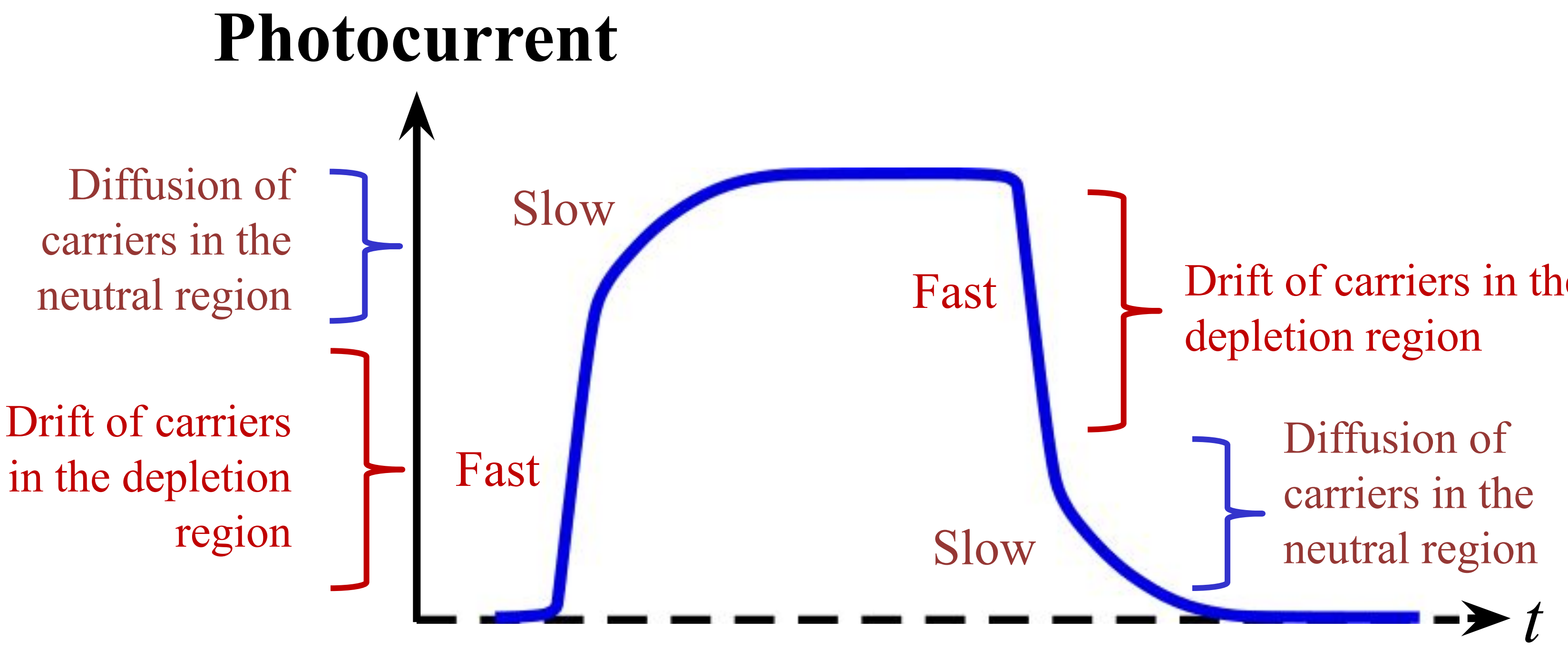
$$\tau = (R_s + R_L)C_t \approx R_L C_t$$

$$f_c \approx \frac{1}{2\pi R_L C_t} = \frac{1}{2\pi\tau} = \frac{0.35}{\tau_F} = \frac{350\,\text{MHz}}{\tau_F(\text{ns})}$$

