Optics and Photonics

Lecture 01: Wave Nature of Light

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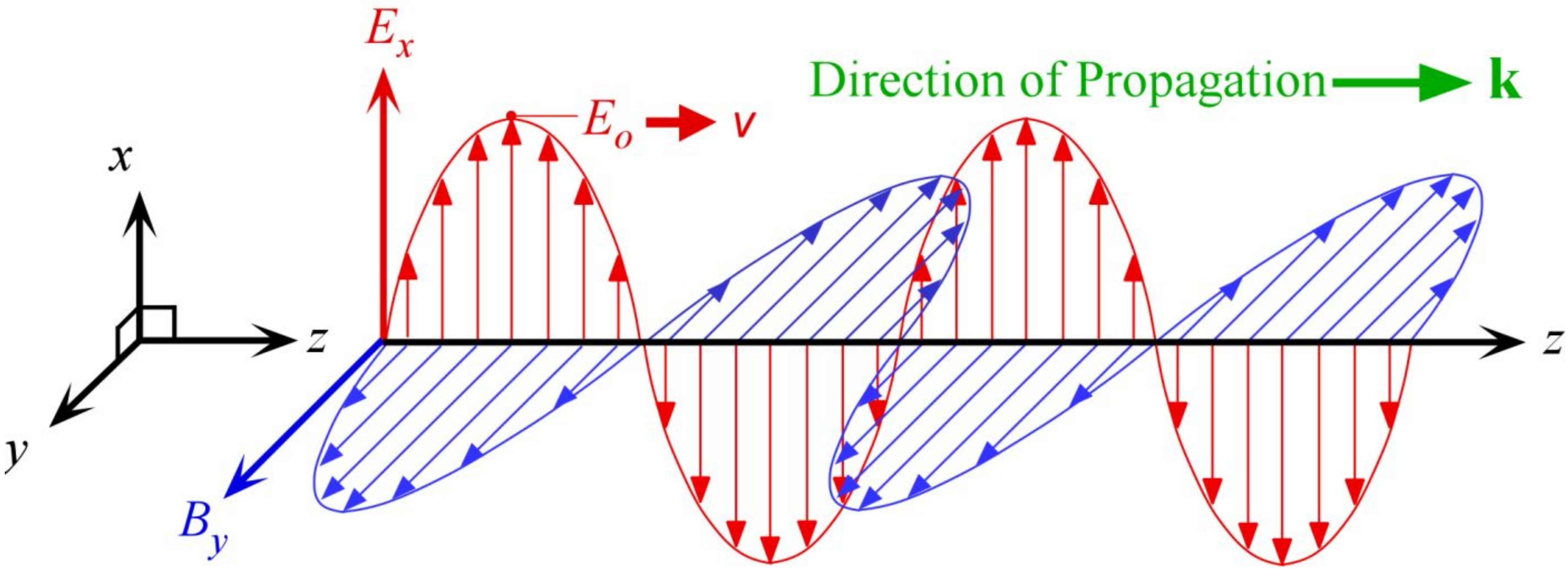
Light is an electromagnetic wave



An electromagnetic wave is a traveling wave that has time-varying electric and magnetic fields that are perpendicular to each other and the direction of propagation *z*.

$$E_x = E_o \cos(\omega t - kz + \varphi_o)$$

E_x = Electric field along x at position z at time t

k = **Propagation constant** = $2\pi/\lambda$

λ = Wavelength

ω = Angular frequency = $2\pi\upsilon$ (υ = frequency)

E_o = Amplitude of the wave

φ_o = Phase constant; at $t = 0$ and $z = 0$, E_x may or may not necessarily be zero depending on the choice of origin.

$(\omega t - kz + \varphi_o) = \varphi$ = Phase of the wave

This is a **monochromatic plane wave** of *infinite extent* traveling in the positive z direction.

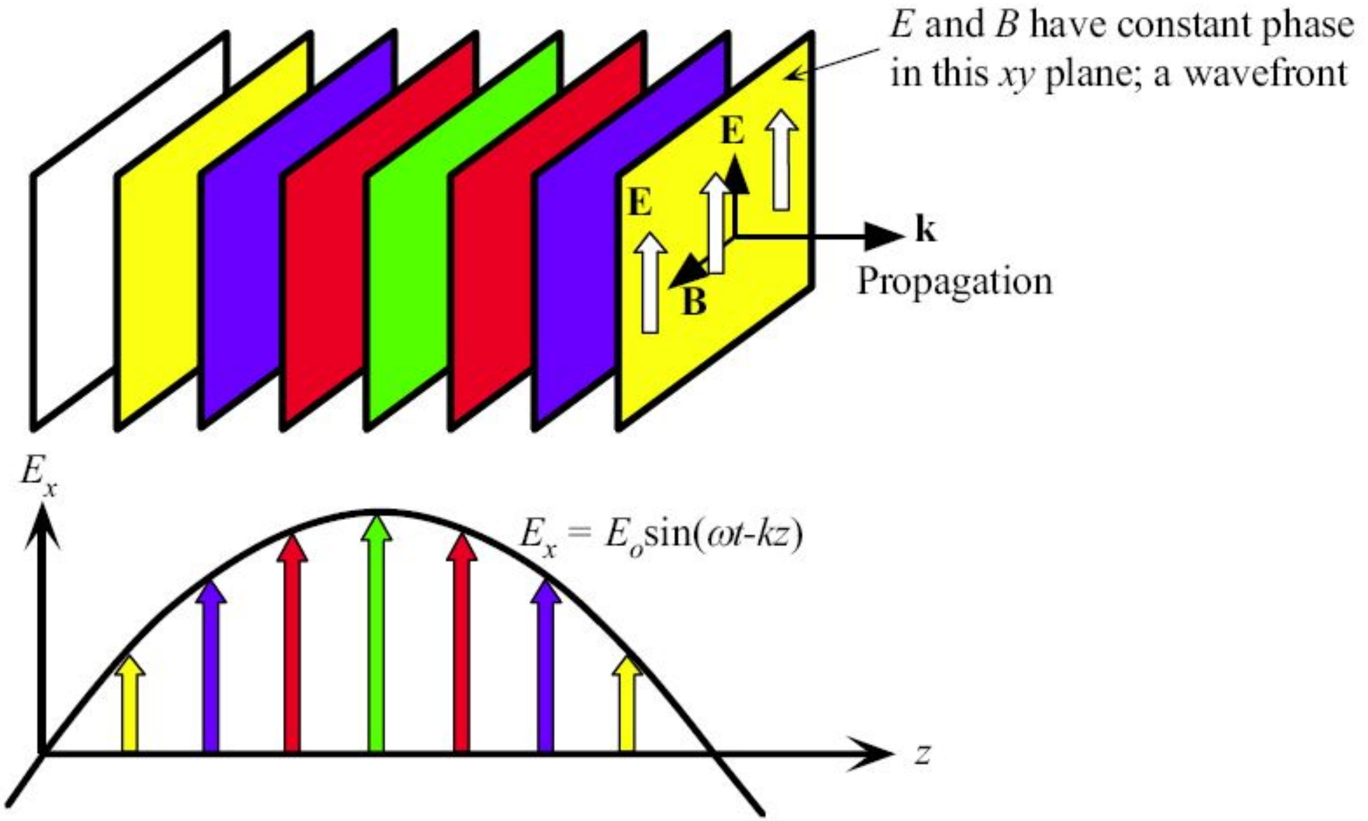
Wavefront

A surface over which the phase of a wave is constant is referred to as a wavefront

A wavefront of a plane wave is a plane perpendicular to the direction of propagation

The interaction of a light wave with a nonconducting medium (conductivity = 0) uses the electric field component E_x rather than B_y.

Optical field refers to the electric field E_x.



A plane EM wave traveling along *z*, has the same E_x (or B_y) at any point in a given *xy* plane. All electric field vectors in a given *xy* plane are therefore in phase. The *xy* planes are of infinite extent in the *x* and *y* directions.

Phase Velocity

The time and space evolution of a given phase φ, for example that corresponding to a maximum field is described by

$$\varphi = \omega t - kz + \varphi_o = \text{constant}$$

During a time interval δt, this constant phase (and hence the maximum field) moves a distance δz. The phase velocity of this wave is therefore $\delta z/\delta t$. The phase velocity v is

$$v = \frac{\delta z}{\delta t} = \frac{\omega}{k} = \upsilon\lambda$$

Phase change over a distance Δz

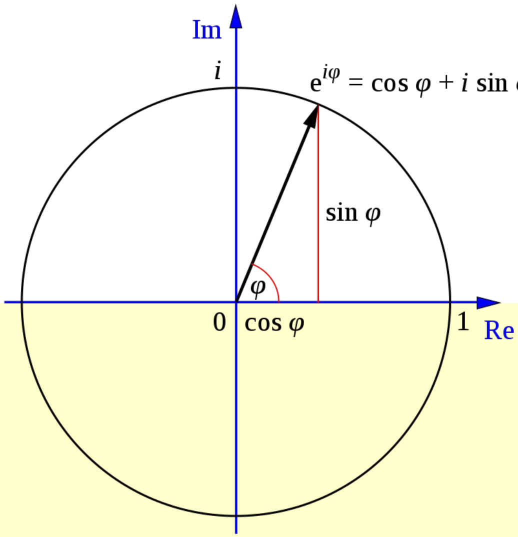
$$\varphi = \omega t - kz + \varphi_o$$

$$\Delta\varphi = k\Delta z$$

The phase difference between two points separated by Δz is simply $k\Delta z$ since ωt is the same for each point

If this phase difference is 0 or multiples of 2π then the two points are in phase. Thus, the phase difference $\Delta\varphi$ can be expressed as $k\Delta z$ or $2\pi\Delta z/\lambda$

Exponential Notation



Recall that

$$\cos\varphi = \mathrm{Re}[\exp(j\varphi)]$$

where Re refers to the real part. We then need to take the real part of any complex result at the end of calculations. Thus,

$$E_x(z,t) = \mathrm{Re}[E_o \exp(j\varphi_o)\exp j(\omega t - kz)]$$

or

$$E_x(z,t) = \mathrm{Re}[E_c \exp j(\omega t - kz)]$$

where $E_c = E_o\exp(j\varphi_o)$ is a complex number that represents the amplitude of the wave and includes the constant phase information φ_o.

Wave Vector or Propagation Vector

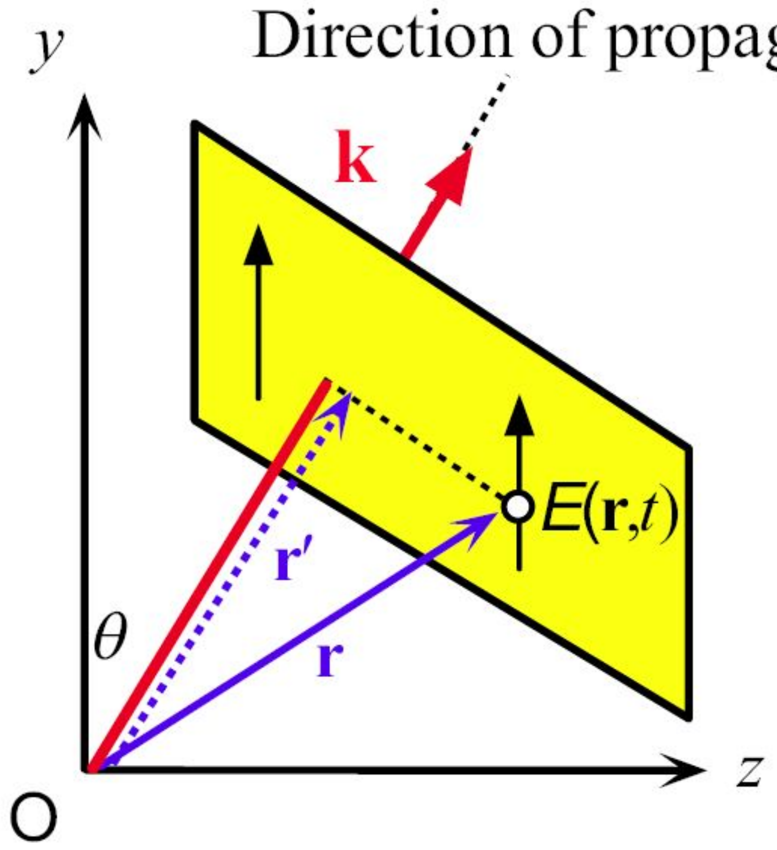
Direction of propagation is indicated with a vector k, called the wave vector, whose magnitude is the *propagation constant*, $k = 2\pi/\lambda$. k is *perpendicular* to constant phase planes.

When the electromagnetic (EM) wave is propagating along some arbitrary direction k, then the electric field $E(\mathbf{r},t)$ at a point r on a plane perpendicular to k is

$$E(\mathbf{r},t) = E_o \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \varphi_o)$$

If propagation is along z, $\mathbf{k}\cdot\mathbf{r}$ becomes kz. In general, if k has components k_x, k_y and k_z along x, y and z, then from the definition of the dot product, $\mathbf{k}\cdot\mathbf{r} = k_x x + k_y y + k_z z$.

Wave Vector k



$$E(\mathbf{r},t) = E_o\cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \varphi_o)$$

A traveling plane EM wave along a direction **k**

Maxwell's Wave Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \varepsilon_o \varepsilon_r \mu_o \frac{\partial^2 E}{\partial t^2} = 0$$

μ_0 is the absolute permeability, ε_0 is the absolute permittivity, and ε_r is the relative permittivity of the medium.

