Lecture 6

Artificial Neural Network

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Hanoi, 2017

Outline

- Artificial Neural Network
- Back Propagation Gradient Descent

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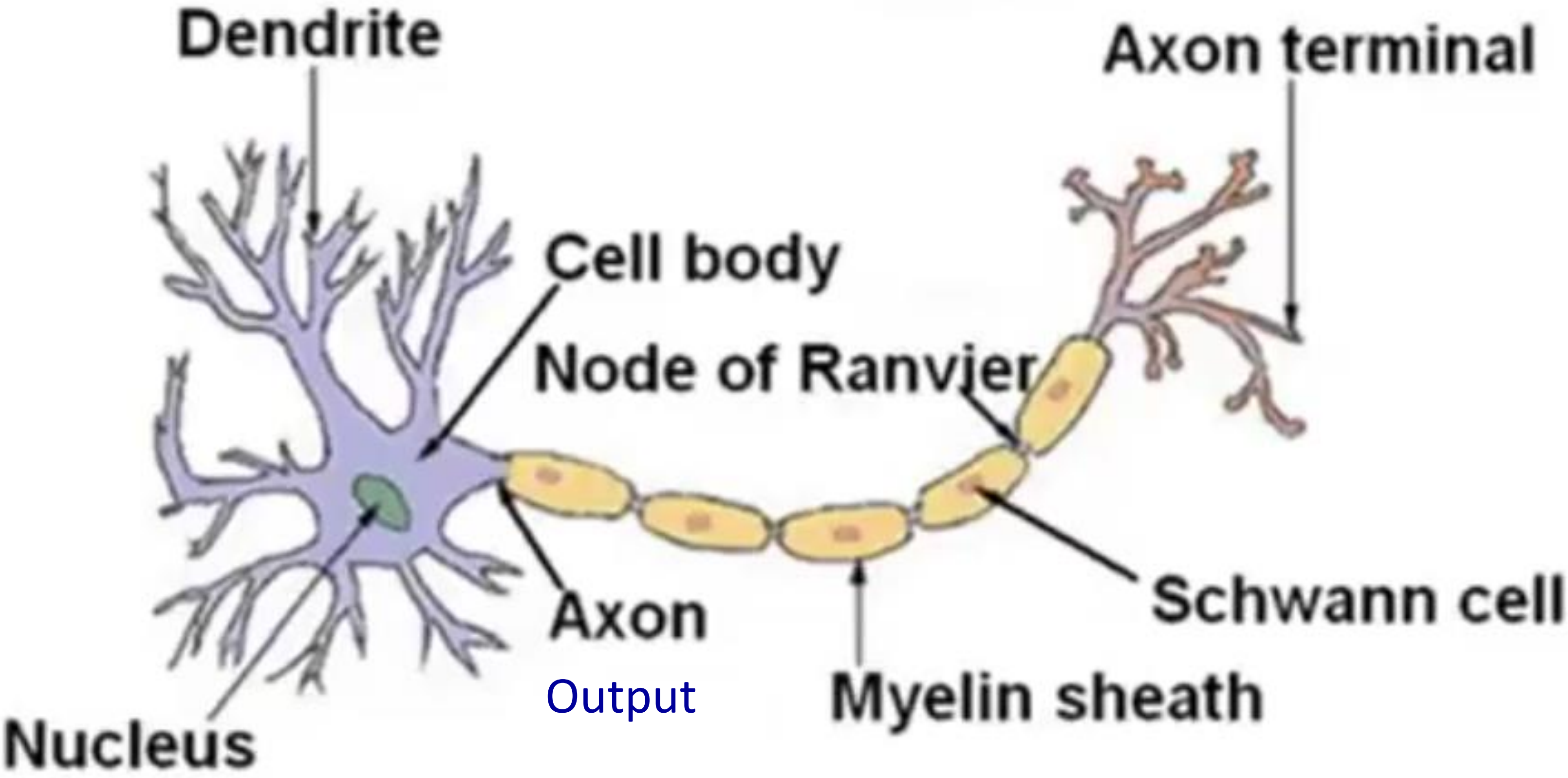
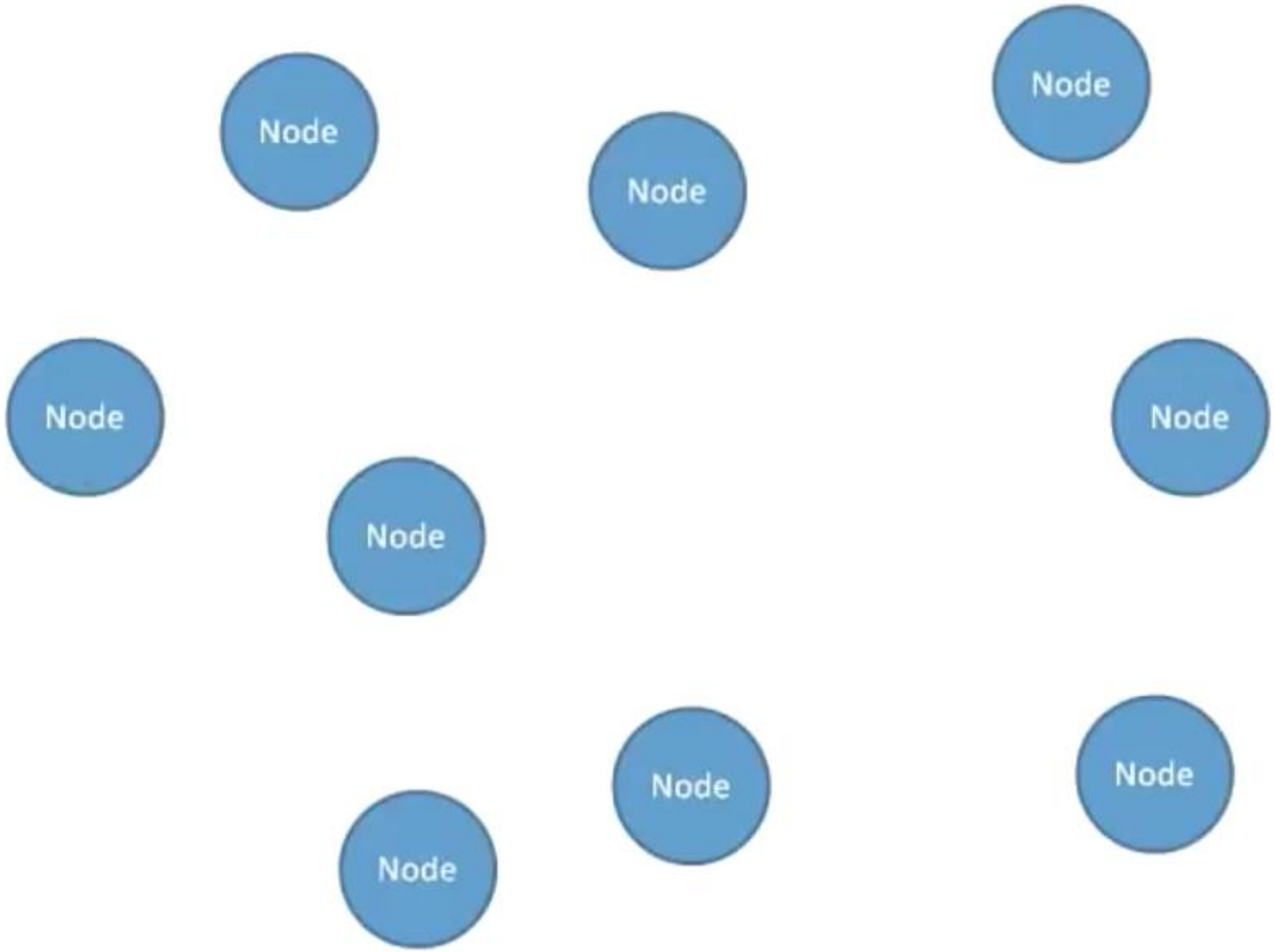
Artificial Neural Network