Pulsed Excitation

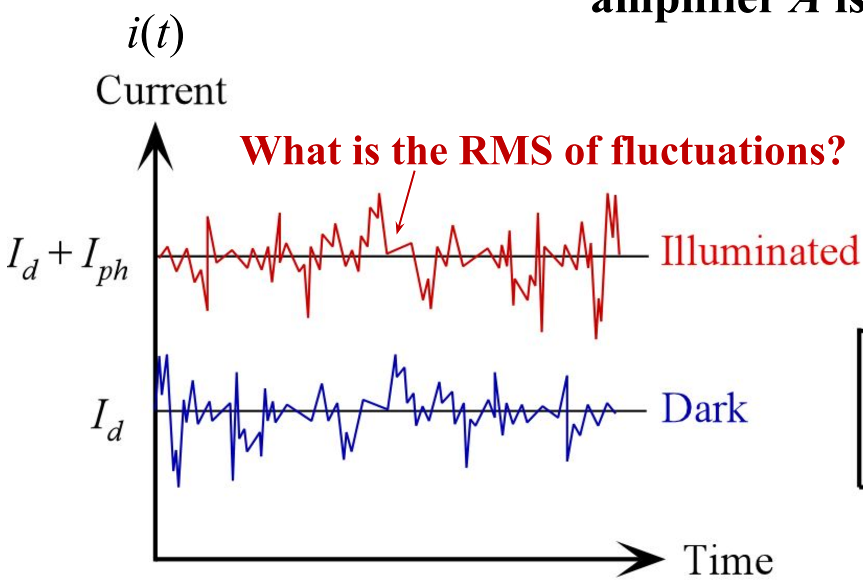
Non-$R_L C_t$ response

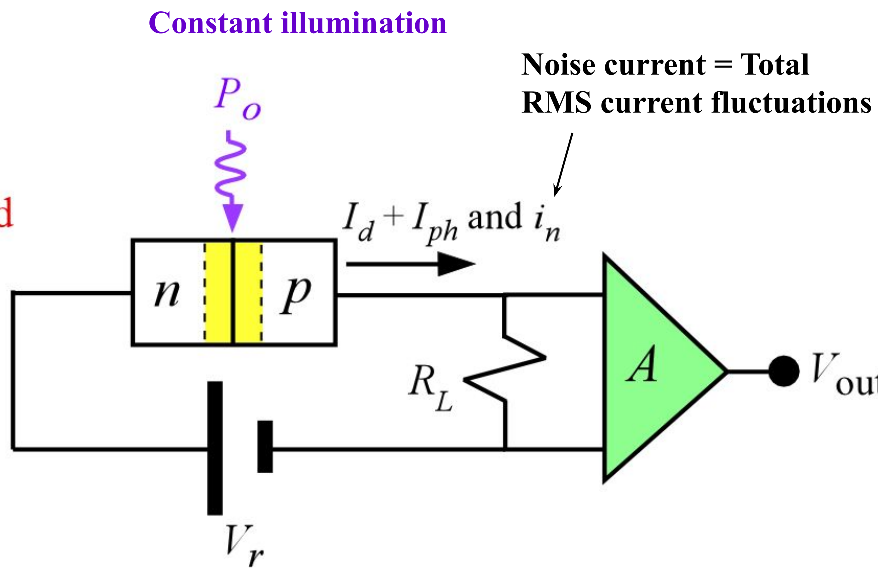
Response due to the diffusion and drift of photogenerated carriers
Assume $R_s + R_L$ is very small so that $(R_s + R_L)C_t$ is negligible



Noise in Photodiodes

Consider a receiver with a photodiode and a sampling resistor R_L The amplifier A is assumed noiseless





Consider **constant illumination P_o**

Total current without noise = Dark current (I_d) + Photocurrent (I_{ph}) = "Constant"