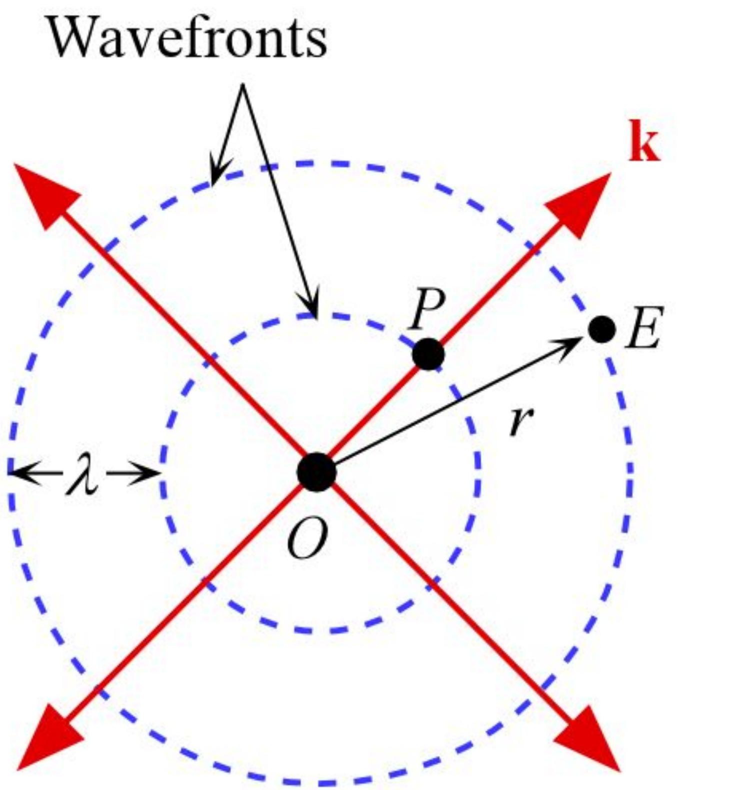
A plane wave is a solution of Maxwell's wave equation

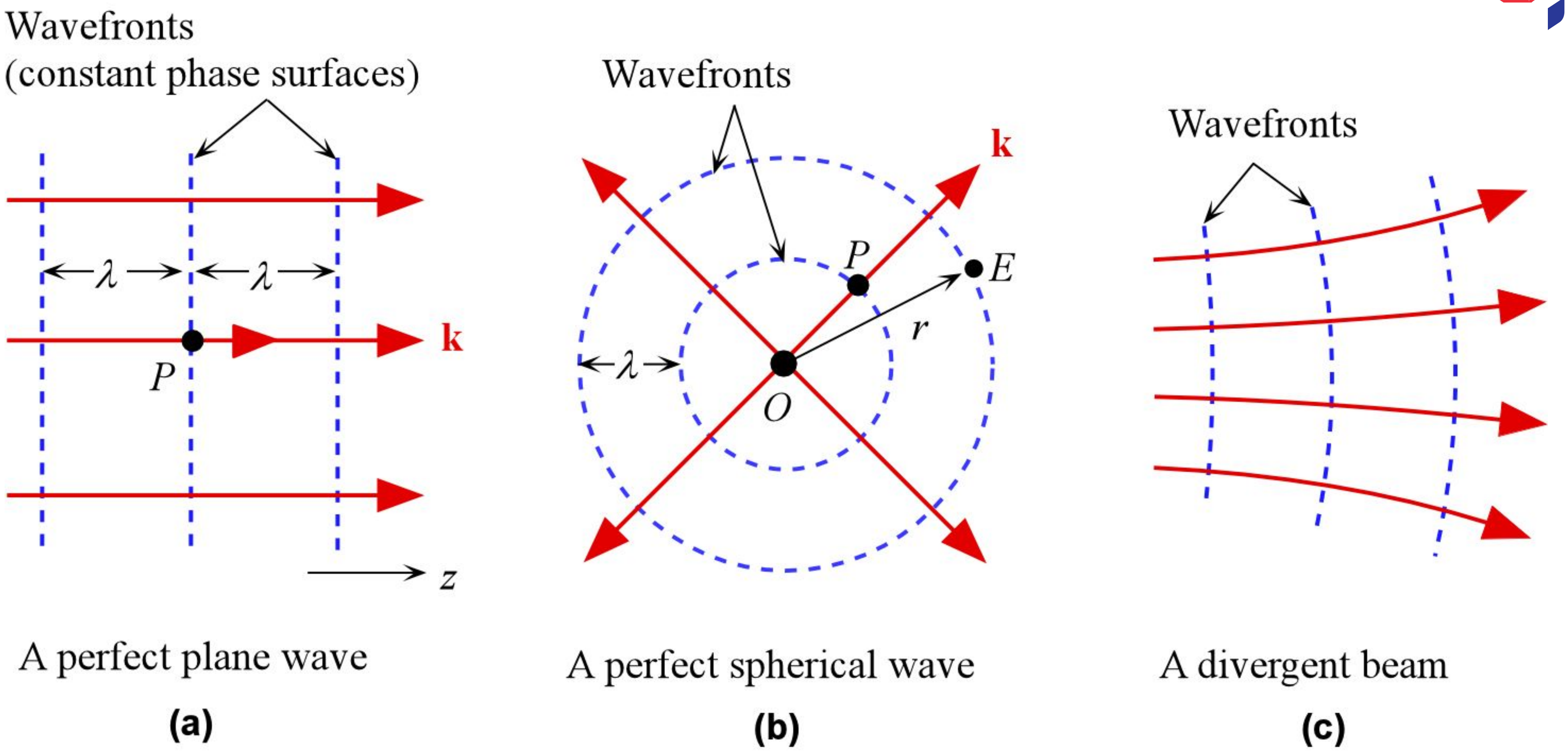
$$E_x = E_o \cos(\omega t - kz + \varphi_o)$$

Substitute into Maxwell's Equation to show that this is a solution.

Spherical Wave

$$E = \frac{A}{r}\cos(\omega t - kr)$$



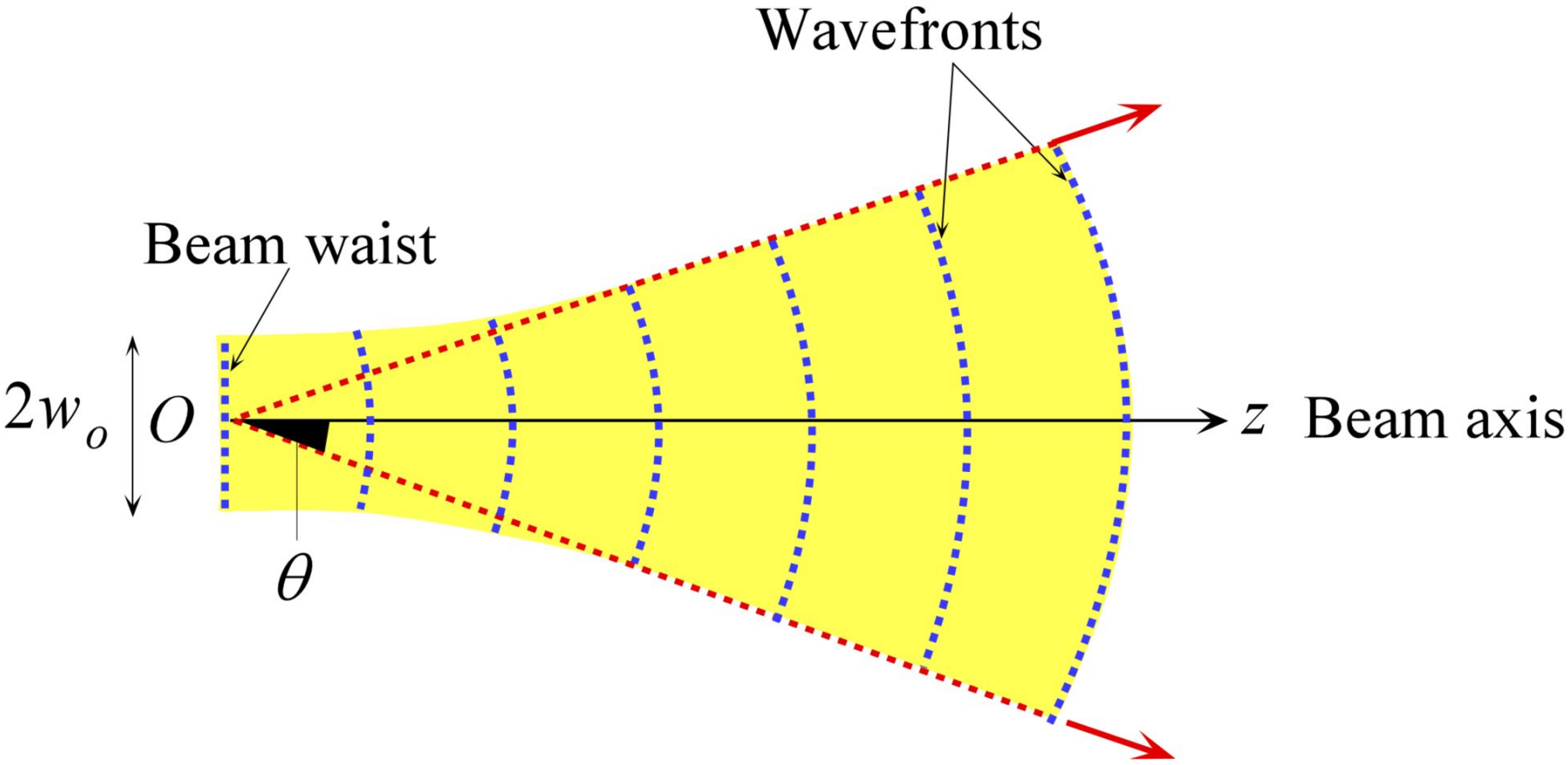


Examples of possible EM waves

Optical divergence refers to the angular separation of wave vectors on a given wavefront.

Gaussian Beam

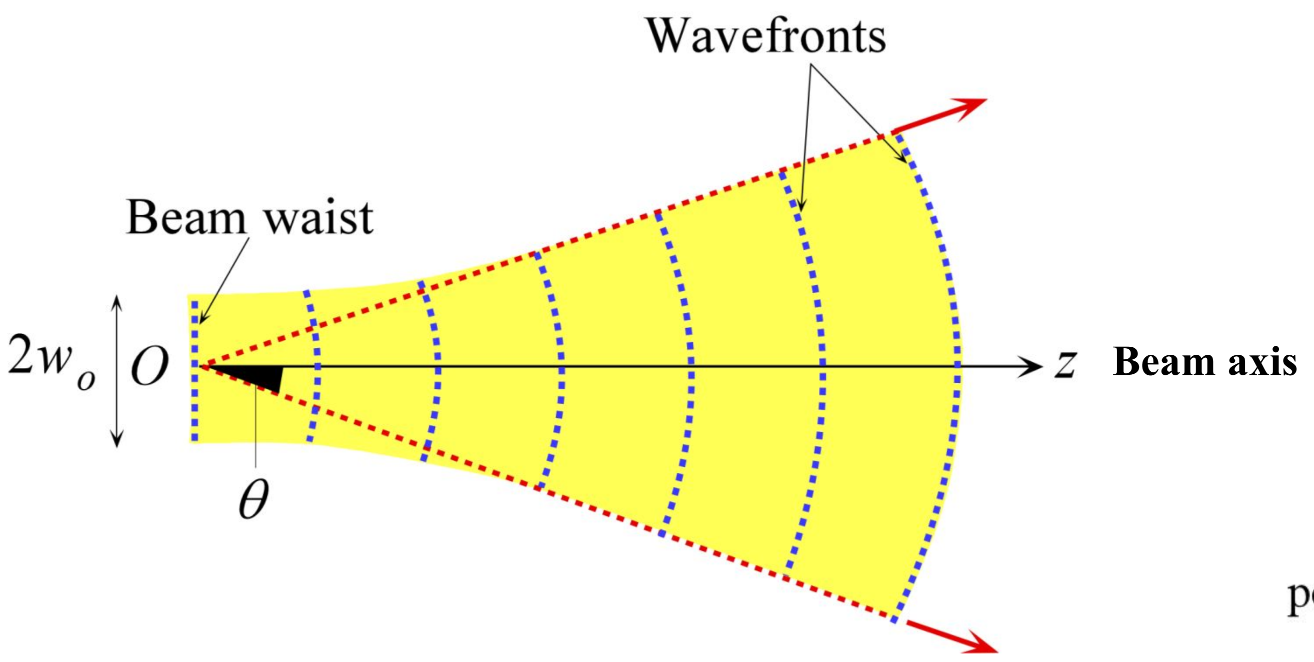
The radiation emitted from a laser can be approximated by a Gaussian beam. Gaussian beam approximations are widely used in photonics.