Input

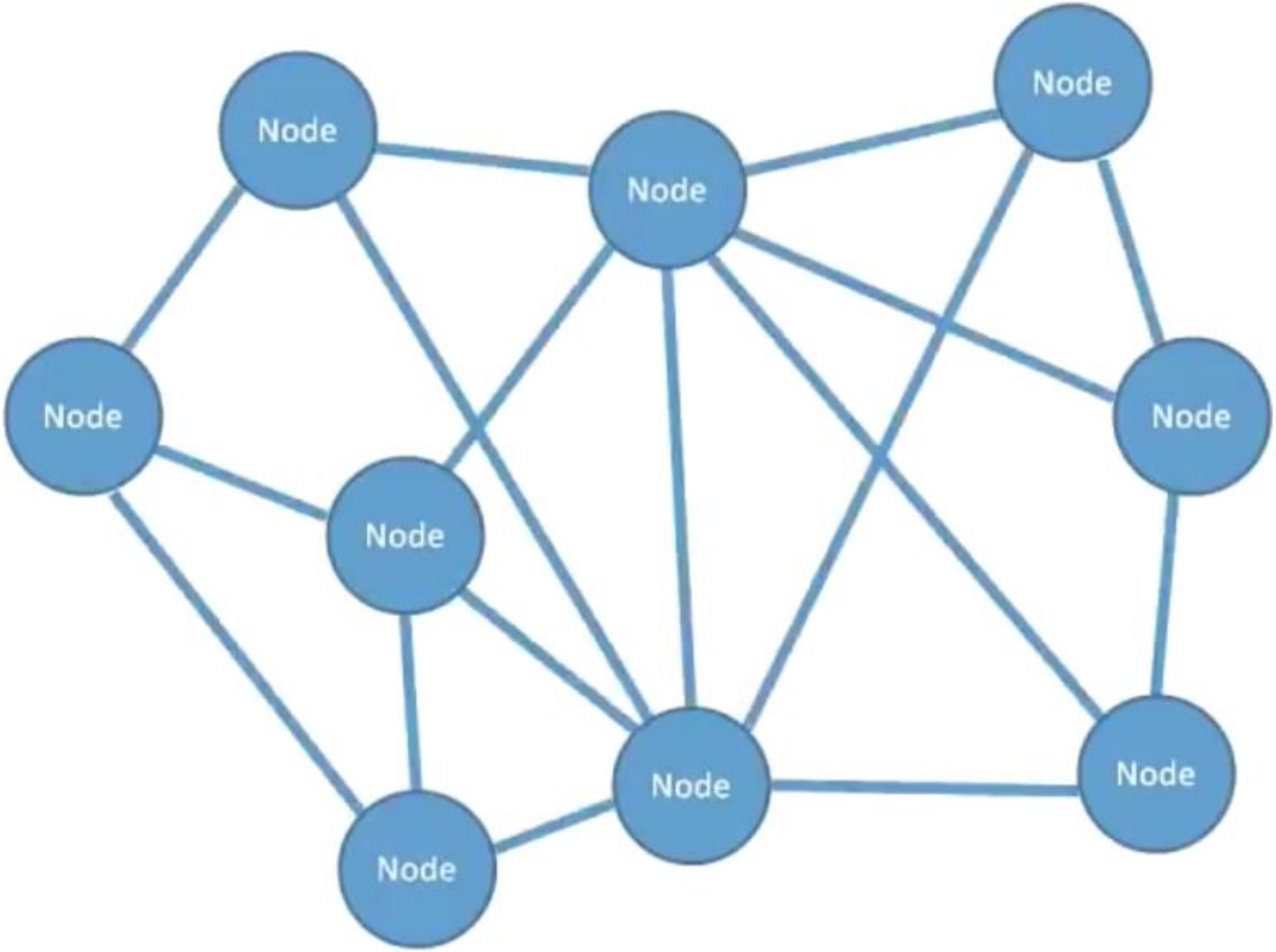


Output

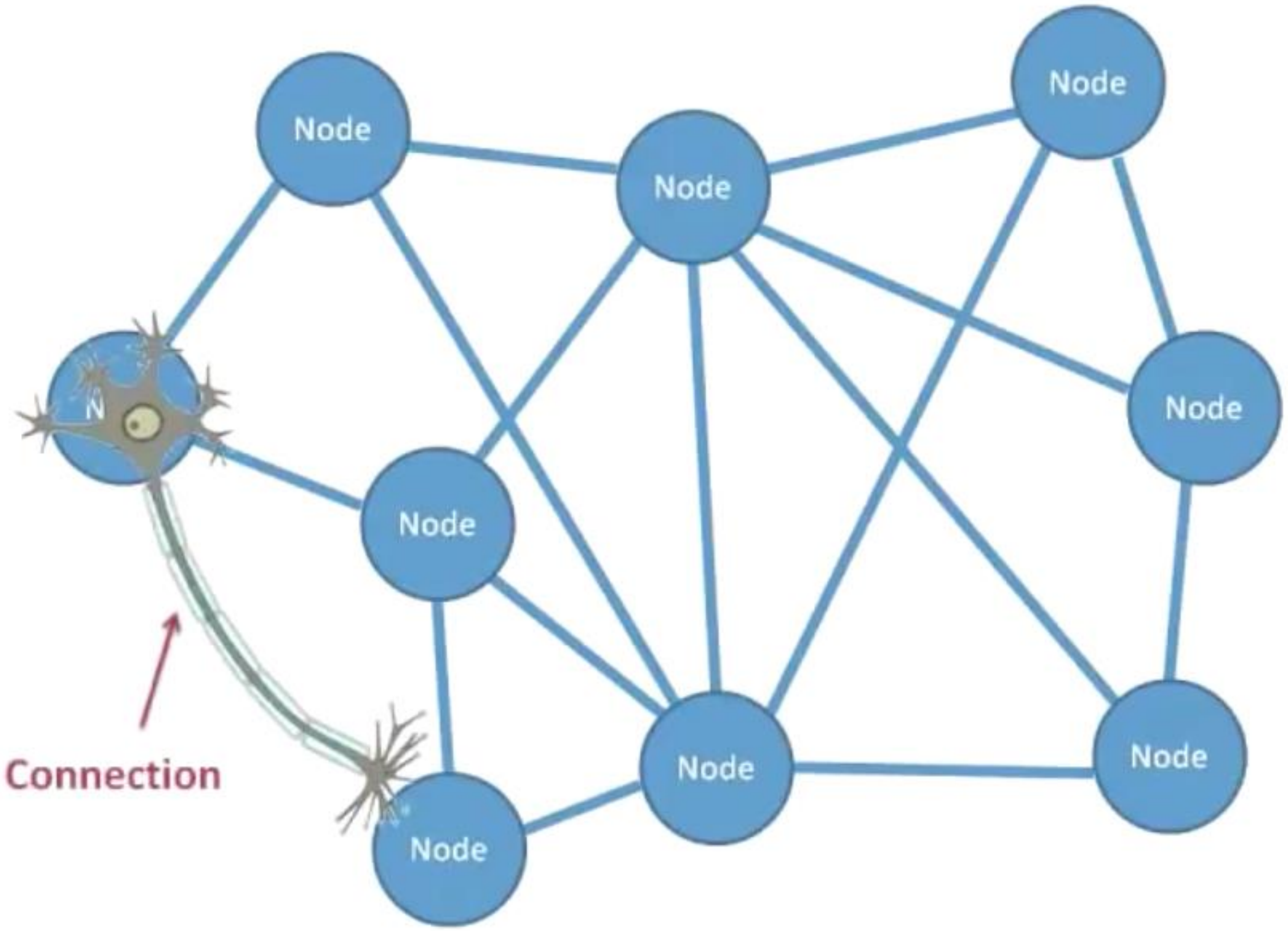
Artificial Neural Network



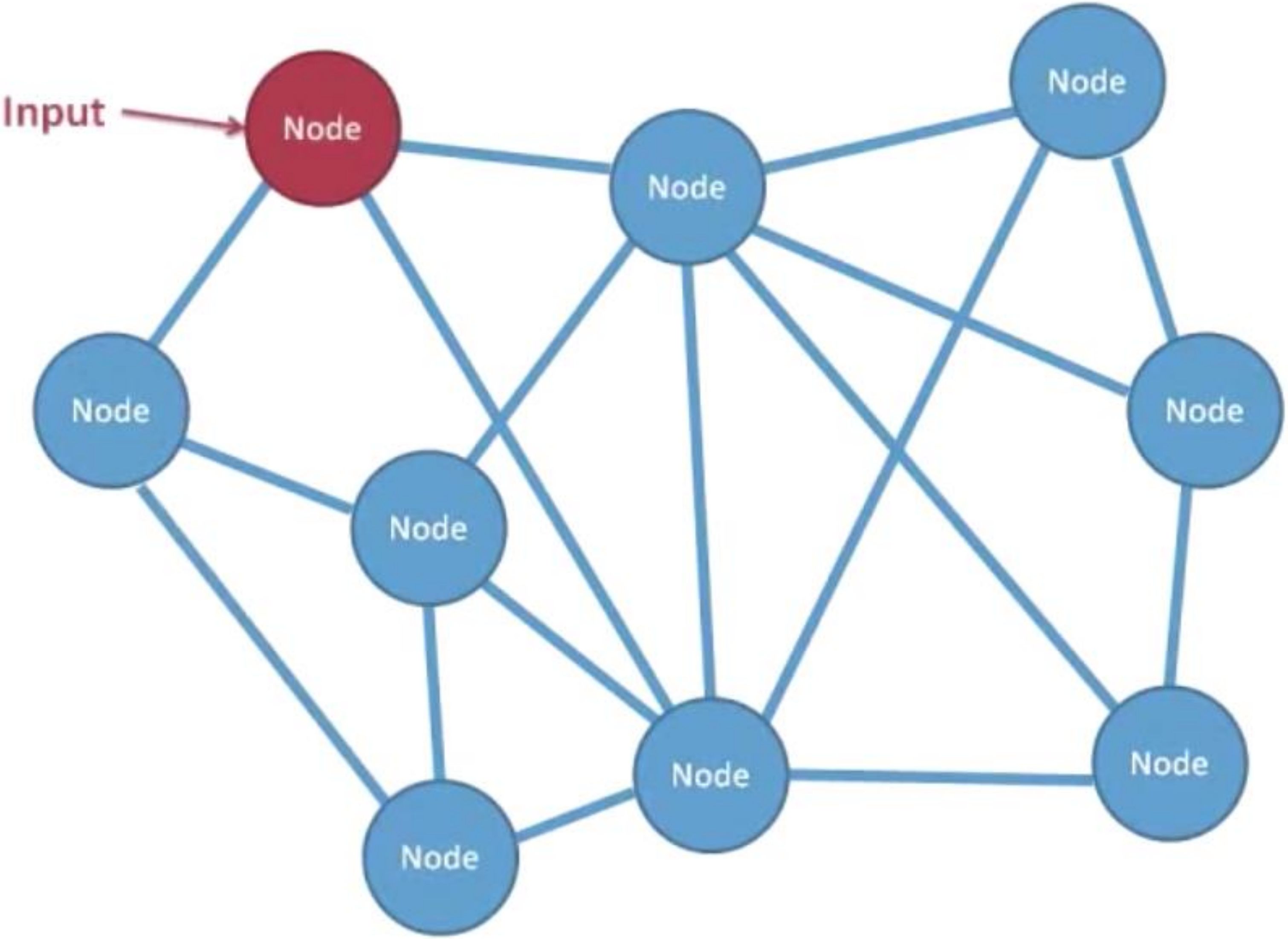
Artificial Neural Network



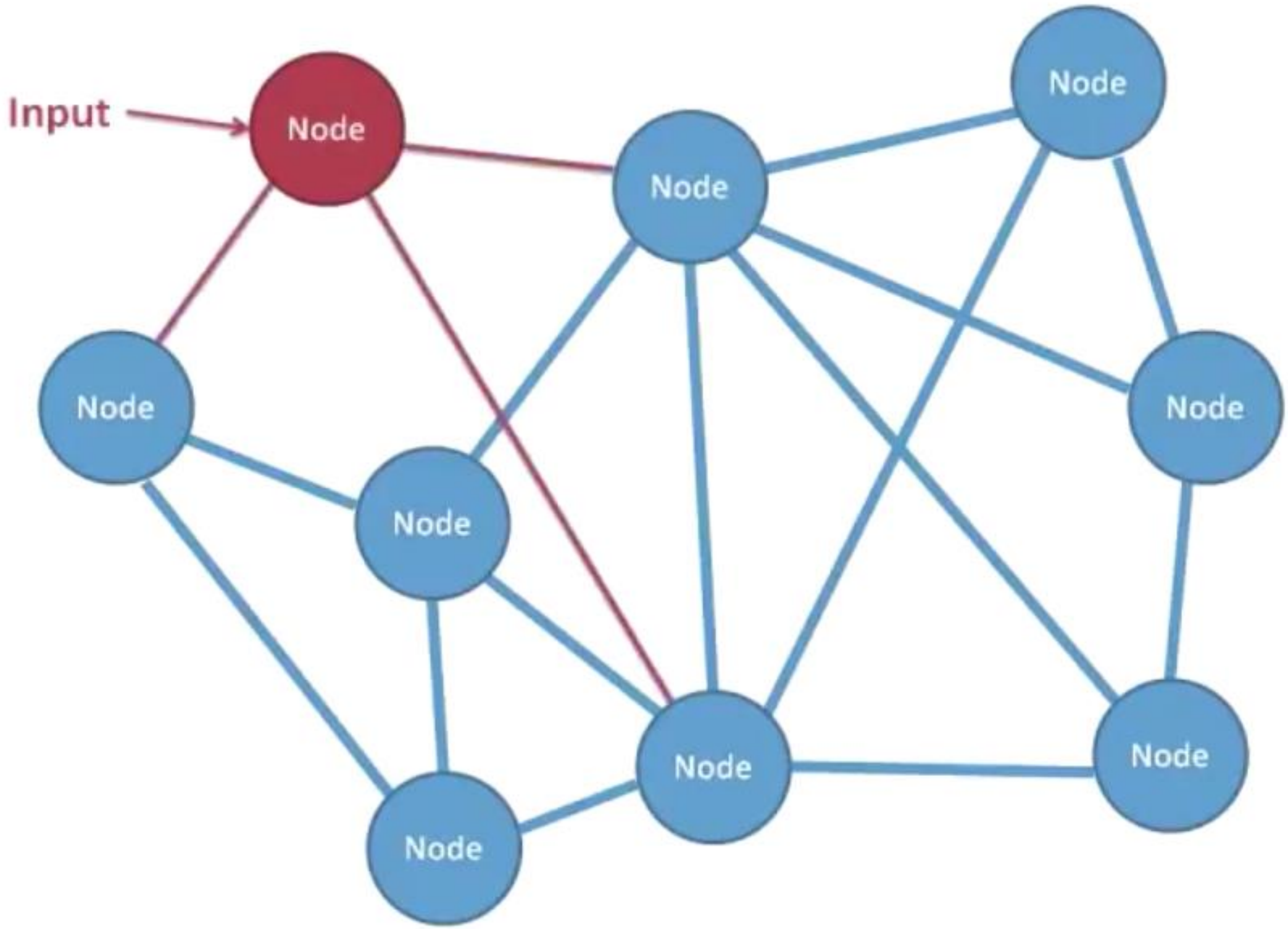
Artificial Neural Network