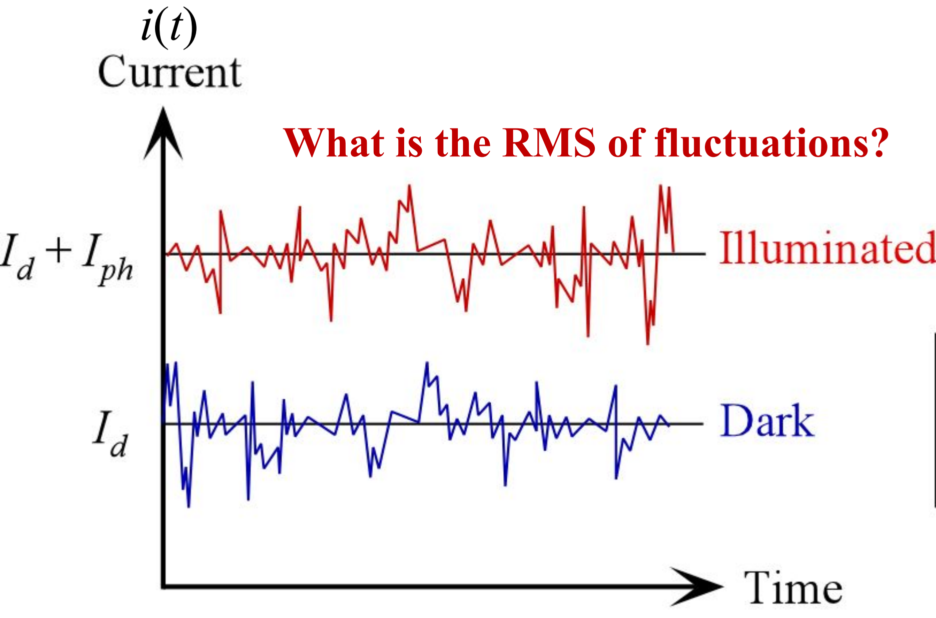
Observed Current = Dark current + Photocurrent **and Fluctuations (Noise)**

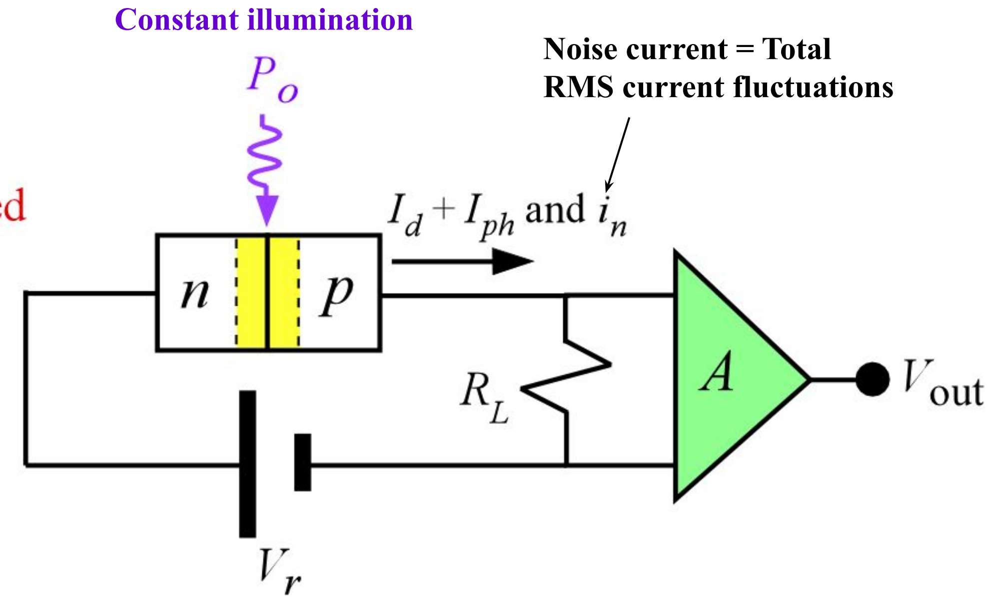
What is this "Noise" ?

