**Probability and Statistics**
**USTH-B2**

**Exercices Chapter 2**

1. Let $X$ be a drv following a Poisson law with parameter $\lambda$, that is $P(X=j)=\frac{\lambda^j}{j!}e^{-\lambda}$, for all $j\in\mathbb{N}$. Find quickly the formula for the mean and variance of $X$.
2. (Sum of two dices). We throw two well balanced dices with 4 faces, and we denote by $X$ the sum of obtained numbers.
   (a) Compute the law of the drv$X$.
   (b) Compute its mean and variance.
   (c) Compute $P(X<5)$, $P(X\leq 5)$ and $P(3\leq X<5)$.
   (d) Compute and draw the distribution function of $X$.
3. Let$X$ be a rv with distribution function
$$F(x)=1-e^{-2x/3}, x>0 \text{ and } 0 \text{ otherwise}$$
   (a) Compute the density of $X$.
   (b) Compute the mean and std of $X$.
4. Rayleigh law. Let $X$ be a rv with distribution function
$$F(x)=1-e^{-\frac{x^2}{2a^2}}, x\geq 0 \text{ and } 0 \text{ othersie}$$
   Compute its density, its mean and its variance.
5. Let $X$ be a normal rv with mean $m$ and variance $\sigma^2$, that is $X\simeq\mathcal{N}(m,\sigma^2)$.
   Compute the density, mean and variance of the rv $Y=aX+b$, with $a$ and $b$ constants, $a\neq 0$. Choose $a$ and $b$ such that $Y$ follows a standard normal law.
6. The law $\mathcal{N}(0,1)$ is given by a table, and gives an approximated value for the error function
$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du$$
   (a) Let $X\simeq\mathcal{N}(0,1)$. Find $\alpha$ such that $P(X>\alpha)=0,95$.
   (b) Let $Y\simeq\mathcal{N}(m,\sigma^2)$. Find $P(Y\in[m-\sigma;m+\sigma])$ and $P(Y\in[m-2\sigma;m+2\sigma])$. Find $\beta$ s.t. $P(Y\in[m-\beta\sigma;m+\beta\sigma])=0,95$.
7. Let $X\simeq Exp(\lambda=2)$.
   (a) Compute $P(X^2<0)$
   (b) Use BT to compute $P(|X-1/2|\geq 2)$. Compare with the exact value.
8. A machine produces devices of diameter X following approximatively a law $N(\mu,\sigma^2=(0,01)^2)$. What should be the value of $\mu$ s.t. no more than 1% of devices have a diameter bigger than $3cm$ ?

9. Let X be the utilisation duration in hours of a computer, in one working day. Assume that X follows a law $N(4,2)$. Find the number $x_0$ s.t. for 90% of days, X is bigger than $x_0$.
10. Let X be the lifetime in months of an electronic device.
    (a) Assume that $X \simeq N(20,16)$. Compute $P(|X| < 24)$.
    (b) Assume that $X \simeq G(\alpha = 2, \lambda)$, that is
    $$f_X(x) = \lambda^2 x e^{-\lambda x} 1_{x>0}$$
    Compute the expectation of $Y$.
11. An airline company notes that, one the average, 4% of persons having booked for a fixed flight, do not check in at the departure counter. Therefore, the policy of the company is to take 75 bookings for a flight with only 73 seats. What is the probability that a person checking in before the departure will have a seat (do all necessary assumptions) ?