Probability and Statistics
USTH-B2

Exercices Chapter 4

1. Let X, Y and Z be three independent r.v. with $var(X) = 1$, $Var(Y) = 4$ and $Var(Z) = 11$. Set $W = 3X + 2Y - Z$. Compute $std(W)$.
2. Let X be a r.v. with $E(X) = Var(X) = 1$. Use CLT to compute apprximatively $P(\sum_{k=1}^{36} X_k < 42)$, with the X_k i.i.d. as X.
3. Let $X \simeq B(n = 25, p = 1/2)$. Use the approximation of a binomial law by a gaussian law to compute $P(X < 12)$.
4. Let X_1, ..., X_{50} be r.v. with the same geometric law with parameter $p = 1/4$ and S their sum.
   (a) Compute $Var(S)$ if the correlation coefficients are all equal to $1/2$.
   (b) We assume that all X_i are independent. Use CLT to compute $P(S < 201)$.
5. Let X_1, ..., X_{50} be i.i.d. r.v. with law $N(0, 1)$. Set $Y = \sum_k X_k^2$.
   (a) Compute the mean of Y.
   (b) Use CLT to obtain $P(Y < 60)$.
6. During a quality control process, one checks the painting of n new cars taken at random among those manufactured by a specific company. Let $X_k = 1$ if the painting of k-th car has at least one defect, and 0 otherwise. Assume that the r.v. X_k are independent and that the probability that the painting of a new car is perfect is $0,75$. Thus X_k follows a Bernoulli law with parameters $p = 0,25$ for all k.
   (a) Compute $\lim_{n\to+\infty} P(|\sum_k X_k - \frac{n}{4}| \geq \frac{n}{2})$.
   (b) Let $Y \equiv X_1 + X_2$ and $Z = X_1 - X_2$. Compute $Cov(Y, Z)$.
   (c) Use CLT to compute $P(\sum_k X_k = 10)$, if $n = 40$.
7. (a) Let X, Y and X be r.v. with $var(X) = 1$, $var(Y) = 4$, $var(Z) = 9$, $cov(X, Y) = 1/2$, $cov(X, Z) = 0$ and $cov(Y, Z) = -1/2$. Compute $var(X - Y/2 + Z/3)$.
   (b) We perform 40 independent observations of the r.v. X. Let N be the number of observations greater than 1. Use CLT to compute the probability that $P(N > 5)$ if $X \simeq N(0, 1)$.