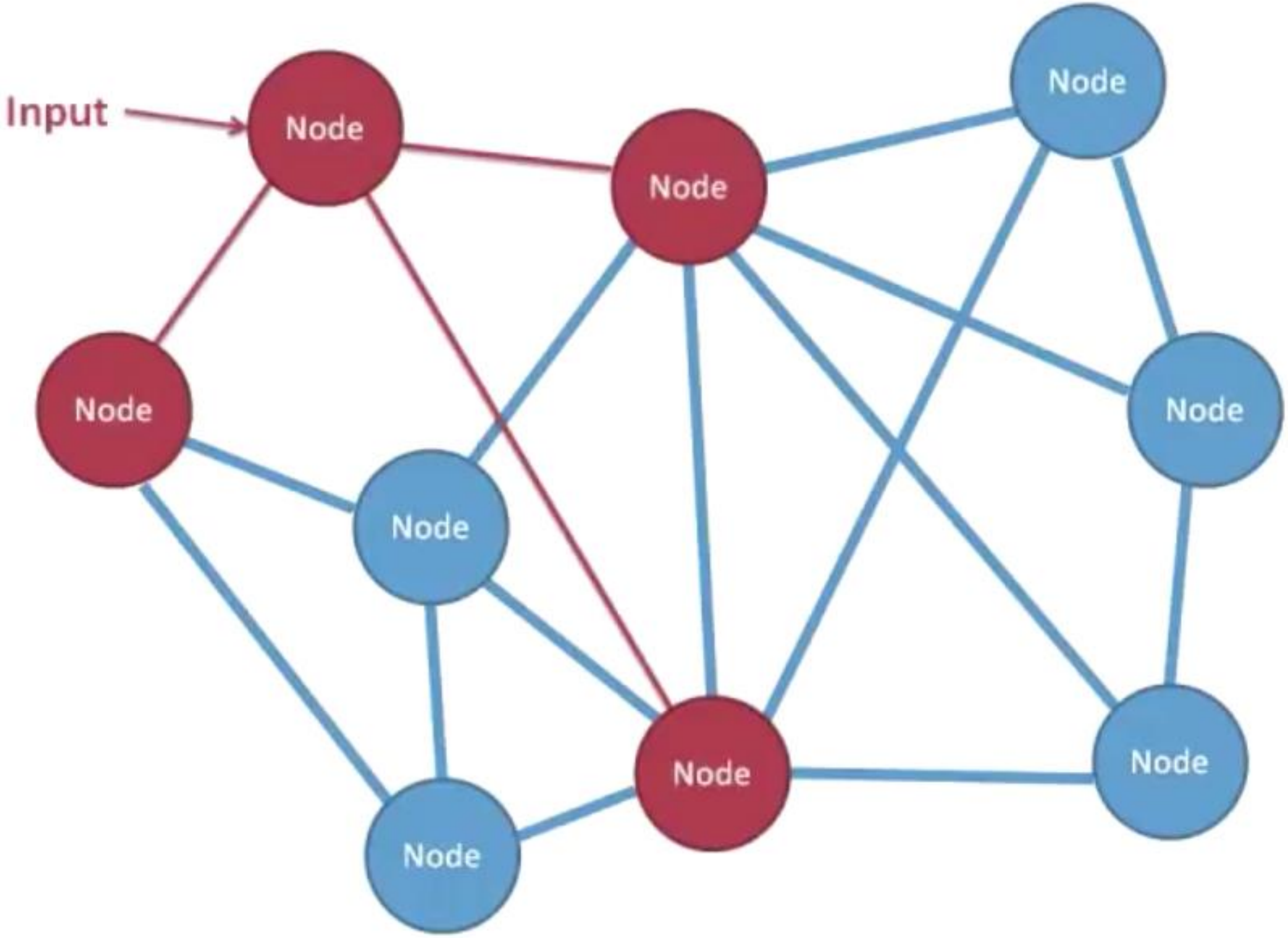
Artificial Neural Network



Artificial Neural Network

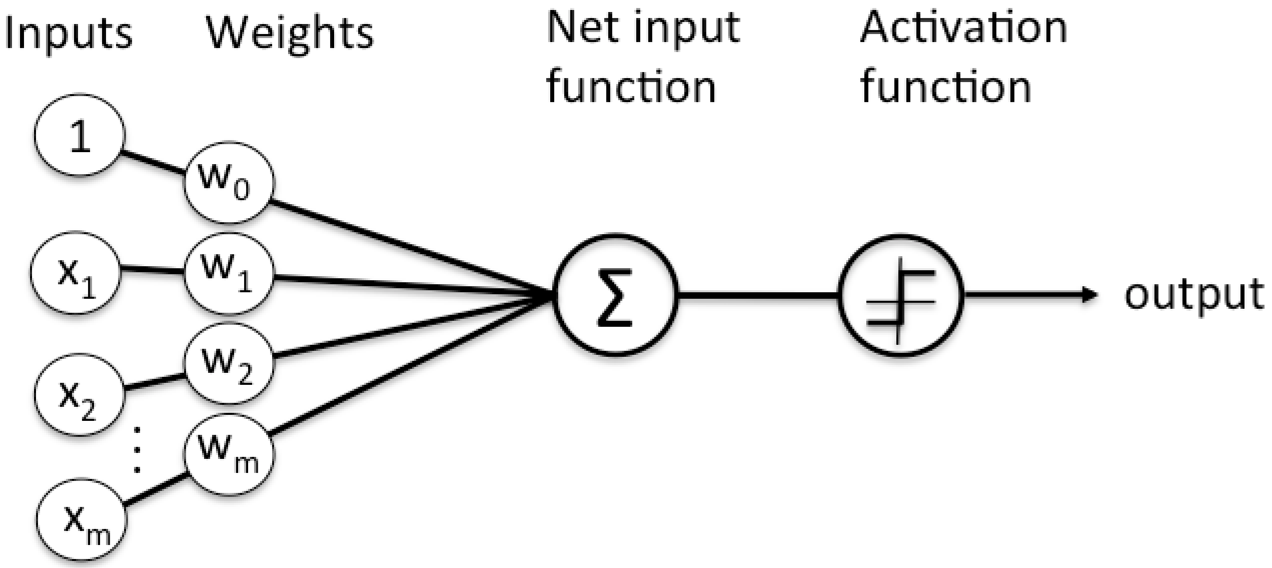


Artificial Neural Network



Artificial Neural Network

Perceptron: mimic the operation of neuron

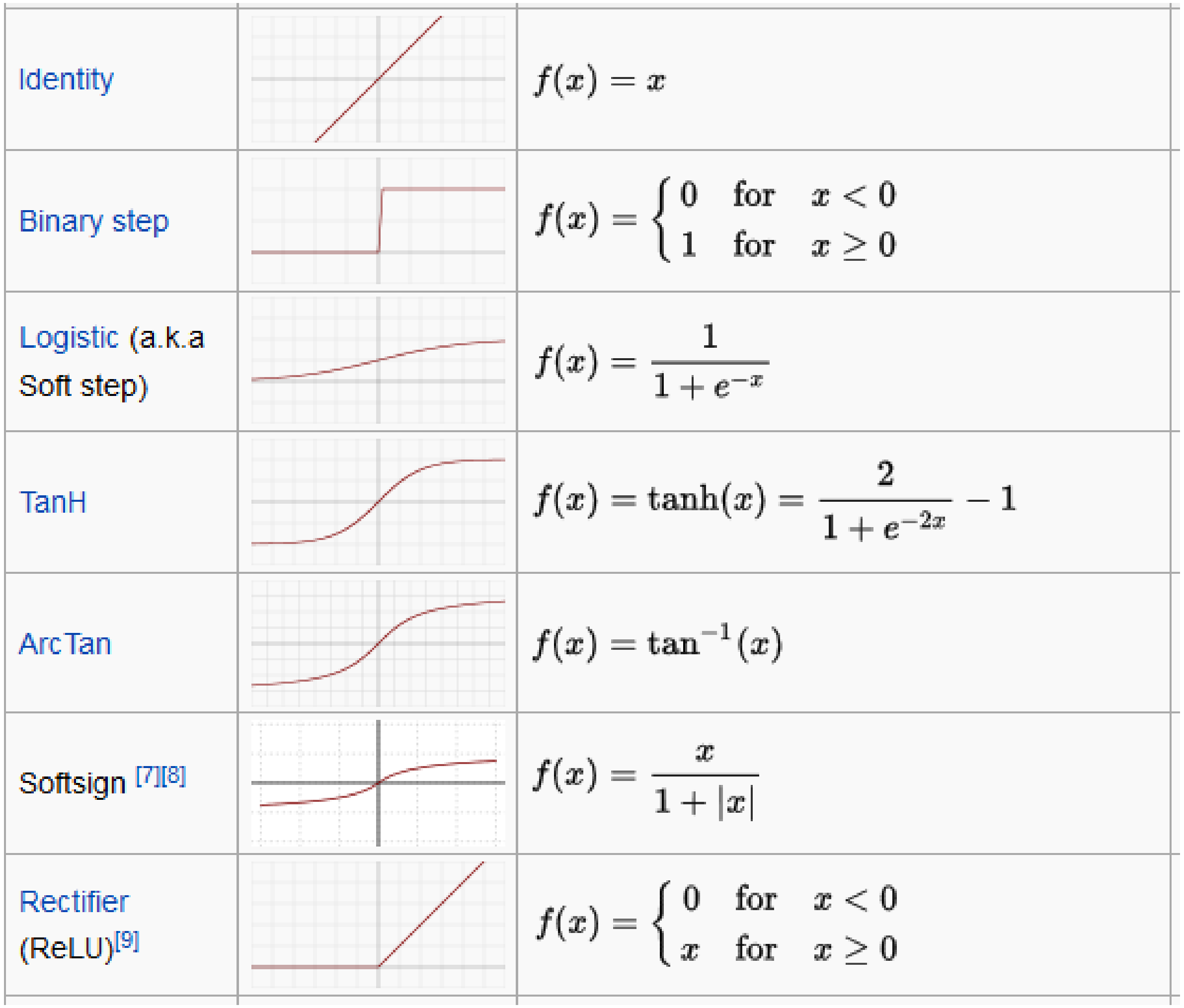


