Algebraic Structure

Tutorial # 4: Isomorphism

Exercise 1:
Show that a mapping from $f : (\mathbb{Z}, +) \to (\mathbb{Z}_k, +)$ is homomorphic but not isomorphic where f(n) = n mod k.

Exercise 2:
Determine whether the mappings defined from G to H are homomorphic/isomorphic and the kernel:

- G, H are the group of non-zero real numbers ($\mathbb{R}^*$) under addition and/or multiplication and $\phi(x) = x^2$ for all $x \in G$
- G, H are the group of non-zero real numbers ($\mathbb{R}^*$) under addition and/or multiplication and $\phi(x) = 2^x$ for all $x \in G$
- G and H are groups of ($\mathbb{R}$,+), $\phi(x) = x + 1$ for all $x \in G$

Exercise 3:
Let $\phi : G_1 \to G_2$ be a homomorphism of groups. Prove that if H_1 is a subgroup of G_1, then $\phi(H_1)$ is a subgroup of G_2.

Exercise 4:
Show that the following maps are group homomorphisms and compute their kernels.
1. $f : (\mathbb{R}^*, .) \to (R^{2\times2}, .)$ given by

$$f(x) = \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix}$$

2. $f : (\mathbb{R}, +) \to (R^{2\times2}, .)$ given by

$$f(x) = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$$

Exercise 5:
Let G be a group and define a map $f : G \to G$ by $f(a) = a^2$ for each $a \in G$. Then prove that G is an abelian group if and only if the map f is a group homomorphism.

Exercise 6:

Let G be a group. Prove that the mapping $\phi(g) = g^{-1}$ for all $g \in G$ is an automorphism if and only if G is abelian.