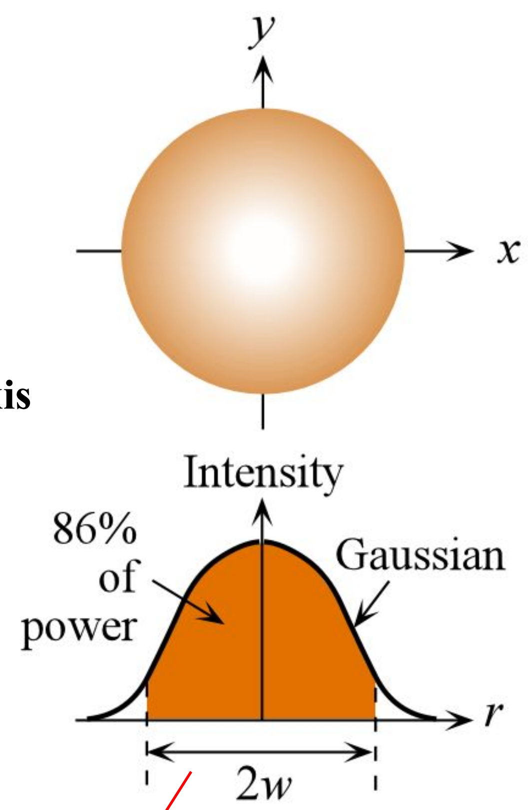


Wavefronts of a Gaussian light beam

Gaussian Beam

The intensity across the beam follows a Gaussian distribution



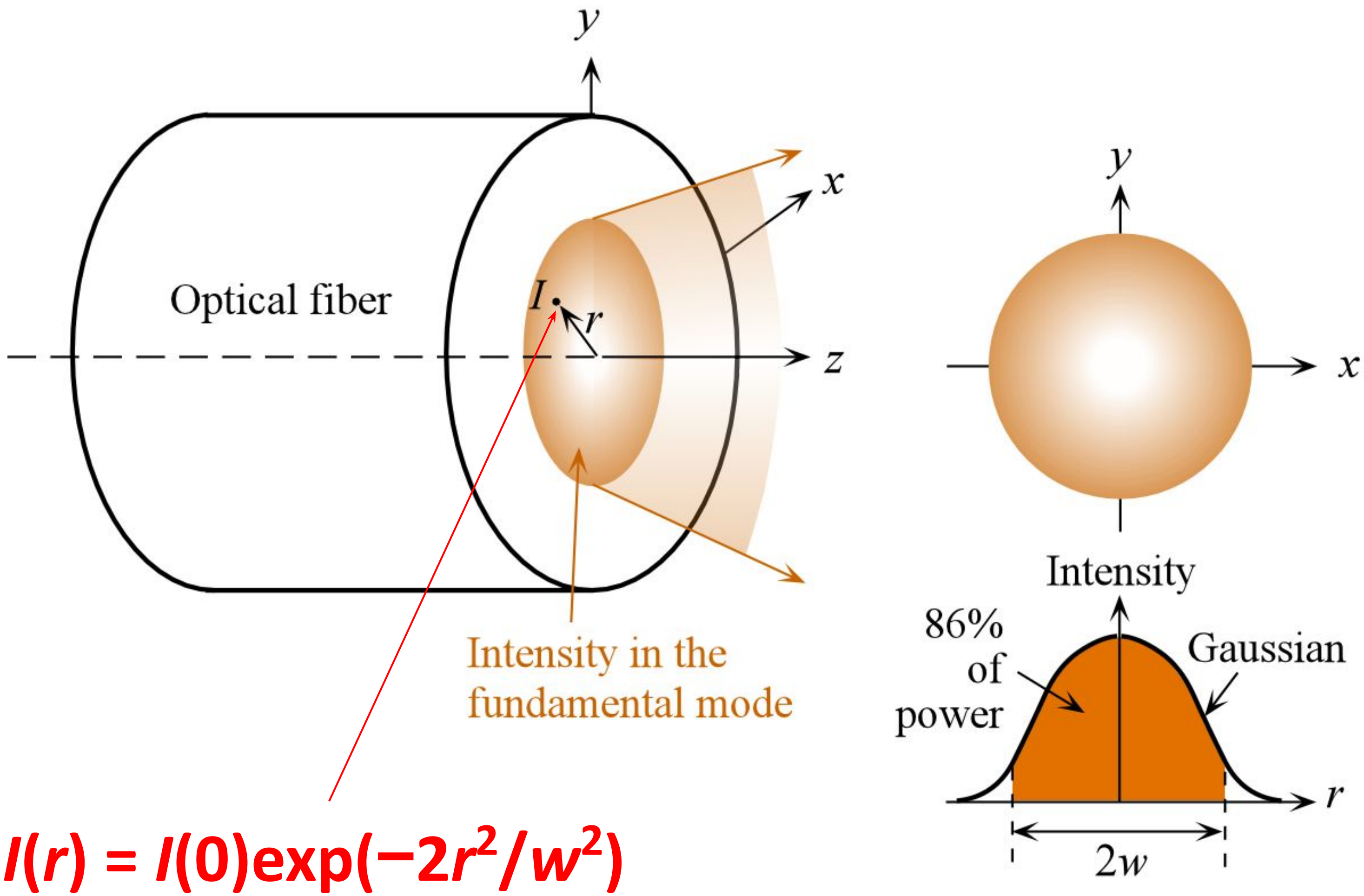


$$\text{Intensity} = I(r,z) = [2P/(\pi w^2)]\exp(-2r^2/w^2)$$

$\theta = w/z = \lambda/(\pi w_o)$ 2θ = Far field divergence

The Gaussian Intensity Distribution is Not Unusual

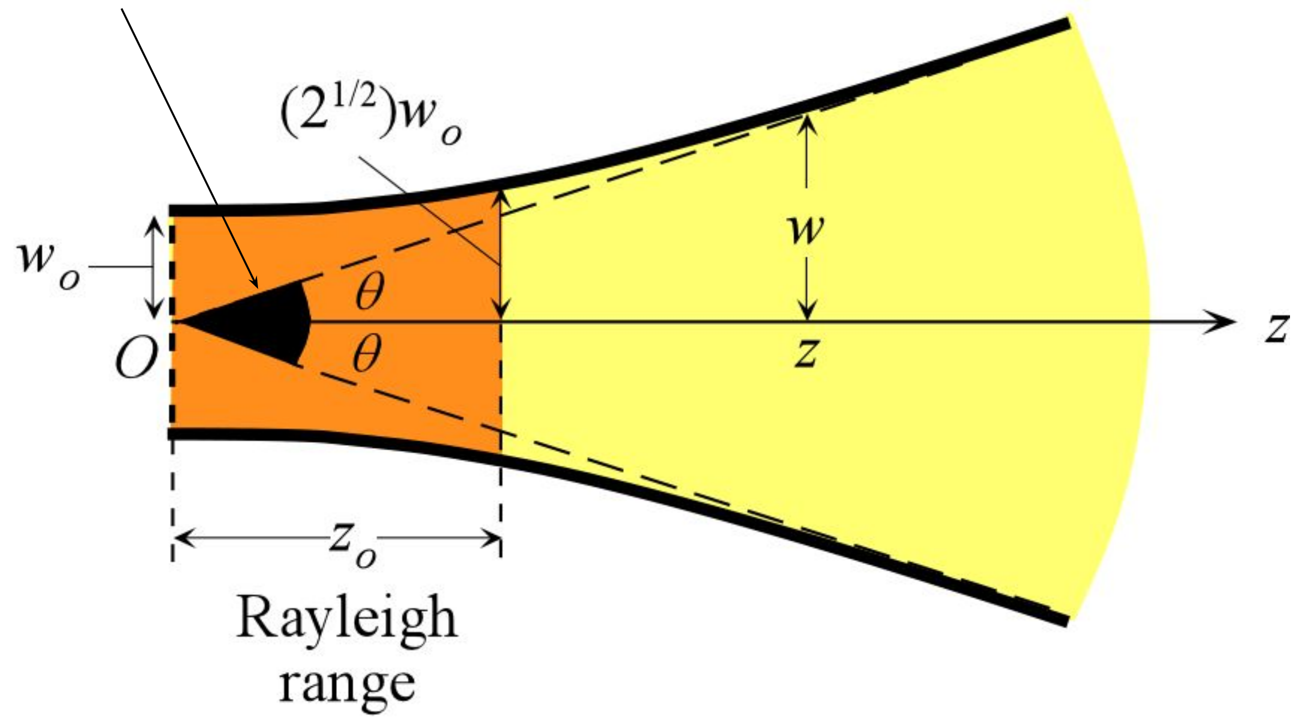
The Gaussian intensity distribution is also used in fiber optics
The fundamental mode in single mode fibers can be approximated with a Gaussian intensity distribution across the fiber core



$$I(r) = I(0)\exp(-2r^2/w^2)$$

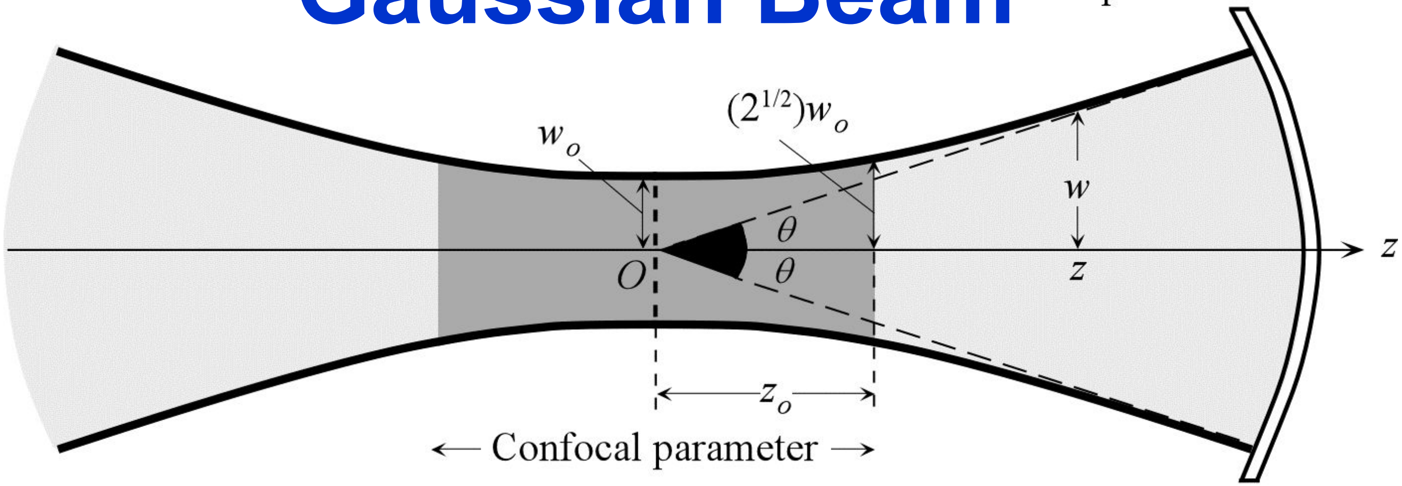
Gaussian Beam

2θ = Far field divergence



$$z_o = \pi w_o^2/\lambda$$

Gaussian Beam



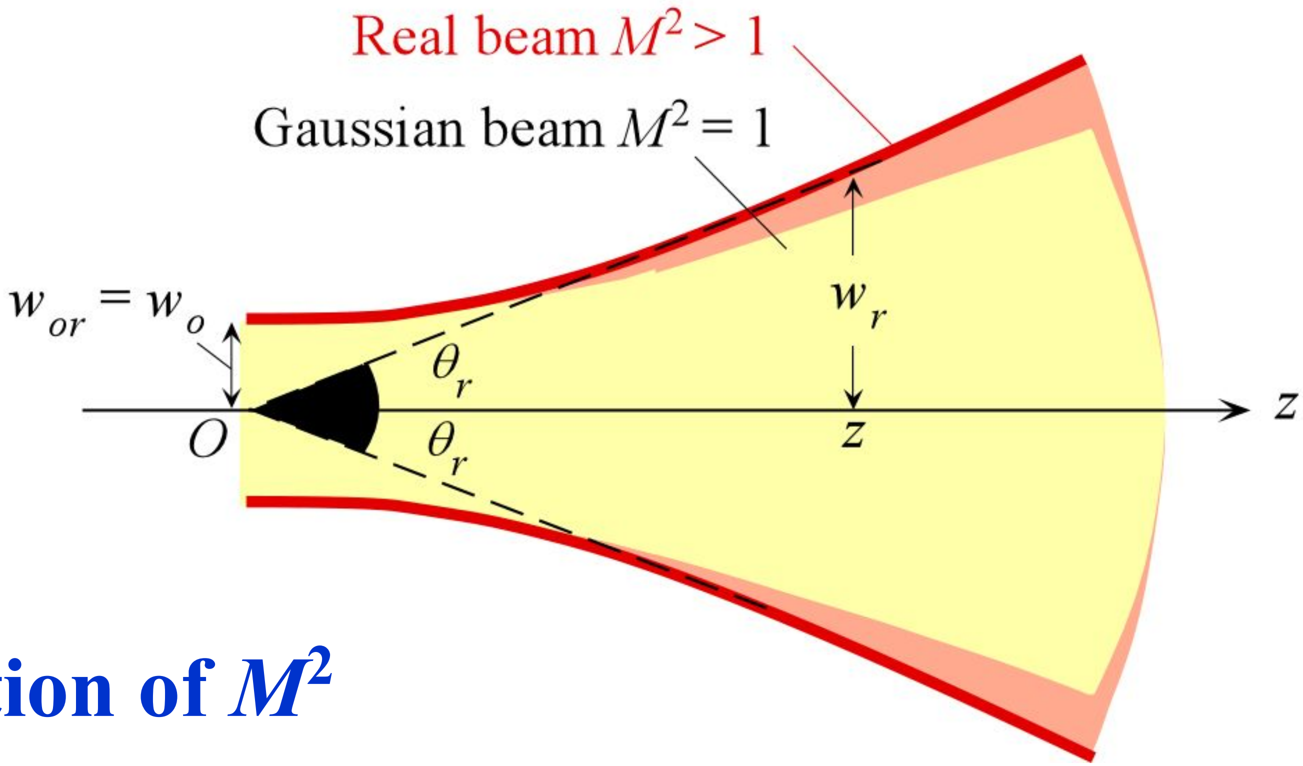
Rayleigh range

$$z_o = \frac{\pi w_o^2}{\lambda}$$

$$2w = 2w_o\left[1 + \left(\frac{z}{z_o}\right)^2\right]^{1/2}$$

$$2w = 2w_o\left[1 + \left(\frac{z\lambda}{\pi w_o^2}\right)^2\right]^{1/2}$$

Real and Ideal Gaussian Beams



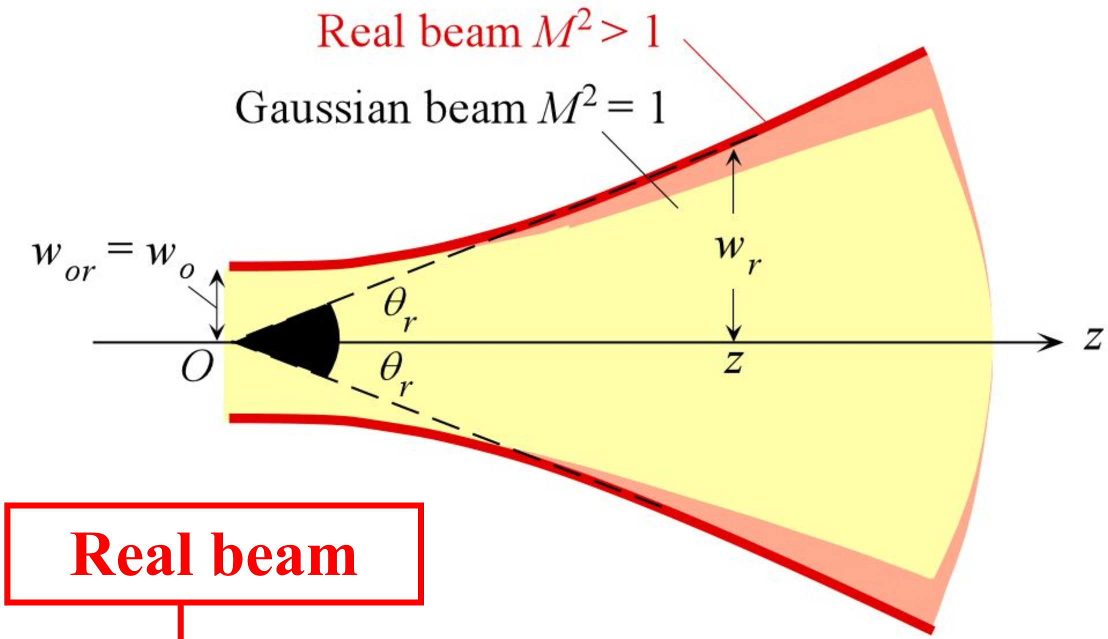
Definition of M^2

$$M^2 = \frac{w_{or}\theta_r}{w_o\theta} = \frac{w_{or}\theta_r}{(\lambda/\pi)}$$

$$2w_r = 2w_{or}\left[1 + \left(\frac{z\lambda M^2}{\pi w_{or}^2}\right)^2\right]^{1/2}$$

The Gaussian beam concept is so useful in photonics that a special quantity, called the **M^2-factor**, has been introduced to compare a given laser beam to an ideal Gaussian beam.

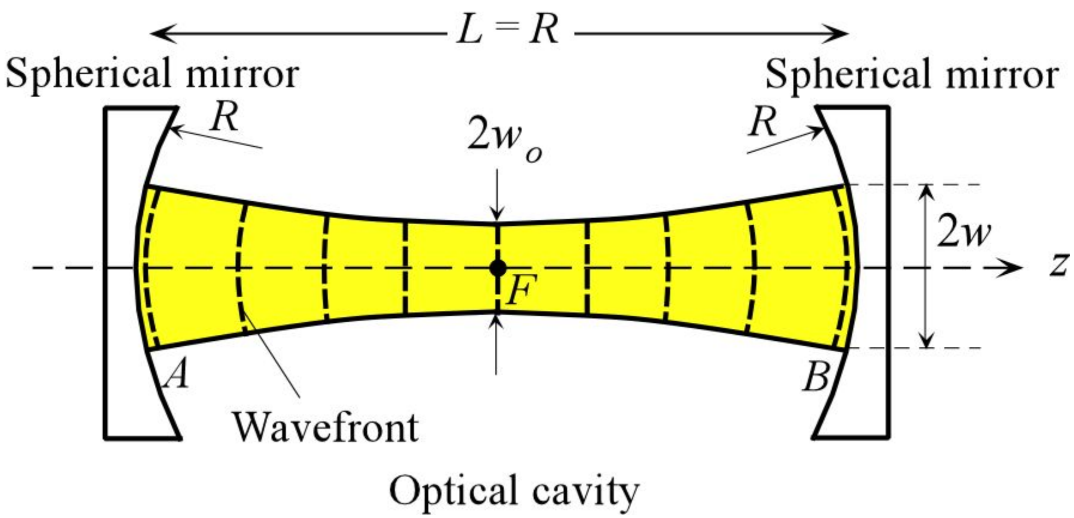
Real Gaussian Beam



$$2w_r = 2w_{or}\left[1+\left(\frac{z\lambda M^2}{\pi w_{or}^2}\right)^2\right]^{1/2}$$

Correction note: Page 10 in textbook, Equation (1.11.1), w should be w_r as above and w_{or} should be squared in the parantheses.

Gaussian Beam in an Optical Cavity



Two spherical mirrors reflect waves to and from each other. The optical cavity contains a Gaussian beam. This particular optical cavity is symmetric and confocal; the two focal points coincide at *F*.

EXAMPLE 1.1.1 A diverging laser beam

Consider a He-Ne laser beam at 633 nm with a spot size of 1 mm. Assuming a Gaussian beam, what is the divergence of the beam? What are the Rayleigh range and the beam width at 25 m?