$$net = 1 * w_0 + x_1 * w_1 + x_2 * w_2 + x_3 * w_3 = w^T x$$

$$y = f(z) : activation\ \ function$$

Artificial Neural Network

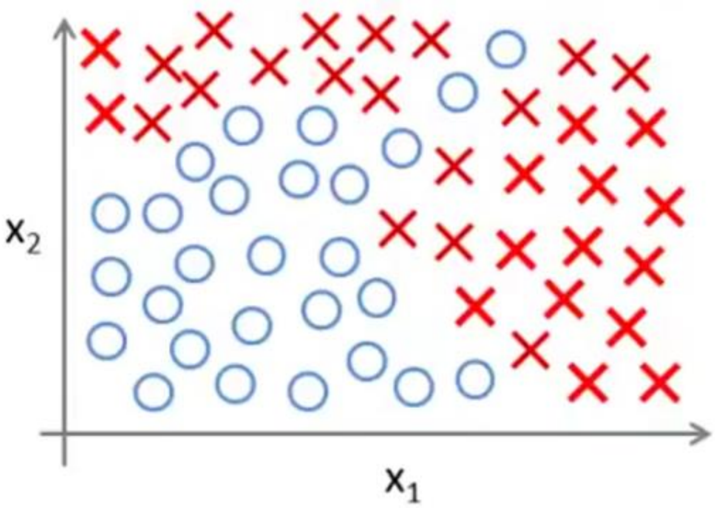
Popular activation function



Identity		$f(x) = x$
Binary step		$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$