We can represent the "noise current" by the RMS of fluctuations

$$\boxed{\text{RMS of fluctuations} = \sqrt{i(t)^2}}$$

Noise in Photodiodes





The dark current has **shot noise** or fluctuations about I_d,

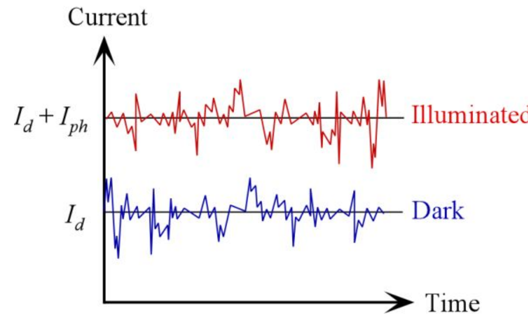
$$i_{n\text{-dark}} = (2eI_dB)^{1/2}$$

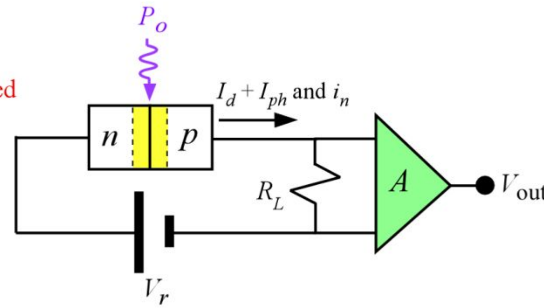
B = Bandwidth

Quantum noise is due to the photon nature of light and its effects are the same as **shot noise**. Photocurrent has quantum noise or shot noise

$$i_{n\text{-quantum}} = (2eI_{ph}B)^{1/2}$$

Noise in Photodiodes





Total shot noise current, i_n

$$i_n^2 = i_{n-\text{dark}}^2 + i_{n-\text{quantum}}^2$$

$$i_n = [2e(I_d + I_{ph})B]^{1/2}$$

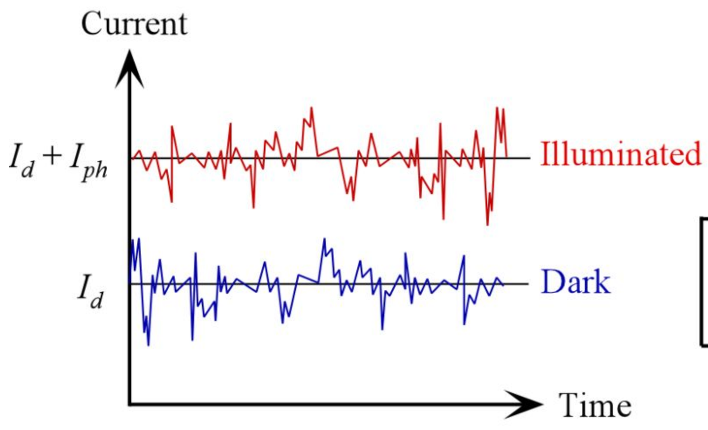
We can conceptually view the photodetector current as

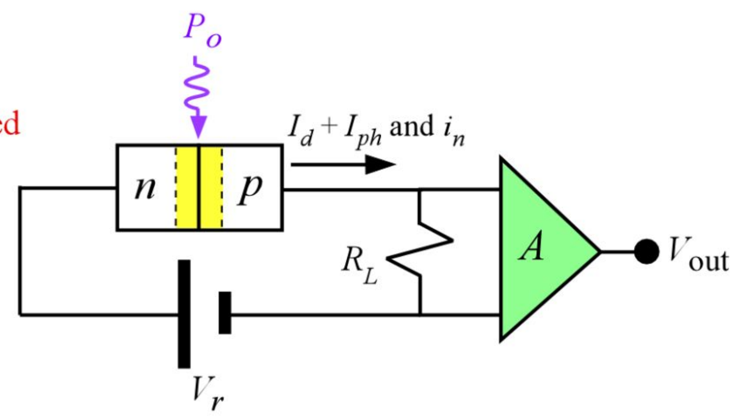
$$I_d + I_{ph} + i_n$$

This flows through a load resistor R_L and voltage across R_L is amplified by A to give V_{out}