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Solution

Using Eq. (1.1.7), we find

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633 \times 10^{-9}\ \text{m})}{\pi(1 \times 10^{-3}\ \text{m})} = 8.06 \times 10^{-4}\ \text{rad} = 0.046°$$

The Rayleigh range is

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{\pi\left[(1 \times 10^{-3}\ \text{m})/2\right]^2}{(633 \times 10^{-9}\ \text{m})} = 1.24\ \text{m}$$

The beam width at a distance of 25 m is

$$2w = 2w_o[1 + (z/z_o)^2]^{1/2} = (1 \times 10^{-3}\ \text{m})\{1 + [(25\ \text{m})/(1.24\ \text{m})]^2\}^{1/2}$$

$$= 0.0202\ \text{m} \quad \text{or} \quad 20\ \text{mm}.$$

$$2w = 2w_o\left[1+\left(\frac{z}{z_o}\right)^2\right]^{1/2} \approx 2w_o\frac{z}{z_o} = (1\,\text{mm})\frac{25\,\text{m}}{1.24\,\text{m}} = 20\,\text{mm}$$

Refractive Index

When an EM wave is traveling in a dielectric medium, the oscillating electric field **polarizes** the molecules of the medium at the frequency of the wave

The **stronger** is the interaction between the field and the dipoles, the **slower** is the propagation of the wave

Maxwell's Wave Equation in an isotropic medium

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}-\varepsilon_o\varepsilon_r\mu_o\frac{\partial^2 E}{\partial t^2}=0$$

A plane wave is a solution of Maxwell's wave equation

$$E_x = E_o\cos(\omega t - kz + \varphi_o)$$

The phase velocity of this plane wave in the medium is given by

$$v=\frac{\omega}{k}=\frac{1}{\sqrt{\varepsilon_o\varepsilon_r\mu_o}}$$

The phase velocity in vacuum is

$$c=\frac{\omega}{k_o}=\frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

Phase Velocity and ε_r

The relative permittivity ε_r measures the ease with which the medium becomes polarized and hence it indicates the extent of interaction between the field and the induced dipoles.

For an EM wave traveling in a nonmagnetic dielectric medium of relative permittivity ε_r, the phase velocity v is given by

$$v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_o \mu_o}}$$

Refractive Index *n*

Phase Velocity and ε_r

$$V = \frac{1}{\sqrt{\varepsilon_r \varepsilon_o \mu_o}}$$

Refractive index *n* definition

$$n = \frac{c}{V} = \sqrt{\varepsilon_r}$$

Low frequency (LF) relative permittivity ε_r(LF) and refractive index *n*.

TABLE 1.1 **Low-frequency (LF) relative permittivity ε_r(LF) and refractive index *n***

Material	ε_r(LF)	$[\varepsilon_r(\mathrm{LF})]^{1/2}$	n (at λ)	Comment
Si	11.9	3.44	3.45 (at 2.15 μm)	Electronic bond polarization up to optical frequencies
Diamond	5.7	2.39	2.41 (at 590 nm)	Electronic bond polarization up to UV light
GaAs	13.1	3.62	3.30 (at 5 μm)	Ionic polarization contributes to ε_r(LF)
SiO_2	3.84	2.00	1.46 (at 600 nm)	Ionic polarization contributes to ε_r(LF)
Water	80	8.9	1.33 (at 600 nm)	Dipolar polarization contributes to ε_r(LF), which is large

Optical frequencies

Typical frequencies that are involved in optoelectronic devices are in the infrared (including far infrared), visible, and UV, and we generically refer to these frequencies as **optical frequencies**

Somewhat arbitrary range:

Roughly 10^{12} Hz to 10^{16} Hz

Refractive Index and Propagation Constant

k_o Free-space propagation constant (wave vector)
$k_o = 2\pi/\lambda_o$

λ_o Free-space wavelength

k Propagation constant (wave vector) in the medium

λ Wavelength in the medium

$$n = \frac{k}{k_o}$$

In noncrystalline materials such as glasses and liquids, the material structure is the same in all directions and n does not depend on the direction. The refractive index is then **isotropic**

Refractive Index and Wavelength

It is customary to drop the subscript *o* on *k* and *λ*

$$k_{\text{medium}} = nk$$

$$\lambda_{\text{medium}} = \lambda / n$$

In free space

Refractive Index and Isotropy

Crystals, in general, have non-isotropic, or anisotropic, properties

Typically noncrystalline solids such as glasses and liquids, and cubic crystals are optically isotropic; they possess only one refractive index for all directions

n depends on the wavelength λ

Dispersion relation: $n = n(\lambda)$

The simplest electronic polarization gives

$$n^2 = 1 + \left(\frac{N_{\text{at}} Z e^2}{\varepsilon_o m_e}\right)\left(\frac{\lambda_o}{2\pi c}\right)^2 \frac{\lambda^2}{\lambda^2 - \lambda_o^2}$$

N_{at} =Number of atoms per unit volume
Z = Number of electrons in the atom (atomic number)

λ_o = A "resonant frequency"

Sellmeier Equation

$$n^2 = 1 + \frac{A_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3 \lambda^2}{\lambda^2 - \lambda_3^2}$$

n depends on the wavelength λ

Cauchy dispersion relation

$$n = n(\upsilon)$$

$$n = n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4$$

n depends on the wavelength *λ*

TABLE 1.2 Sellmeier and Cauchy coefficients

Sellmeier	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO_2 (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO_2-13.5%GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO_2	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	–	0.1750	0.1060	–

Cauchy	Range of $h\nu$ (eV)	n_{-2} (eV^2)	n_0	n_2 (eV^{-2})	n_4 (eV^{-4})
Diamond	0.05–5.47	-1.07×10^{-5}	2.378	8.01×10^{-3}	1.04×10^{-4}
Silicon	0.002–1.08	-2.04×10^{-8}	3.4189	8.15×10^{-2}	1.25×10^{-2}
Germanium	0.002–0.75	-1.0×10^{-8}	4.003	2.2×10^{-1}	1.4×10^{-1}

Source: Sellmeier coefficients combined from various sources. Cauchy coefficients from D. Y. Smith *et al.*, *J. Phys. CM*, *13*, 3883, 2001.

EXAMPLE 1.2.1 Sellmeier equation and diamond

Using the Sellmeier coefficients for diamond in Table 1.2, calculate its refractive index at 610 nm (red light) and compare with the experimental quoted value of 2.415 to three decimal places.

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Solution

The Sellmeier dispersion relation for diamond is

$$n^2 = 1 + \frac{0.3306\lambda^2}{\lambda^2 - 175\text{ nm}^2} + \frac{4.3356\lambda^2}{\lambda^2 - 106\text{ nm}^2}$$

$$n^2 = 1 + \frac{0.3306(610\text{ nm})^2}{(610\text{ nm})^2 - (175\text{ nm})^2} + \frac{4.3356(610\text{ nm})^2}{(610\text{ nm})^2 - (106\text{ nm})^2} = 5.8308$$

So that

$$n = 2.4147$$

which is 2.415 to three decimal places and matches the experimental value.

EXAMPLE 1.2.2 Cauchy equation and diamond

Using the Cauchy coefficients for diamond in Table 1.2, calculate the refractive index at 610 nm.

EXAMPLE 1.2.2 Cauchy equation and diamond

Using the Cauchy coefficients for diamond in Table 1.2, calculate the refractive index at 610 nm.

Solution

At $\lambda = 610$ nm, the photon energy is

$$h\upsilon = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s})(2.998 \times 10^{8}\,\mathrm{m\,s^{-1}})}{(610 \times 10^{-9}\,\mathrm{m})} \times \frac{1}{1.602 \times 10^{-19}\,\mathrm{J\,eV^{-1}}} = 2.0325\,\mathrm{eV}$$

Using the Cauchy dispersion relation for diamond with coefficients from Table 1.2,

$$\begin{aligned} n &= n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4 \\ &= (-1.07 \times 10^{-5})(2.0325)^{-2} + 2.378 + (8.01 \times 10^{-3})(2.0325)^2 \\ &\quad + (1.04 \times 10^{-4})(2.0325)^4 \\ &= 2.4140 \end{aligned}$$

which is slightly different than the value calculated in Example 1.2.1; one reason for the discrepancy is due to the Cauchy coefficients quoted in Table 1.2 being applicable over a wider wavelength range at the expense of some accuracy. Although both dispersion relations have four parameters, A_1, A_2, λ_1, λ_2 for Sellmeier and n_{-2}, n_0, n_2, n_4 for Cauchy, the functional forms are different.

Group Velocity and Group Index

There are no perfect monochromatic waves

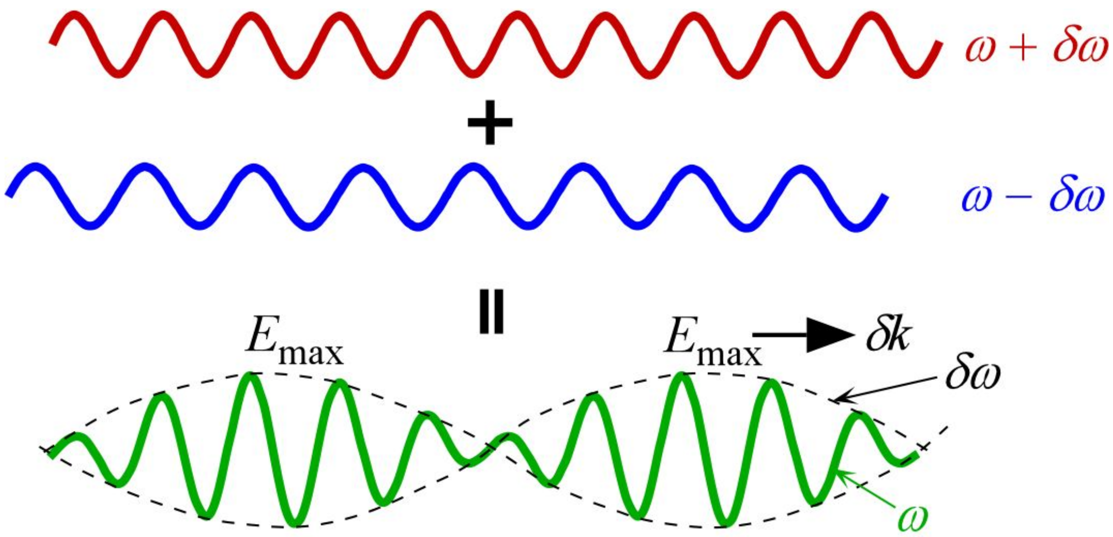
We have to consider the way in which a group of waves differing slightly in wavelength travel along the *z*-direction

Group Velocity and Group Index

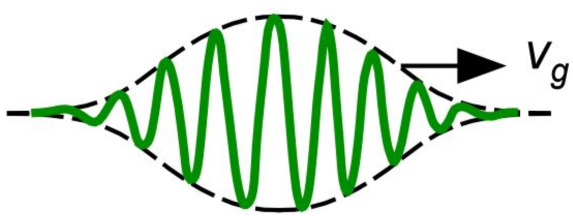
When two perfectly harmonic waves of frequencies $\omega - \delta\omega$ and $\omega + \delta\omega$ and wavevectors $k - \delta k$ and $k + \delta k$ interfere, they generate a wave packet which contains an oscillating field at the mean frequency ω that is amplitude modulated by a slowly varying field of frequency $\delta\omega$. The maximum amplitude moves with a wavevector δk and thus with a group velocity that is given by

$$v_g = \frac{d\omega}{dk}$$

Group Velocity

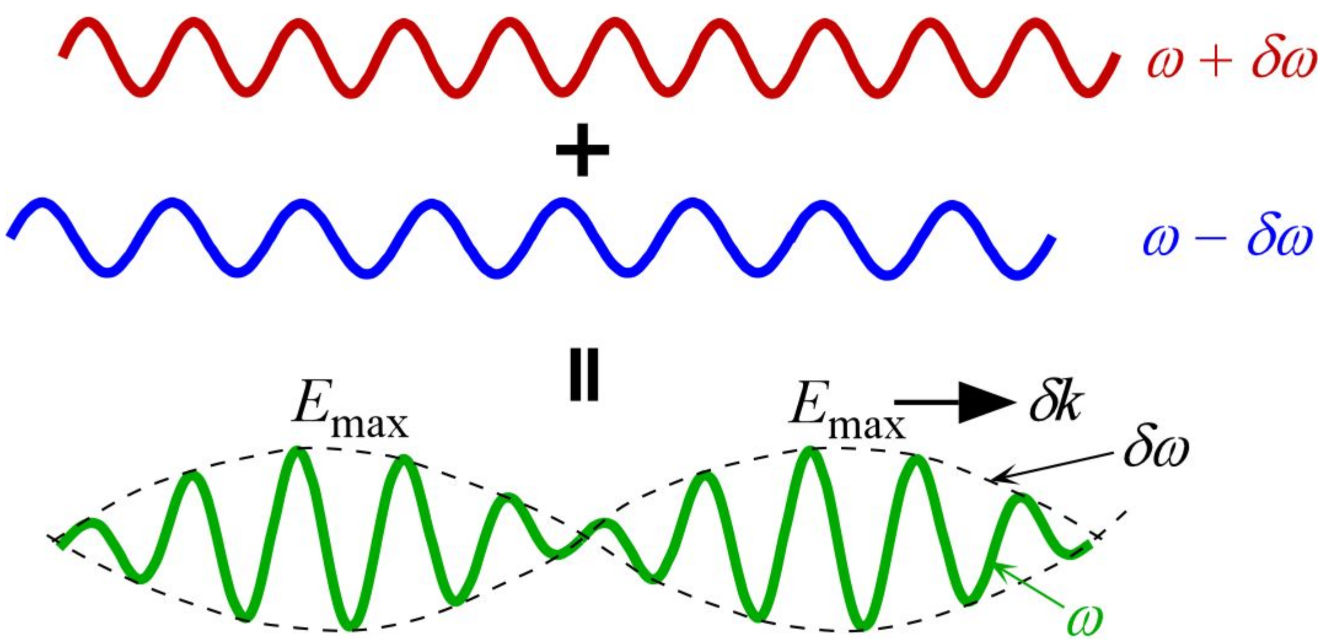


A periodic wave packet

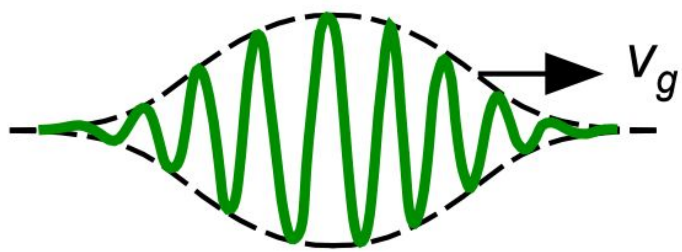


A single wave packet

Two slightly different wavelength waves traveling in the same direction result in a wave packet that has an amplitude variation that travels at the group velocity.



A periodic wave packet



$$v_g = \frac{d\omega}{dk}$$

A single wave packet

Group Velocity

Consider two sinusoidal waves that are close in frequency, that is, they have frequencies $\omega - \delta\omega$ and $\omega + \delta\omega$. Their wavevectors will be $k - \delta k$ and $k + \delta k$. The resultant wave is

$$E_x(z,t) = E_o\cos[(\omega - \delta\omega)t - (k - \delta k)z] + E_o\cos[(\omega + \delta\omega)t - (k + \delta k)z]$$

By using the trigonometric identity

$$\cos A + \cos B = 2\cos[{}^1/_2(A - B)]\cos[{}^1/_2(A + B)]$$

we arrive at

$$E_x(z,t) = 2E_o\cos[(\delta\omega)t - (\delta k)z][\cos(\omega t - kz)]$$

$$E_x(z,t) = 2E_o\cos[(\delta\omega)t - (\delta k)z][\cos(\omega t - kz)]$$

This represents a sinusoidal wave of frequency ω . This is amplitude modulated by a very slowly varying sinusoidal of frequency $\delta\omega$. This system of waves, *i.e.* the modulation, travels along z at a speed determined by the modulating term, $\cos[(\delta\omega)t - (\delta k)z]$. The maximum in the field occurs when $[(\delta\omega)t - (\delta k)z] = 2m\pi$ = constant (m is an integer), which travels with a velocity

$$\frac{dz}{dt} = \frac{\delta\omega}{\delta k} \quad \text{or} \quad v_g = \frac{d\omega}{dk}$$

This is the **group velocity** of the waves because it determines the speed of propagation of the maximum electric field along *z*.