ArcTan		$f(x) = \tan^{-1}(x)$
Softsign [7][8]		$f(x) = \frac{x}{1 + \lvert x \rvert}$
Rectifier (ReLU)[9]		$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x & \text{for} \quad x \geq 0 \end{cases}$

Artificial Neural Network

Neural Network and logistic regression



$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 + \theta_4 x_1^2 x_2 + \theta_5 x_1^3 x_2 + \theta_6 x_1 x_2^2 + \dots)$$

Many features, and their relation is complex

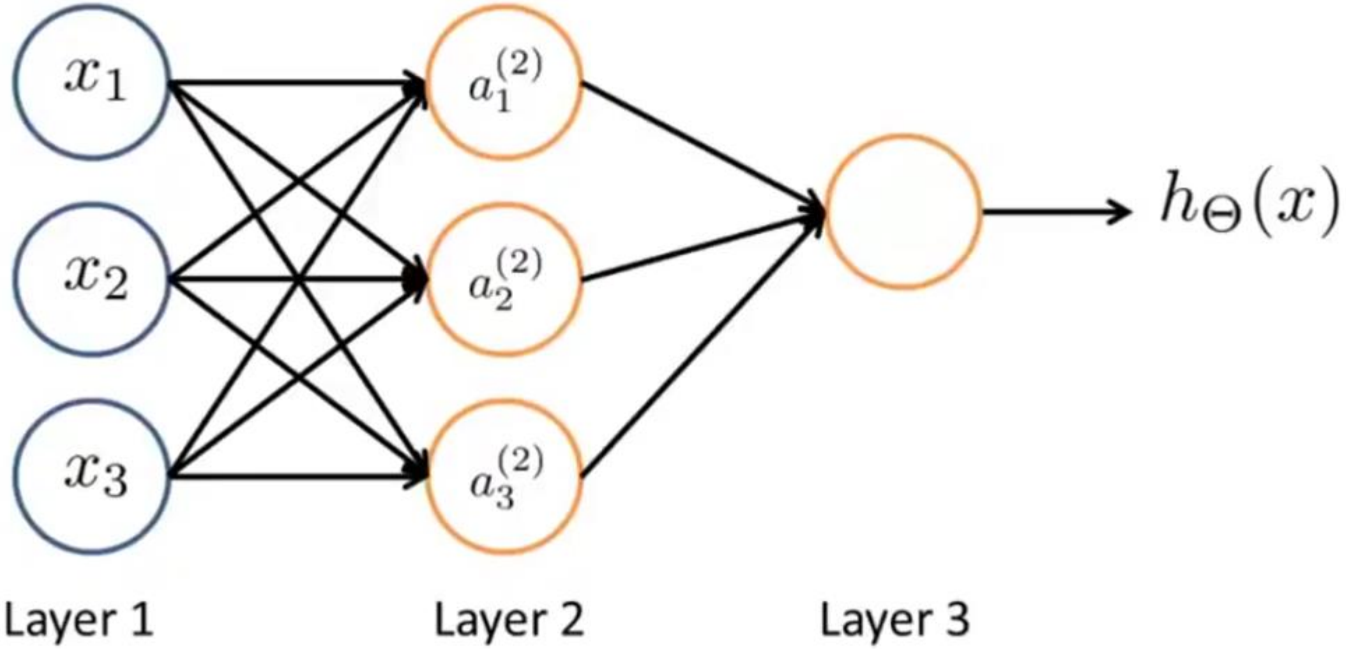
100 original features => 5000 second order features

