Probability and Statistics
USTH-B2

Exercices Chapter 3

1. Let X and Y two dr variables given by their laws

x	0	1	2
$p_X(x)$	1/4	1/4	1/2

and

y	-1	0	1
$p_Y(y)$	1/3	1/2	1/6

Assume that X and Y are independent. Compute $P_Z(z)$ with $Z \equiv XY$.

2. Let (X, Y) be the discrete random vector whose joint law is given by

$X..Y$	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(a) Compute the marginal laws.
(b) Compute the means of X and Y.
(c) Compute $Cov(X, Y)$, σ_X, σ_Y and $\varrho(X, Y)$.

3. Let X be the discrete random variable given by its law

x_i	-2	-1	1	2
p_i	1/8	3/8	3/8	1/8

Set $Y = X^2$.
(a) Compute the law of the drv Y.
(b) Compute the law of (X, Y).
(c) Compute $Cov(X, Y)$ and $\varrho(X, Y)$.

4. We draw at random and with reset two numbers in $\{0, 1, 2, 3\}$. Let X be smallest of these two numbers, and Y the biggest.
(a) Compute the law of the couple (X, Y).
(b) Compute the marginal laws.
(c) Are these rv independent ?
(d) Compute $Cov(X, Y)$.

5. Let (X, Y) be the dr vector whose probability law is given by

$$p_{ij} = cij \text{ if } (i, j) \in \{1, 2, 3\}^2 \text{ and 0 otherwise}$$

where c is a constant.
(a) Compute c.
(b) Compute $P(1 \leq X \leq 2, Y \leq 2)$, $P(X \geq 2)$, $P(Y < 2)$.

(c) Find the marginal laws of X and Y.

(d) Are the rv X and Y independent ?

6. Let
$$f_{X,Y}(x,y) = 3/2x \text{ if } 1 \leq x \leq y \leq 2$$
and 0 elsewhere.

(a) Compute the marginal densities f_X and f_Y.

(b) Compute $P(X^2 < Y)$.

7. Let
$$f_{X,Y}(x,y) = (4xy)^{-1/2} \text{ if } 0 < y < x < 1$$
and 0 elsewhere. Compute a) $f_X(x)$ and $f_Y(y)$, b) $P(X > 1/2, Y > 1/2)$.

8. Let
$$f_{X,Y}(x,y) = 2 - x - y \text{ if } 0 < x < 1, 0 < y < 1$$
and 0 elsewhere. Compute a) $F_{X,Y}(x,y)$, b) $f_X(x)$, c) $cov(X,Y)$ and d) $P(X+Y < 1)$.

9. Let the random couple (X,Y) defined by its joint density
$$f(x,y) = 2 \text{ if } x \in D \text{ and 0 elsewhere}$$
with $D = \{(x,y) \in \mathbb{R}^2, x \geq 0, y \geq 0, x+y \leq 1\}$. Compute the marginal densities.

10. Let the random vector (X,Y) with density
$$f(x,y) = cxy \text{ if } (x,y) \in [1,4] \times [1,5] \text{ and 0 otherwise}$$
where c is a constant.

(a) Compute c.

(b) Compute $P((X,Y) \in]1,2[\times]2,3[$.

(c) Compute the marginal laws of X and Y.

(d) Compute the joint distribution function of (X,Y).

(e) Compute $P(X+Y < 3)$.

11. Let the random vector (X,Y) with density
$$f(x,y) = c(x^2 + y^2) \text{ if } (x,y) \in [0,1] \times [0,1] \text{ and 0 elsewhere}$$
where c is a constant.

(a) Compute c.

(b) Compute $P(X < 1/2, Y > 1/2)$.

(c) Compute $P((X,Y) \in B(0,1)$.

(d) Compute the marginal laws of X and Y.

12. Let the random vector (X, Y) with density

$$f(x, y) = 1 \text{ if } (x, y) \in [0, 1]^2 \text{ and } 0 \text{ elsewhere}$$

Compute the distribution function of $Z = X + Y$.

13. The input of a communication channel is a rv X which follows a gaussian law $N(0, 1)$. The output Y is given by $Y = X + N$, where N is the noise, following a law $N(0, \sigma_N^2)$. Moreover, we assume that X and N are independent.

Compute the coefficient correlation $\varrho_{X,Y}$ of X and Y.

14. A computer generates random numbers X according to a law $N(0, \sigma^2)$. Set

$$Y = X \text{ if } X > 0 \text{ and } 0 \text{ otherwise}$$

$$Z = X \text{ if } X \leq 0 \text{ and } 0 \text{ otherwise}$$

(a) Is the sum $Y + Z$ gaussian ?

(b) Is the couple (Y, Z) following a bi-normal law ?