The **group velocity** therefore defines the speed with which energy or information is propagated.

$$v_g = \frac{d\omega}{dk}$$

$\omega = 2\pi c/\lambda_o$ and $k = 2\pi n/\lambda_o$, λ_o is the free space wavelength.

Differentiate the above equations in red

$$d\omega = -(2\pi c/\lambda_o^{\,2})d\lambda_o$$

$$dk = 2\pi n(-1/\lambda_o^2)d\lambda_o + (2\pi/\lambda_o)\left(\frac{dn}{d\lambda_o}\right)d\lambda_o$$

$$dk = -(2\pi/\lambda_o^2)\left(n - \lambda_o\frac{dn}{d\lambda_o}\right)d\lambda_o$$

$$\therefore \qquad v_g = \frac{d\omega}{dk} = \frac{-(2\pi c/\lambda_o^2)d\lambda_o}{-(2\pi/\lambda_o^2)\left(n - \lambda_o\frac{dn}{d\lambda_o}\right)d\lambda_o} = \frac{c}{n - \lambda_o\frac{dn}{d\lambda_o}}$$

Group Velocity and Group Index

where $n = n(\lambda)$ is a function of the wavelength. The group velocity v_g in a medium is given by,

$$v_g(\text{medium}) = \frac{d\omega}{dk} = \frac{c}{n - \lambda \dfrac{dn}{d\lambda}}$$

This can be written as

$$v_g(\text{medium}) = \frac{c}{N_g}$$

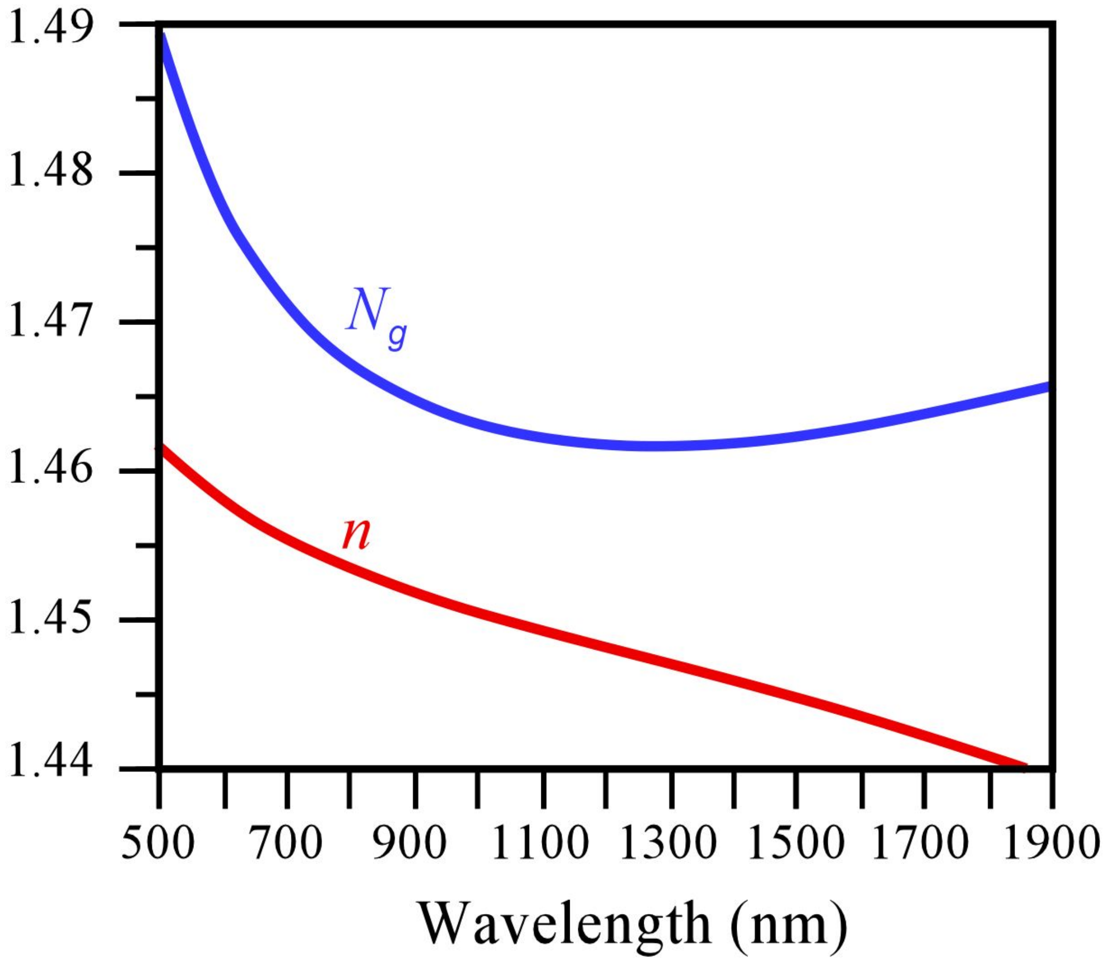
Group Index

$$N_g = n - \lambda \frac{dn}{d\lambda}$$

is defined as the **group index of the medium**

In general, for many materials the refractive index n and hence the group index N_g depend on the wavelength of light. Such materials are called **dispersive**

Refractive Index and Group Index



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

Magnetic Field, Irradiance and Poynting Vector

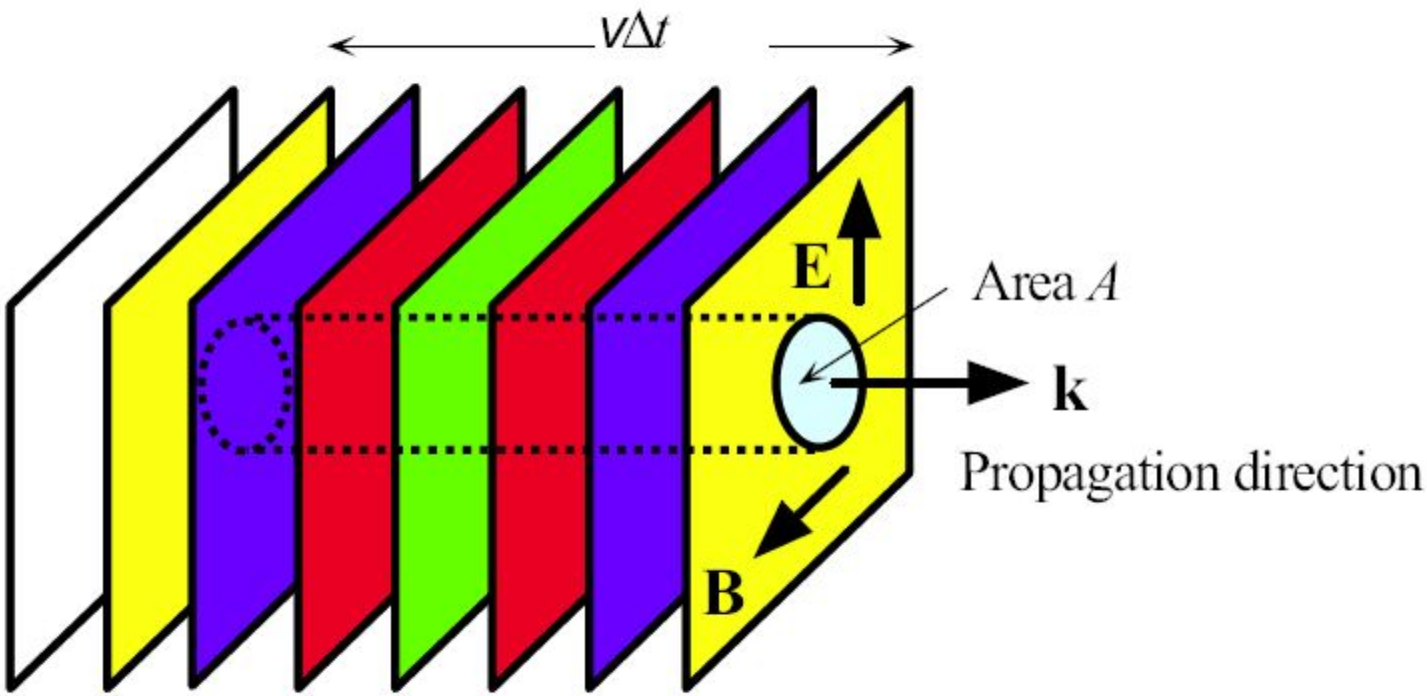
The magnetic field (magnetic induction) component B_y always accompanies E_x in an EM wave propagation.

If v is the **phase velocity** of an EM wave in an isotropic dielectric medium and n is the refractive index, then

$$E_x = vB_y = \frac{c}{n}B_y$$

where $v = (\varepsilon_o\varepsilon_r\mu_o)^{-1/2}$ and $n = \varepsilon^{1/2}$

EM wave carries energy along the direction of propagation **k**. What is the radiation power flow per unit area?



A plane EM wave traveling along **k** crosses an area A at right angles to the direction of propagation. In time Δt, the energy in the cylindrical volume $Av\Delta t$ (shown dashed) flows through A.

Energy Density in an EM Wave

As the EM wave propagates in the direction of the wavevector **k**, there is an energy flow in this direction. The wave brings with it electromagnetic energy.

The energy densities in the E_x and B_y fields are the same,

$$\frac{1}{2}\varepsilon_o\varepsilon_r E_x^2 = \frac{1}{2\mu_o}B_y^2$$

The total energy density in the wave is therefore $\varepsilon_o\varepsilon_r E_x{}^2$.

Poynting Vector and EM Power Flow

If S is the EM power flow per unit area,

S = Energy flow per unit time per unit area

$$S = \frac{(A v \Delta t)(\varepsilon_o \varepsilon_r E_x^2)}{A \Delta t} = v \varepsilon_o \varepsilon_r E_x^2 = v^2 \varepsilon_o \varepsilon_r E_x B_y$$

In an isotropic medium, the energy flow is in the direction of wave propagation. If we use the vectors **E** and **B** to represent the electric and magnetic fields in the EM wave, then the EM power flow per unit area can be written as

$$\mathbf{S} = v^2 \varepsilon_o \varepsilon_r \mathbf{E} \times \mathbf{B}$$

Poynting Vector and Intensity

where S, called the Poynting vector, represents the energy flow per unit time per unit area in a direction determined by E×B (direction of propagation). Its magnitude, power flow per unit area, is called the irradiance (instantaneous irradiance, or intensity).

The average irradiance is

$$I = S_{\text{average}} = \frac{1}{2} v \varepsilon_o \varepsilon_r E_o^2$$

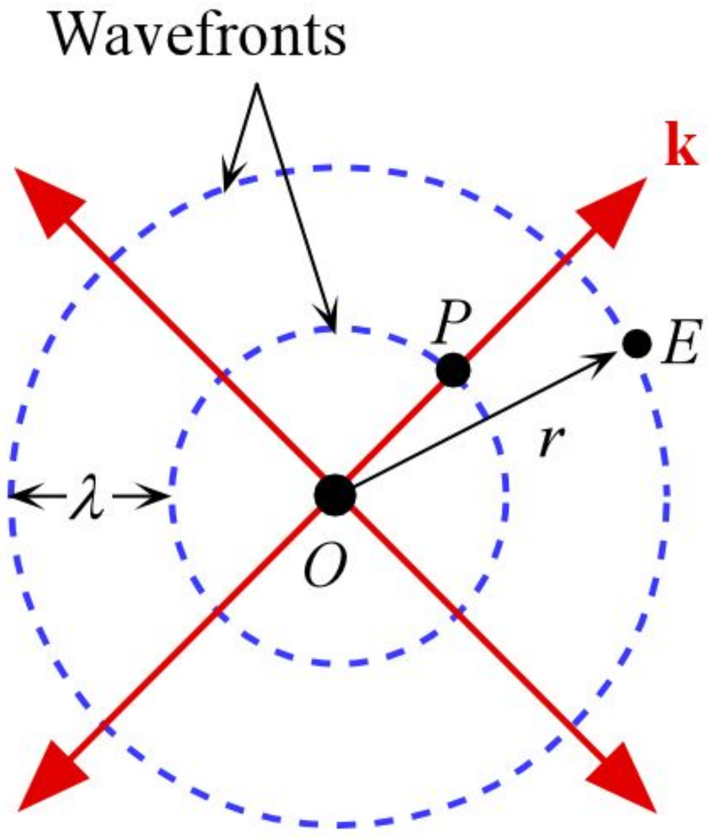
Average Irradiance or Intensity

Since $\boldsymbol{v} = c/n$ and $\varepsilon_r = n^2$ we can write

$$I = S_{\text{average}} = \tfrac{1}{2} c\varepsilon_o n E_o^2 = (1.33 \times 10^{-3}) n E_o^2$$

The instantaneous irradiance can only be measured if the power meter can respond more quickly than the oscillations of the electric field. Since this is in the optical frequencies range, **all practical measurements yield the average irradiance** because all detectors have a response rate much slower than the frequency of the wave.