The noise voltage (RMS) due to shot noise in PD = $i_n R_L A$

Noise in Photodiodes





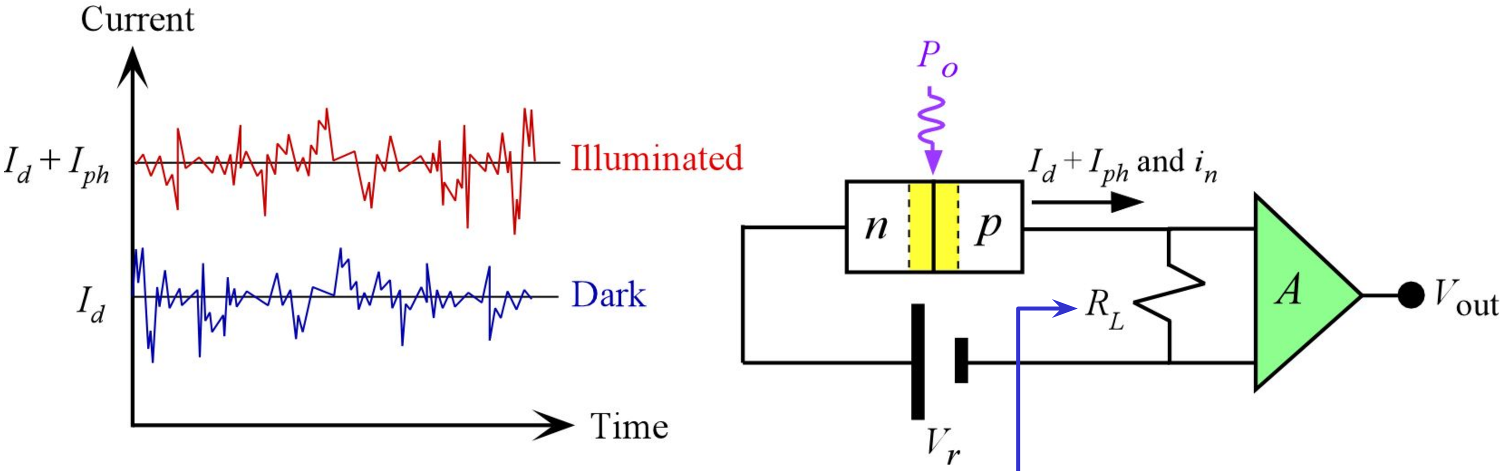
Total current flowing into R_L has three components:

I_d = Dark current. In principle, we can subtract this or block it with a capacitor *if* I_{ph} is an ac (transient) signal.

I_{ph} = Photocurrent. This is the signal. We need this. It could be a steady or varying (ac or transient) signal.

i_n = Total shot noise. Due to shot noise from I_d and I_{ph}. We cannot eliminate this.