=> 170.000 third order features

=> Not really a good way to introduce too many features to build non-linear classification

Artificial Neural Network

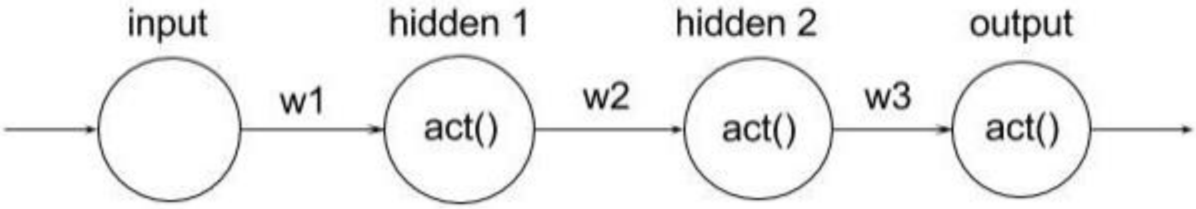
Artificial Neural Network: neurons connect to each others



Input Layer Hidden Layer Output layer

Artificial Neural Network

Neural Network: feed forward structure



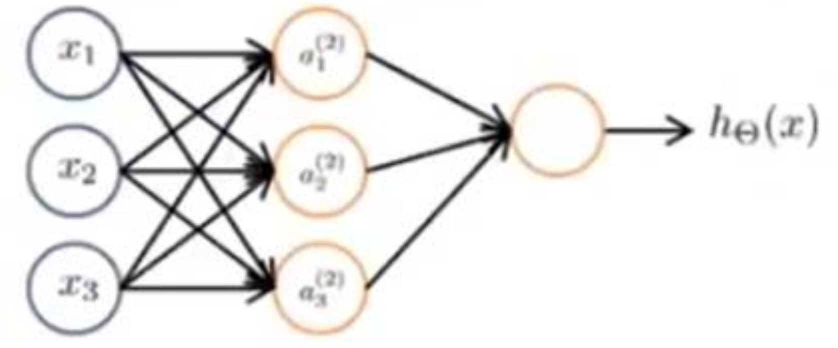
$$a^1 = g(w^1 x)$$

$$a^2 = g(w^2 a^1)$$

$$output = a^3 = g(w^3 a^2)$$

Artificial Neural Network

Neural Network: feed forward structure



$a_i^{(j)}$ = "activation" of unit i in layer j

w^j = matrix of weights controlling function mapping from layer j to layer $j+1$

$$a_1^2 = g(w_{10}^1 x_0 + w_{11}^1 x_1 + w_{12}^1 x_2 + w_{13}^1 x_3)$$

$$a_2^2 = g(w_{20}^1 x_0 + w_{21}^1 x_1 + w_{22}^1 x_2 + w_{23}^1 x_3)$$

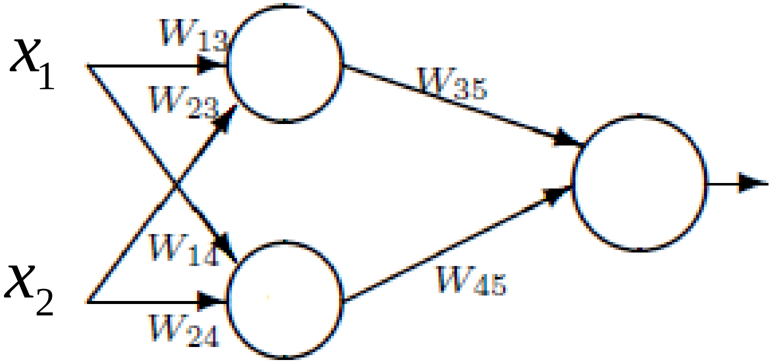
$$a_3^2 = g(w_{30}^1 x_0 + w_{31}^1 x_1 + w_{32}^1 x_2 + w_{33}^1 x_3)$$

$$h(x) = a_1^3 = g(w_{10}^2 a_0^2 + w_{11}^2 a_1^2 + w_{12}^2 a_2^2 + w_{13}^2 a_3^2)$$

Artificial Neural Network

Exercise: Calculate the output of the following network with x_1=1, x_2=0;

$w_{13} = 2$ $w_{23} = -3$	$w_{35} = 2$ $w_{45} = -1$
$w_{14} = 1$ $w_{24} = 4$	



Given the following activation function

$$f(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Outline

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Artificial Neural Network

How to train the network: Gradient descents algorithm

Cost function

$$E(w) = \frac{1}{2m} \sum \; (y - h(x))^2$$

Where y is the expected value of the output

h(x) is the predicted value of the output

$$w := w - \alpha \frac{\partial E}{\partial_w}$$

Artificial Neural Network

Back propagation: Update the weighting w layer by layer, starting from the last layer of the network.

Apply the chain rule derivation

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$\frac{\partial E}{\partial w} = \frac{\partial E}{\partial a} \cdot \frac{\partial a}{\partial net} \cdot \frac{\partial net}{\partial w}$$