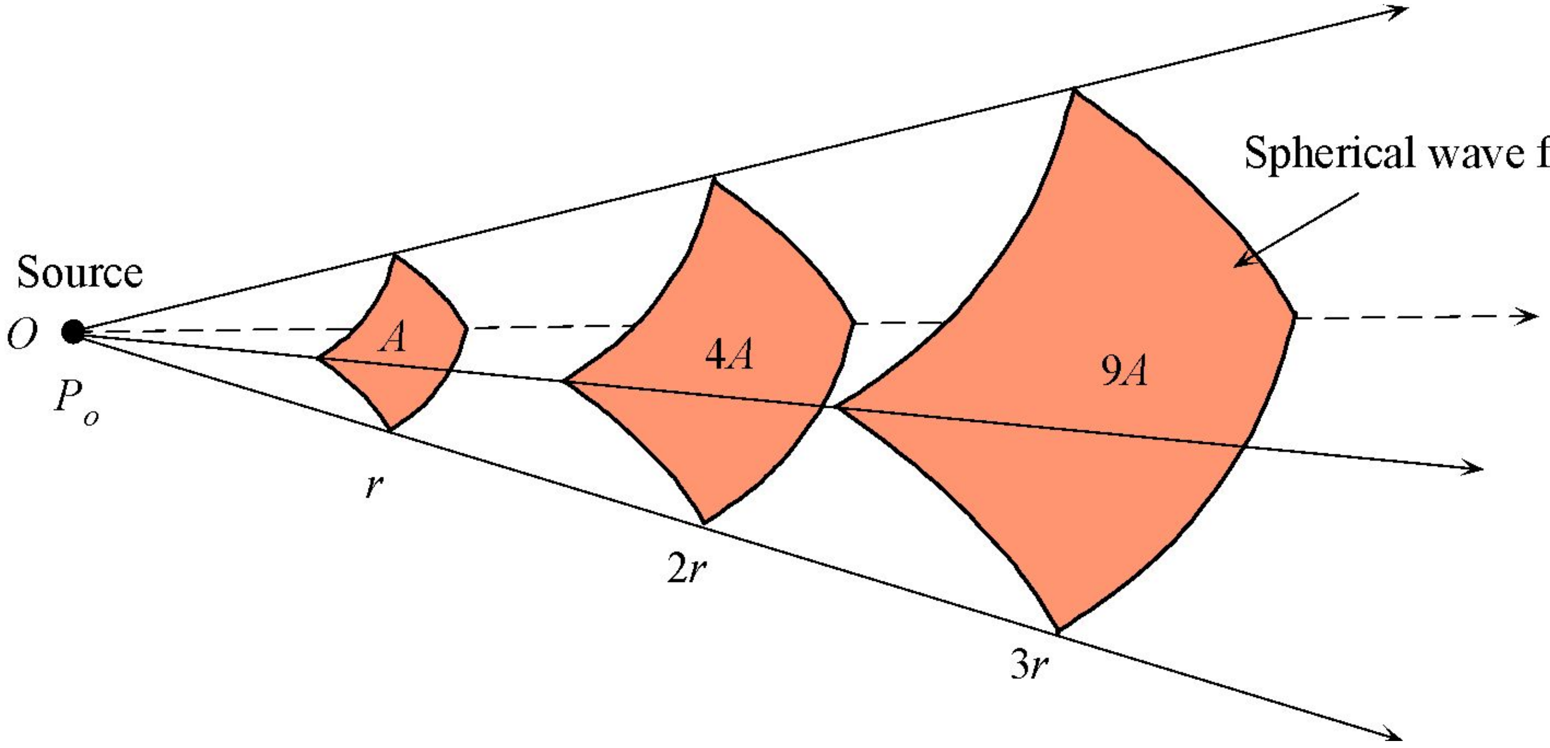
Irradiance of a Spherical Wave



Perfect spherical wave

$$I = \frac{P_o}{4\pi r^2}$$

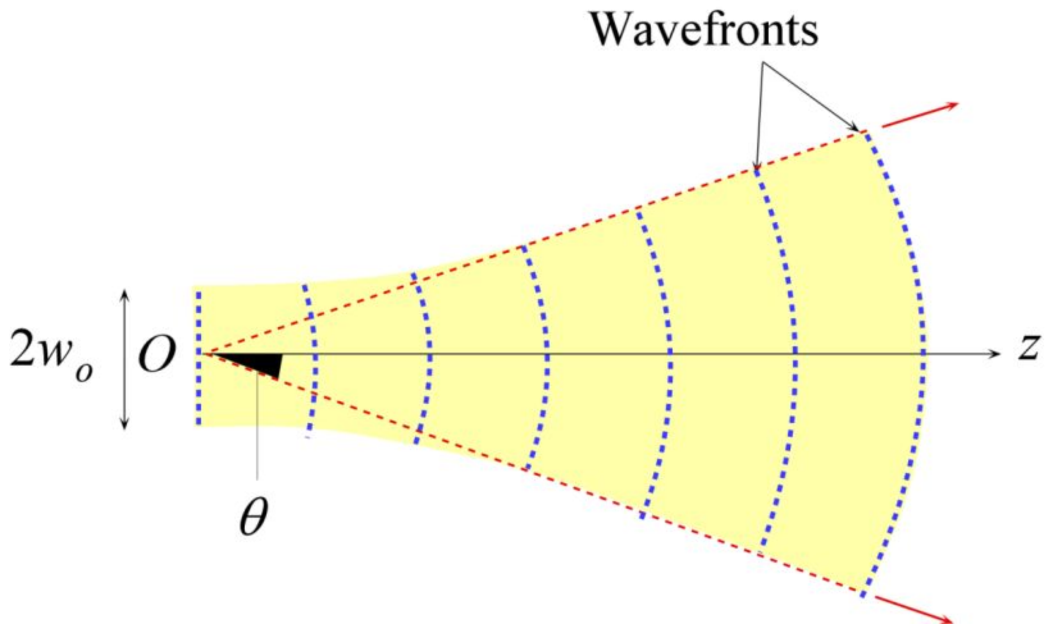
Irradiance of a Spherical Wave

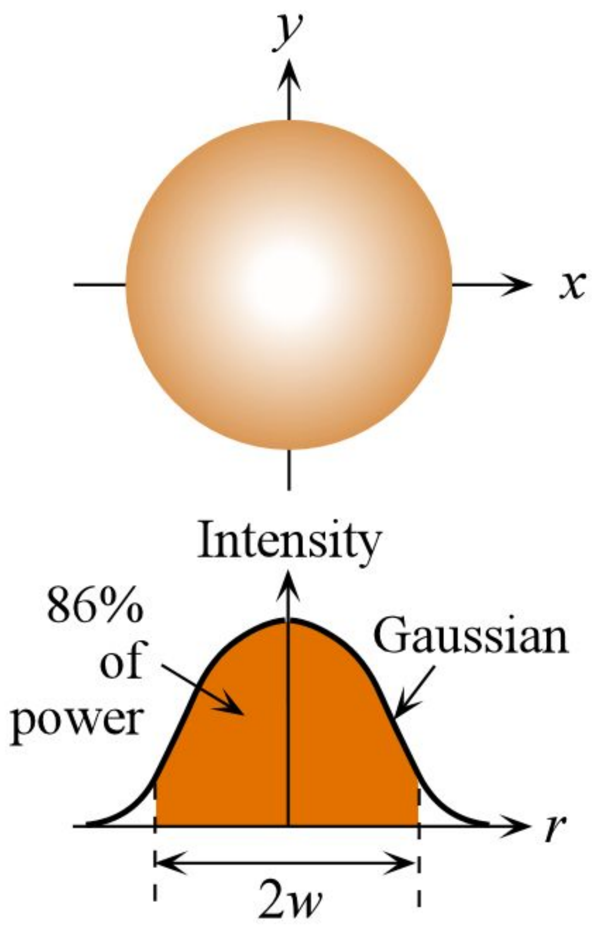


$$I = \frac{P_o}{4\pi r^2}$$

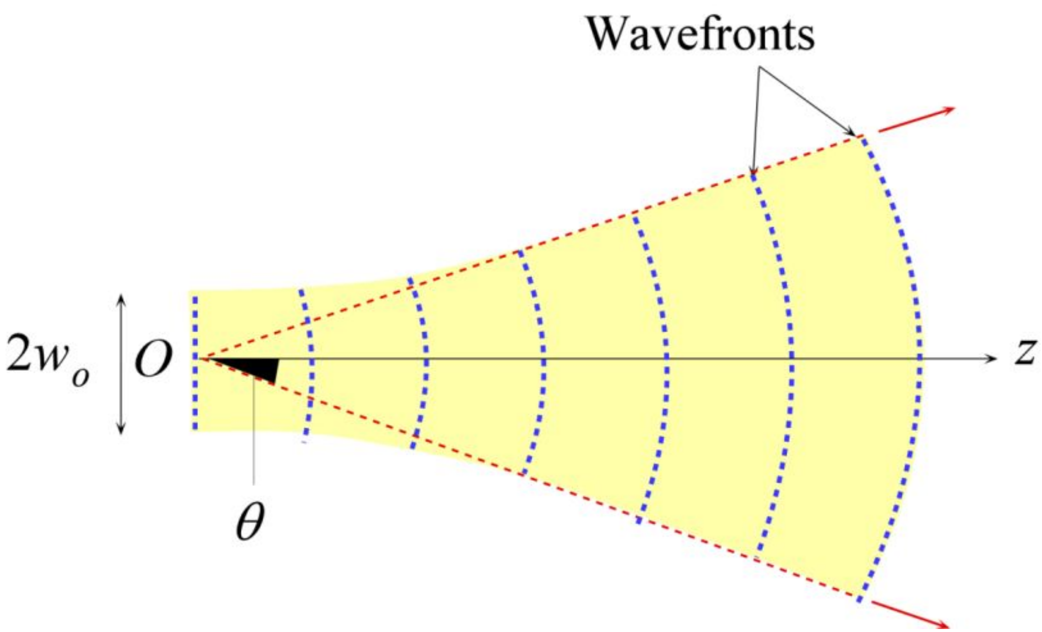
A Gaussian Beam

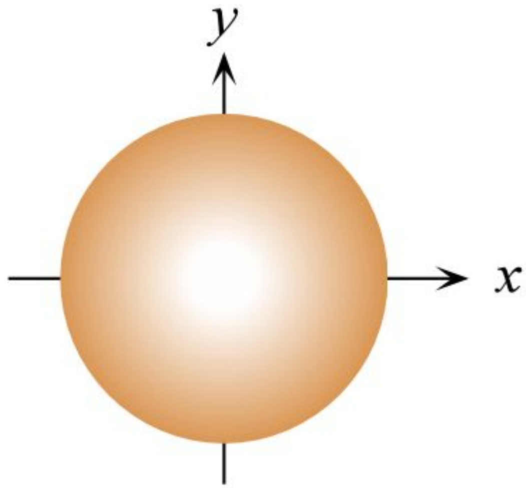
$$I(r, z) = I_{max}\exp(-2r^2/w^2)$$

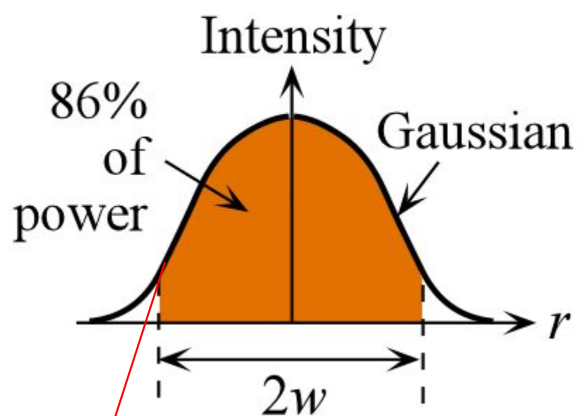




A Gaussian Beam







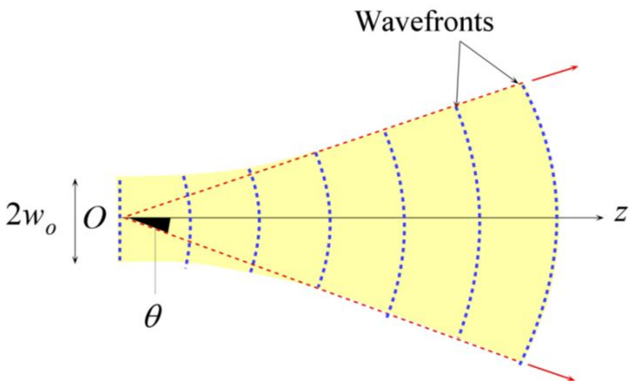
I_{max}

$$I(r,z) = [2P_o/(\pi w^2)]\exp(-2r^2/w^2)$$

$\theta = w/z = \lambda/(\pi w_o)$ 2θ = Far field divergence

A Gaussian Beam

$$I(r,z) = I_{max}\exp(-2r^2/w^2)$$



$$I_{max} = \frac{P_o}{\frac{1}{2}\pi w^2}$$

I_o = Maximum irradiance at the center $r = 0$ at the waist

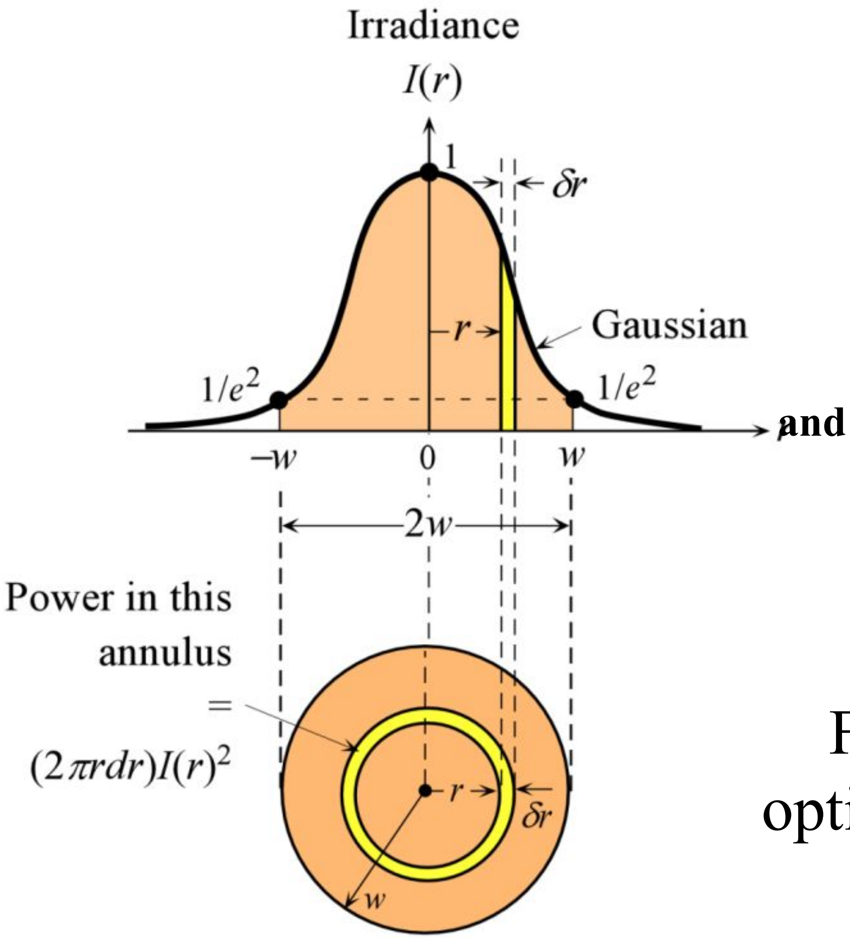
$$I(z,r) = I_o\left(\frac{w_o^2}{w^2}\right)\exp\left(-\frac{2r^2}{w^2}\right)$$

Beyond the Rayleigh range

$z > z_o$

$$2w = 2w_o\left(\frac{z}{z_o}\right) \Rightarrow I(z,0) = I_{max} = I_o\frac{w_o^2}{w^2} = I_o\frac{z_o^2}{z^2}$$

Power in a Gaussian Beam



$$I(r)^2 = I(0)^2 \exp[-2(r/w)^2]$$

Area of a circular thin strip (annulus) with radius r is $2\pi r dr$. Power passing through this strip is proportional to $I(r)\,(2\pi r)dr$

$$\text{Fraction of optical power within } 2w = \frac{\int_0^w I(r)2\pi r dr}{\int_0^\infty I(r)2\pi r dr} = 0.865$$

Gaussian Beam

I_o = Maximum irradiance at the center $r = 0$ at the waist

Example on

$$I_o = \frac{P_o}{\frac{1}{2}\pi w_o^2}$$

Corrected

Example 1.4.2 Power and irradiance of a Gaussian beam

Consider a 5 mW HeNe laser that is operating at 633 nm, and has a spot size that is 1 mm. Find the maximum irradiance of the beam and the axial (maximum) irradiance at 25 m from the laser.