Noise in Photodiodes



The resistor R_L exhibits **thermal noise (Johnson noise)**

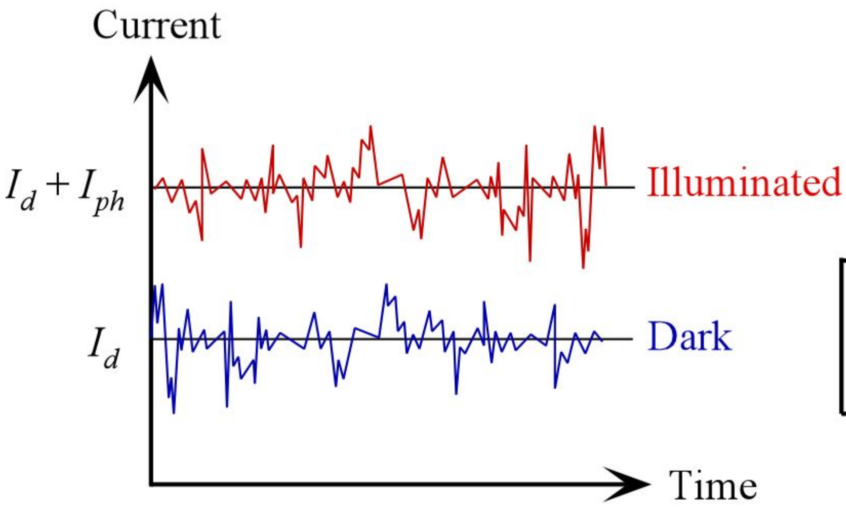
Power in thermal fluctuations in $R_L = 4k_BTB$

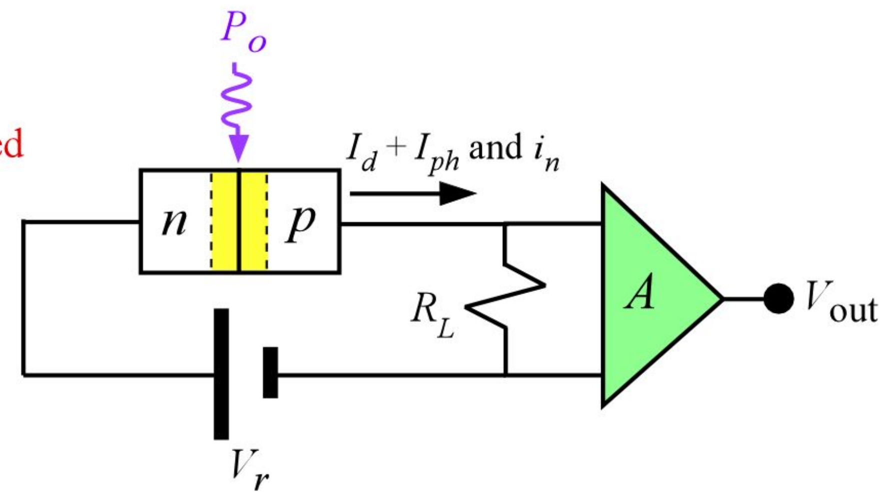
$$\sqrt{\overline{i^2}}$$

$$\therefore \quad R_L\overline{i^2} = 4k_BTB \qquad i = \text{Current in } R_L$$

$$i_{\text{th}} = \text{Thermal noise current from } R_L = \left[\frac{4k_BTB}{R_L}\right]^{1/2}$$

Summary of Noise in PD and R_L





Power in shot noise in PD $= i_n^{\ 2} R_L = [2e(I_d + I_{ph})B]R_L$

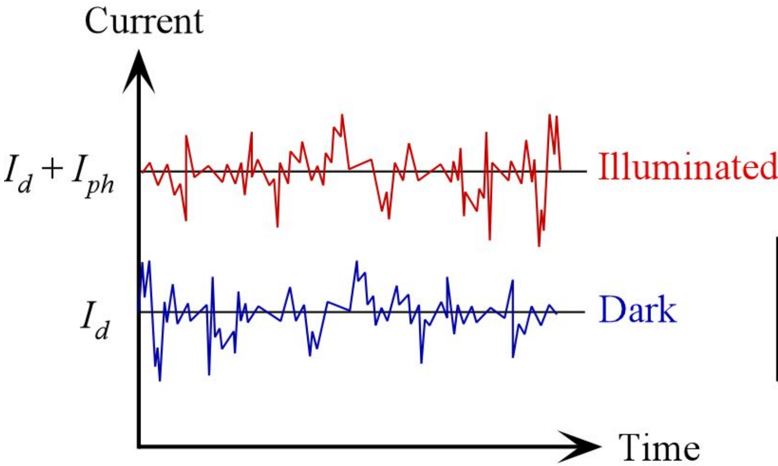
Power in thermal fluctuations in $R_L = 4k_BTB$