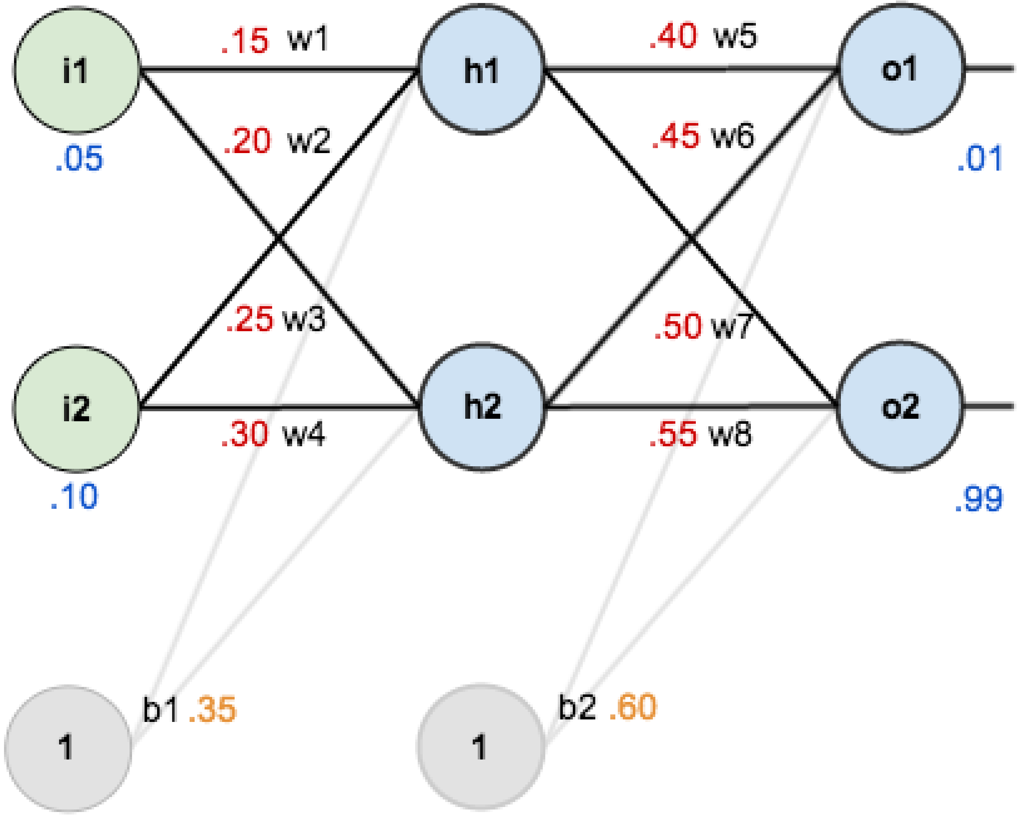
Artificial Neural Network

Example: Given the following network with follow with initial value and training data. Apply the back propagation to update the weighting w

The output of

the network

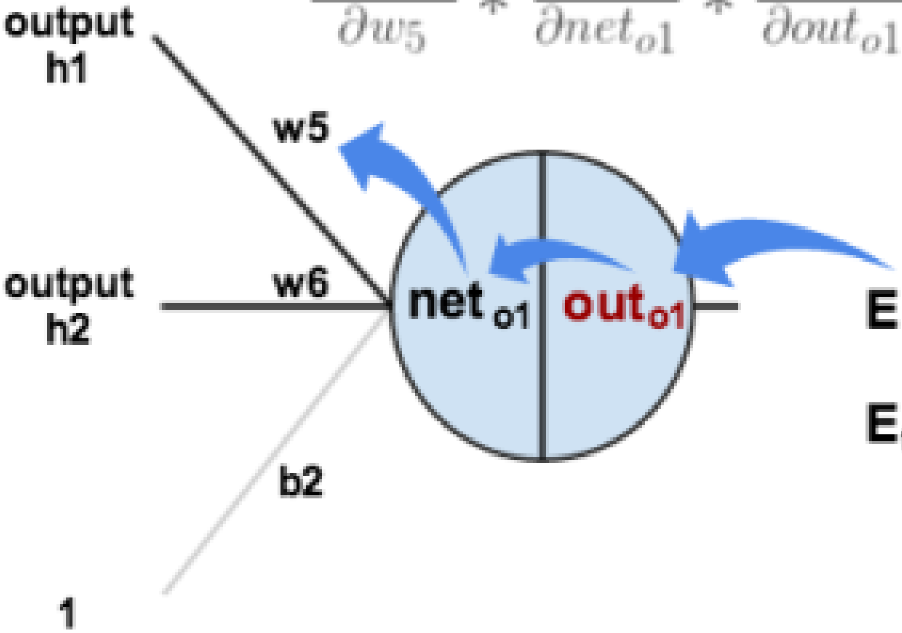
The total error



Artificial Neural Network

Using back propagation gradient descent to update w_5

$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



$$E_{o1} = \tfrac{1}{2}(target_{o1} - out_{o1})^2$$

$$E_{total} = E_{o1} + E_{o2}$$

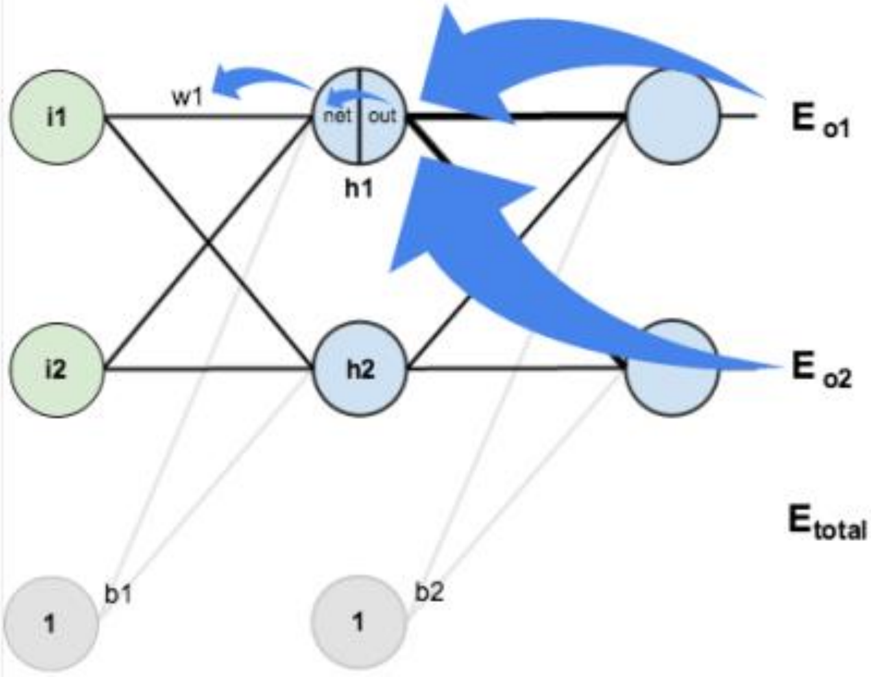
Artificial Neural Network

Using back propagation gradient descent to update w_1

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\downarrow$$

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$



$$E_{total} = E_{o1} + E_{o2}$$