Solution

The 5 mW rating refers to the total optical power P_o available, and 633 nm is the free space output wavelength λ. Apply

$$P_o = I_o(\tfrac{1}{2}\pi w_o^2)$$

$$\therefore \quad 5\times10^{-3}\ \text{W} = I_o[\tfrac{1}{2}\pi(0.5\times10^{-3}\ \text{m})^2]$$

$$I_o = 1.273\ \text{W cm}^{-2}$$

Gaussian Beam

Example on

$$I(z,0) = I_{max} = I_o \frac{w_o^2}{w^2} = I_o \frac{z_o^2}{z^2}$$

The Rayleigh range z_o was calculated previously, but we can recalculate

$$z_o = \pi w_o^2/\lambda = \pi(0.5\times10^{-3}\ \text{m})^2/(633\times10^{-9}\ \text{m}) = 1.24\ \text{m}.$$

The beam width at 25 m is

$$2w = 2w_o[1 + (z/z_o)]^{1/2} = 20\ \text{mm}$$

The irradiance at the beam axis is

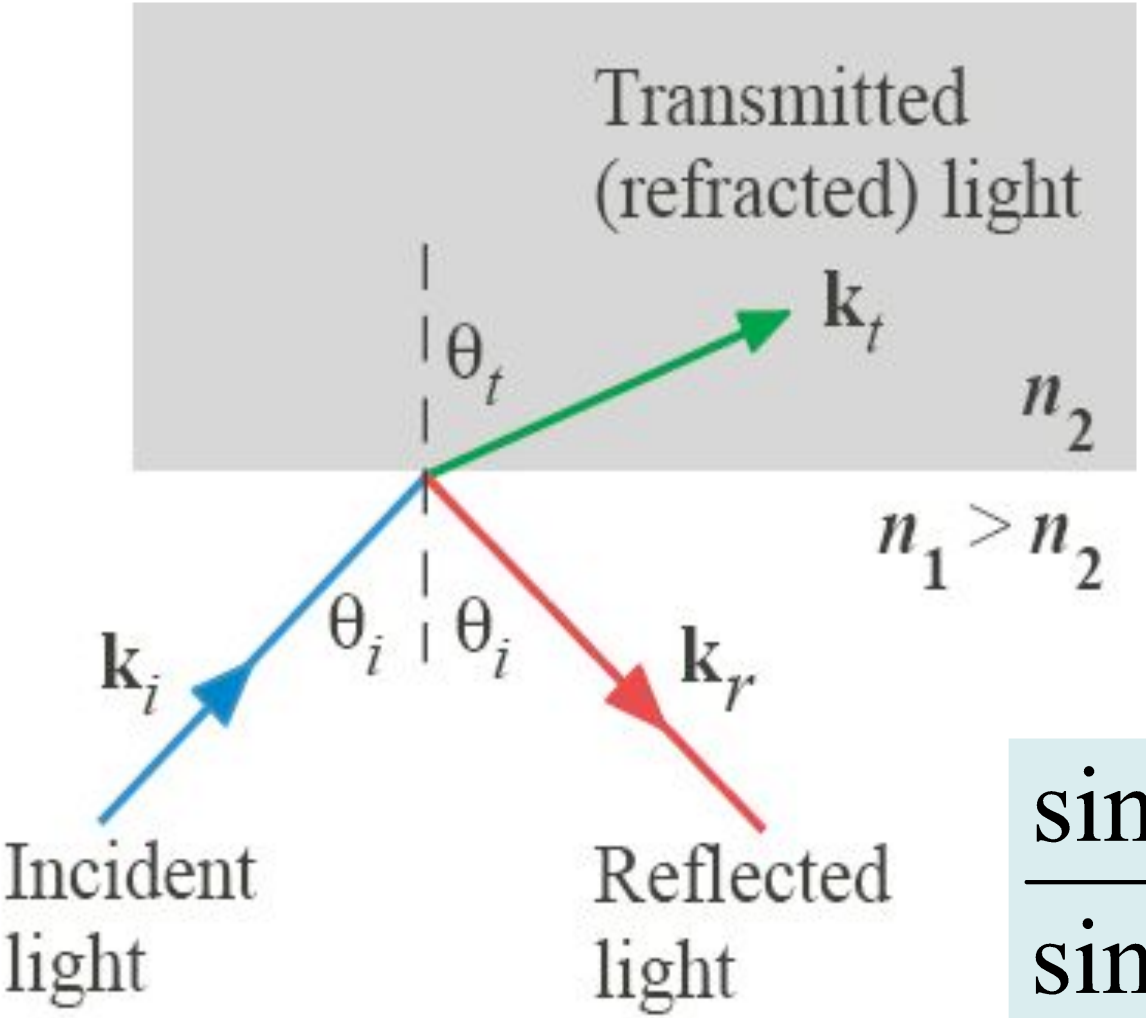
$$I_{axis} = I_o \frac{z_o^2}{z^2} = (1.273\,\text{W}\,\text{cm}^{-2})\frac{(1.24\,\text{m})^2}{(25\,\text{m})^2} = 3.14\,\text{mW}\,\text{cm}^{-2}$$

Snell's Law or Descartes's Law?

Willebrord Snellius (Willebrord Snel van Royen, 1580–1626) was a Dutch astronomer and a mathematician, who was a professor at the University of Leiden. He discovered his law of refraction in 1621 which was published by Réne Descartes in France 1637; it is not known whether Descartes knew of Snell's law or formulated it independently. *(Courtesy of AIP Emilio Segre Visual Archives, Brittle Books Collection.)*

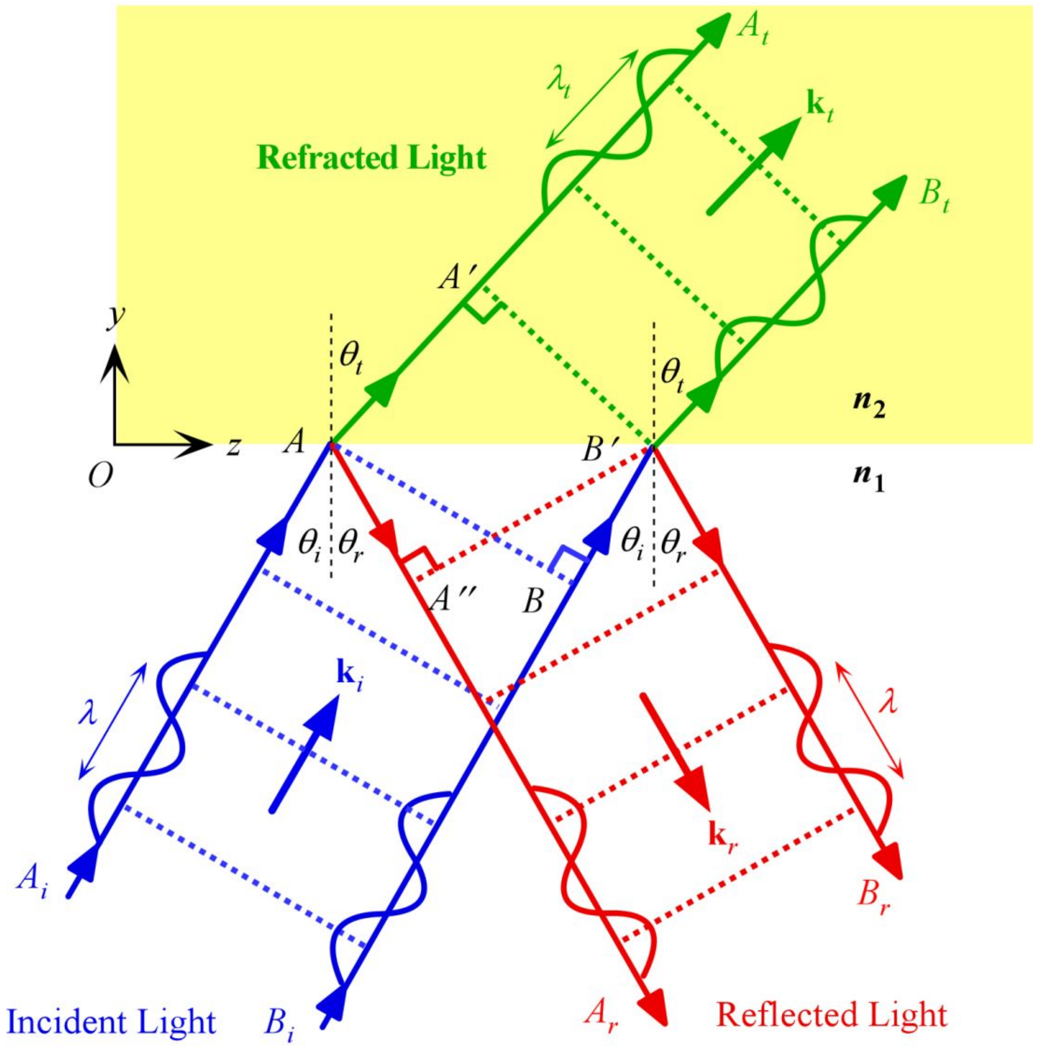
René Descartes (1596–1650) was a French philosopher who was also involved with mathematics and sciences. He has been called the "Father of Modern Philosophy." Descartes was responsible for the development of Cartesian coordinates and analytical geometry. He also made significant contributions to optics, including reflection and refraction. *(Courtesy of Georgios Kollidas/Shutterstock.com.)*

Snell's Law



$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$

Derivation of Snell's Law



A light wave traveling in a medium with a greater refractive index ($n_1 > n_2$) suffers reflection and refraction at the boundary. (Notice that λ_t is slightly longer than λ.)

Snell's Law

We can use *constructive interference* to show that there can only be one reflected wave which occurs at an angle equal to the incidence angle. The two waves along A_i and B_i are in phase.

When these waves are reflected to become waves A_r and B_r then they must still be in phase, otherwise they will interfere destructively and destroy each other. The only way the two waves can stay in phase is if $\theta_r = \theta_i$. All other angles lead to the waves A_r and B_r being out of phase and interfering destructively.

Snell's Law

Unless the two waves at A' and B' still have the same phase, there will be no transmitted wave. A' and B' points on the front are only in phase for one particular transmitted angle, θ_t.

It takes time t for the phase at B on wave B_i to reach B'

$BB' = v_1 t = ct/n_1$

During this time t, the phase A has progressed to A'

$AA' = v_2 t = ct/n_2$

A' and B' belong to the same front just like A and B so that AB is perpendicular to k_i in medium 1 and $A'B'$ is perpendicular to k_t in medium 2. From geometrical considerations,

$AB' = BB'/\sin\theta_i$ **and** $AB' = AA'/\sin\theta_t$ **so that** or $$AB' = \frac{v_1 t}{\sin\theta_i} = \frac{v_2 t}{\sin\theta_t}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$n \sin\theta = \text{constant}$$

This is **Snell's law** which relates the angles of incidence and refraction to the refractive indices of the media.

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

When $n_1 > n_2$ then obviously the transmitted angle is greater than the incidence angle as apparent in the figure. When the refraction angle θ_t reaches 90°, the incidence angle is called the **critical angle** θ_c which is given by

$$\sin\theta_c = \frac{n_2}{n_1}$$

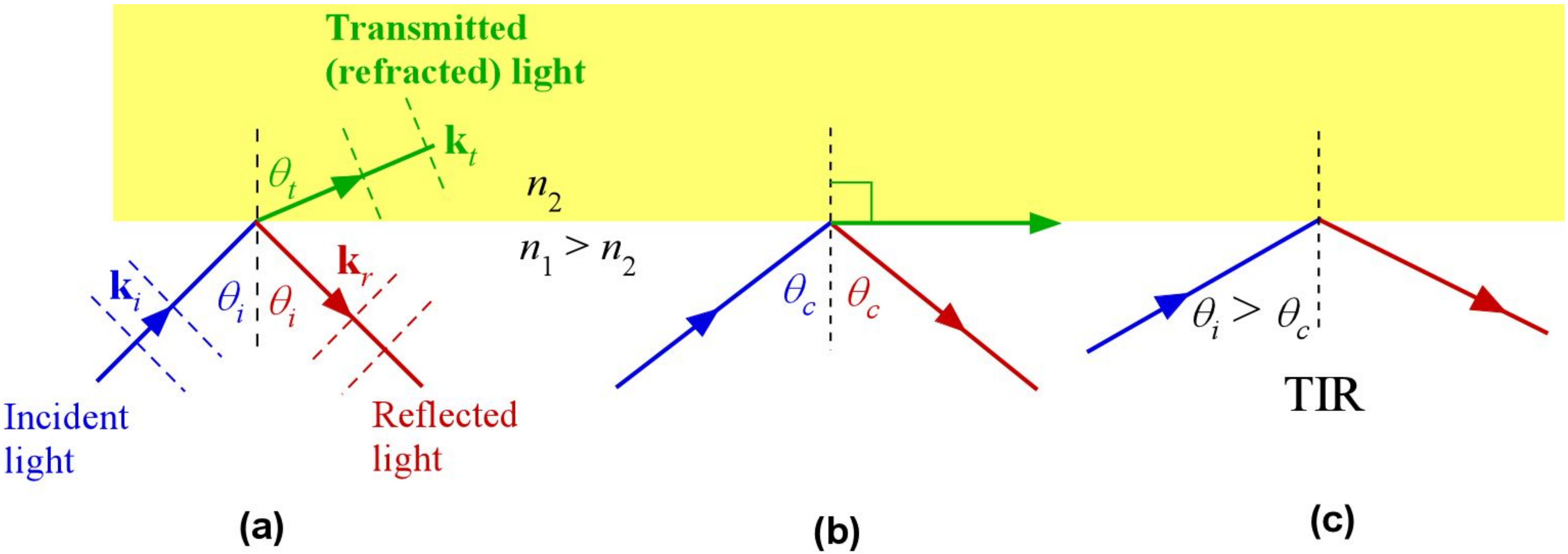
Snell's Law

$$\sin\theta_c = \frac{n_2}{n_1}$$

When the incidence angle θ_i exceeds θ_c then there is no transmitted wave but only a reflected wave. The latter phenomenon is called total internal reflection (TIR). TIR phenomenon that leads to the propagation of waves in a dielectric medium surrounded by a medium of smaller refractive index as in optical waveguides, *e.g.* optical fibers.

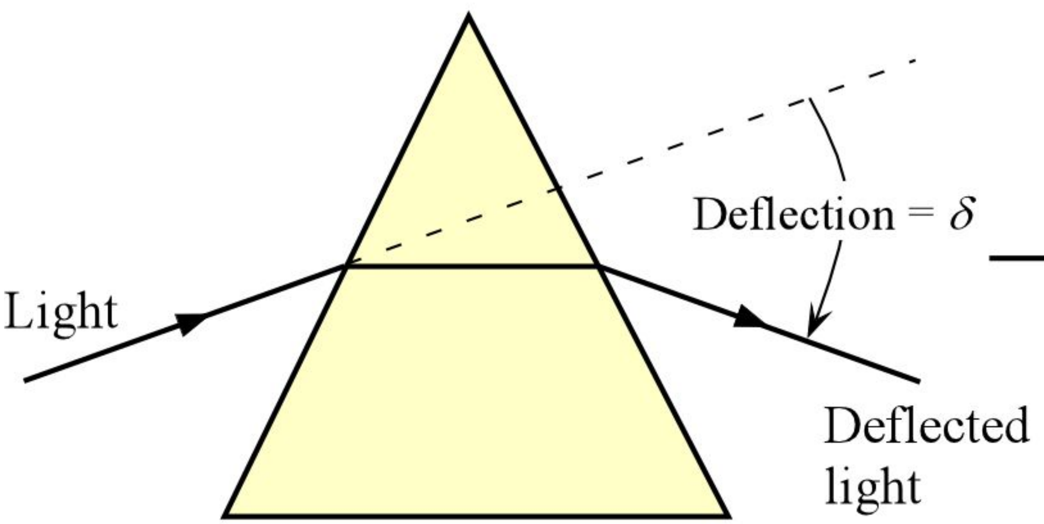
Although Snell's law for $\theta_i > \theta_c$ shows that $\sin\theta_t > 1$ and hence θ_t is an "imaginary" angle of refraction, there is however an attenuated wave called the evanescent wave.