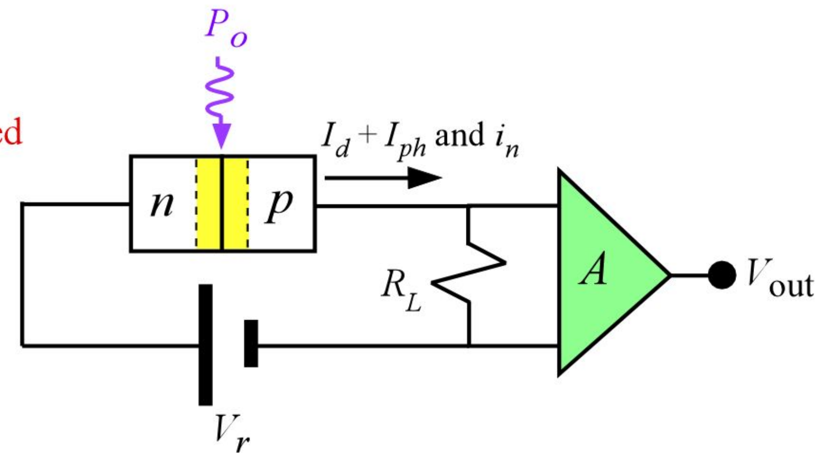
Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations *e.g.* **power in shot noise + power in thermal noise**

Noise in the amplifier *A* must also be included

See advanced textbooks

Signal to Noise Ratio





$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

$$\text{SNR} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_B TB} = \frac{I_{ph}^2}{\left[2e(I_d + I_{ph})B\right] + \dfrac{4k_B TB}{R_L}}$$

Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations *e.g.* **power in shot noise + power in thermal noise**

Noise Equivalent Power: NEP

Definition

$$\text{NEP} = \frac{\text{Input power for SNR} = 1}{\sqrt{\text{Bandwidth}}} = \frac{P_1}{B^{1/2}}$$

NEP is defined as the required optical input power to achieve a SNR of 1 within a bandwidth of 1 Hz

$$\text{NEP} = \frac{P_1}{B^{1/2}} = \frac{1}{R}\left[2e(I_d + I_{ph})\right]^{1/2}$$

Units for NEP are W Hz$^{-1/2}$

Detectivity, *D*

Definition

$$\text{Detectivity} = \frac{1}{\text{NEP}}$$

Specific detectivity *D**

$$D^* = \frac{A^{1/2}}{\text{NEP}}$$

Units for D* are cm $Hz^{-1/2}$ W^{-1}, or Jones

NEP and Detectivity of Photodetectors

Typical noise characteristics of a few selected commercial photodetectors. PC means a photoconductive detector, whose photoconductivity is used to detect light. For PC detectors, what is important is the dark resistance R_d, which depends on the temperature.

Photodiode	GaP Schottky	Si *pin*	Ge *pin*	InGaAs *pin*	PbS (PC) −10°C	PbSe (PC) −10 °C	InSb (PC) −10°C
λ_{peak} (µm)	0.44	0.96	1.5	1.55	2.4	4.1	5.5
I_d or R_d	10 pA	0.4 nA	3 µA	5 nA	0.1−1 MΩ	0.1−1 MΩ	1−10 kΩ
NEP W Hz$^{-1/2}$	5.4×10^{-15}	1.6×10^{-14}	1×10^{-12}	4×10^{-14}	-	-	
D^* cm Hz$^{1/2}$/ W	1×10^{13}	1×10^{12}	1×10^{11}	5×10^{12}	1×10^{9}	5×10^{9}	1×10^{9}

$$\mathrm{NEP} = \frac{P_1}{B^{1/2}} = \frac{1}{R}\left[2e(I_d + I_{ph})\right]^{1/2}$$