Total Internal Reflection

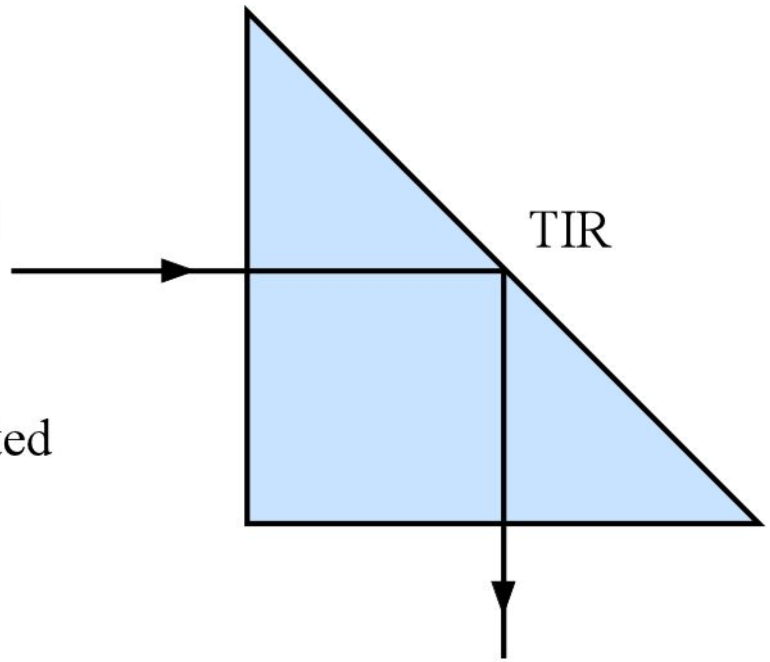


Light wave traveling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to θ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and total internal reflection (TIR).

Prisms

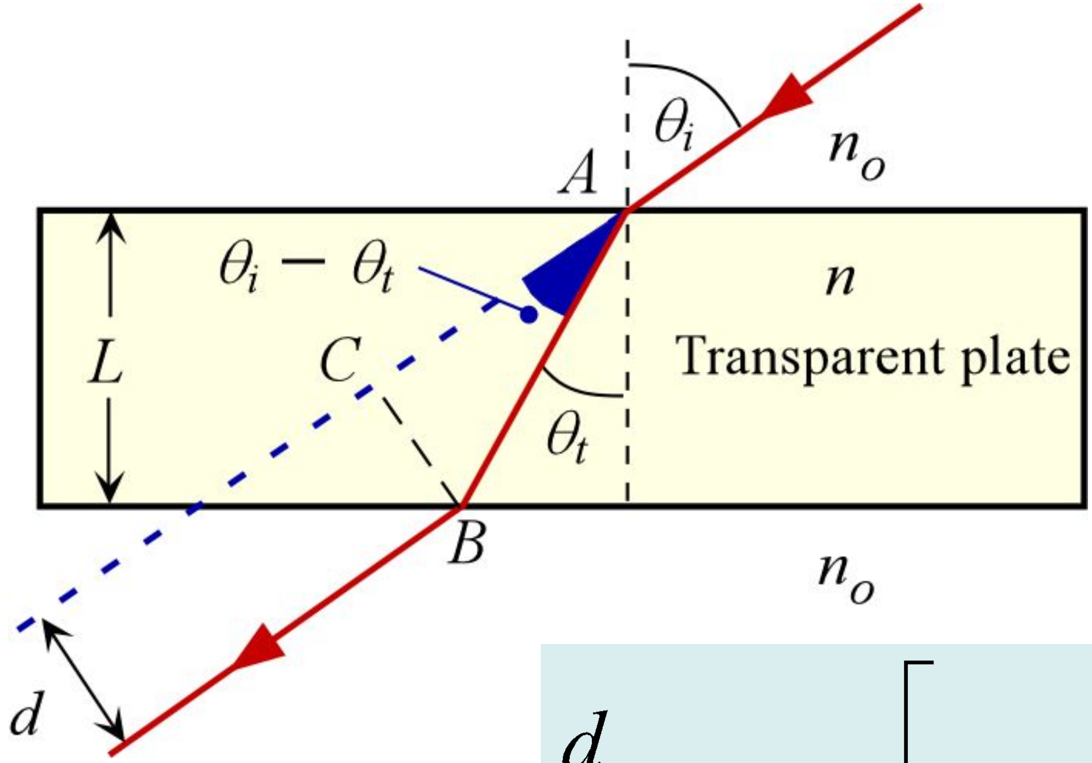


Refracting prism



Reflecting prism

Lateral Displacement



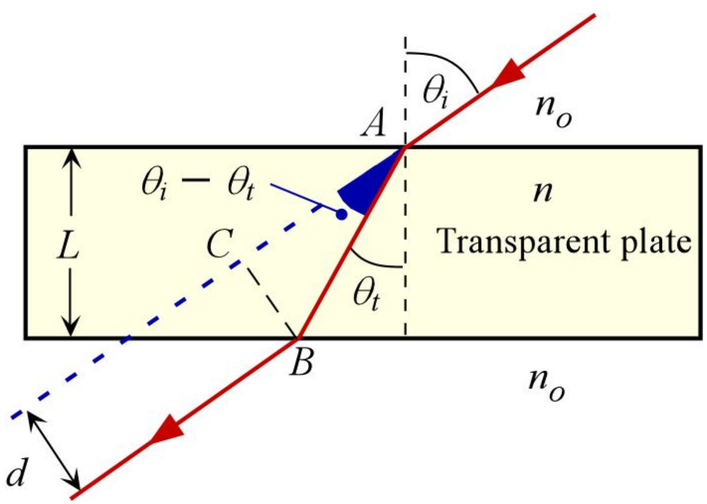
$$\frac{d}{L} = \sin\theta_i \left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}} \right]$$

Example: Lateral Displacement

Lateral displacement of light, or, **beam displacement,** occurs when a beam if light passes obliquely through a plate of transparent material, such as a glass plate. When a light beam is incident on a plate of transparent material of refractive index *n*, it emerges from the other side traveling parallel to the incident light but displaced from it by a distance *d*, called *lateral displacement.* Find the displacement *d* in terms of the incidence angle the plate thickness *L*. What is *d* for a glass of *n* = 1.600, *L* = 10 mm if the incidence angle is 45°

Solution

The displacement $d = BC = AB\sin(\theta_i - \theta_t)$. Further, $L/AB = \cos\theta_t$ so that combining these two equation we find

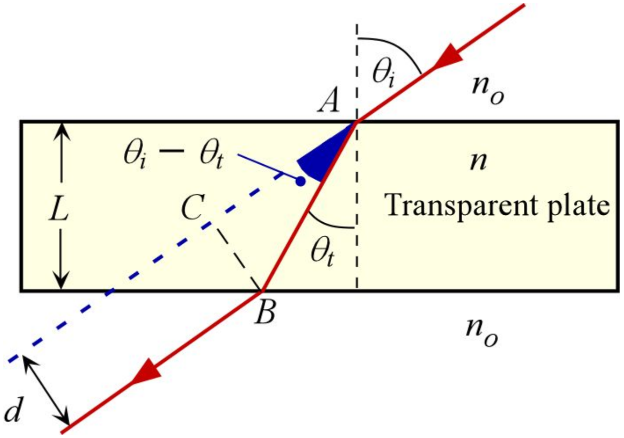


$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

Example: Lateral Displacement (Continued)

Solution (Continued)

Expand $\sin(\theta_i - \theta_t)$ and eliminate $\sin\theta_t$ and $\sin\theta_t$



$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

$$\sin(\theta_i - \theta_t) = \sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

Snell's law $n\sin\theta_t = n_o\sin\theta_i$

$$\frac{d}{L} = \sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}}\right]$$

Example: Lateral Displacement (Continued)

Solution (Continued)

$$\frac{d}{L} = \sin\theta_i \left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}} \right]$$

$L = 10 \text{ mm}$

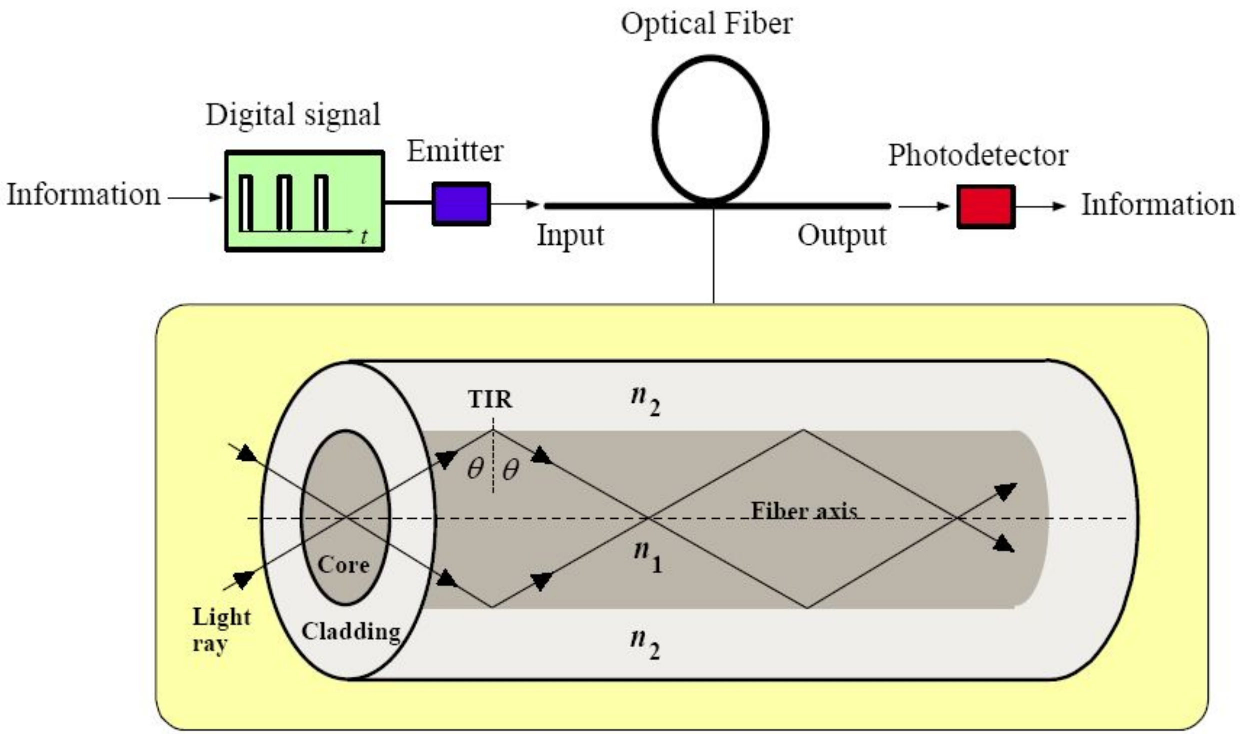
$\theta_i = 45°$

$n = 1.600$

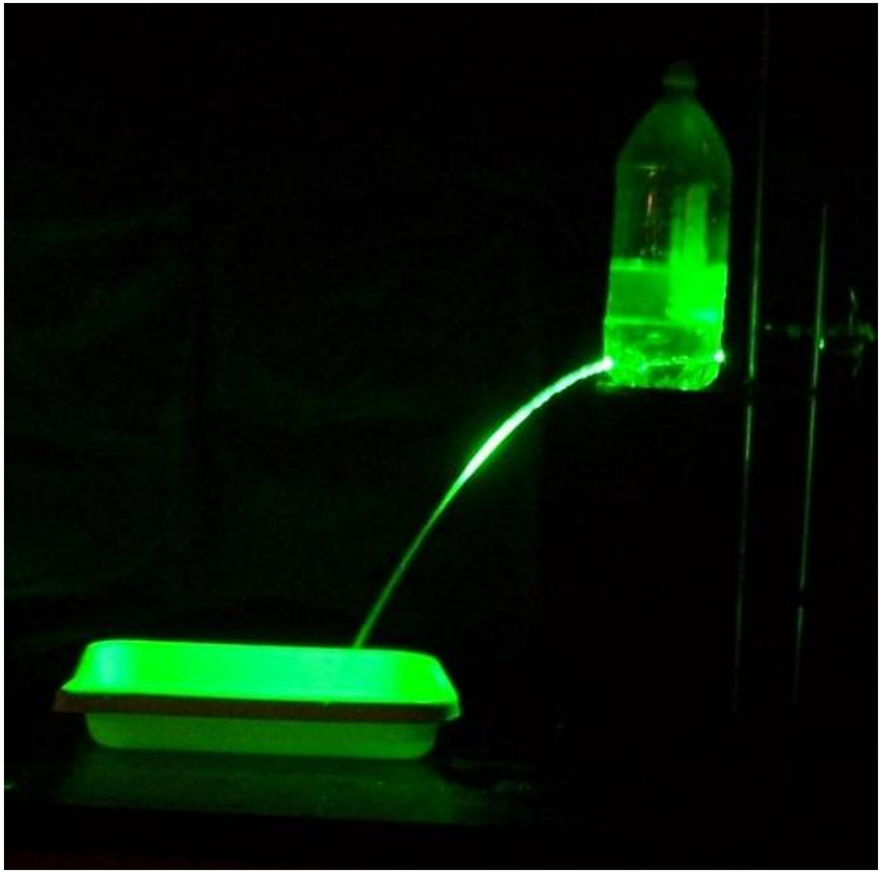
$n_o = 1$

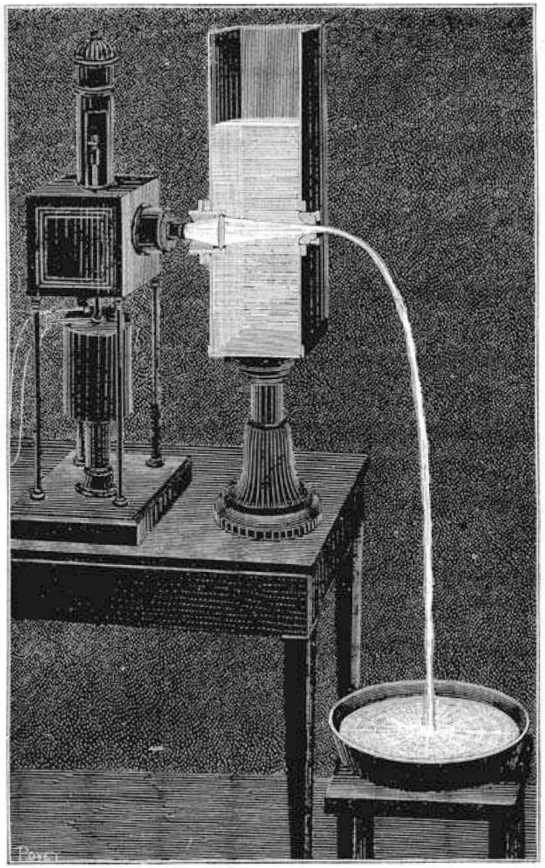
$$\boldsymbol{d = 3.587 \text{ mm}}$$

Light travels by total internal reflection in optical fibers



An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.





A small hole is made in a plastic bottle full of water to generate a water jet. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. The light guiding by a water jet was first demonstrated by Jean-Daniel Colladan, a Swiss scientist (Water with air bubbles was used to increase the visibility of light. Air bubbles scatter light.) [Left: Copyright: S.O. Kasap, 2005] [Right: *Comptes Rendes*, *15*, 800–802, October 24, 1842; *Cnum, Conservatoire Numérique des Arts et Métiers, France*

Physicists use the wave theory on Mondays, Wednesdays and Fridays and the particle theory on Tuesdays, Thursdays and Saturdays.

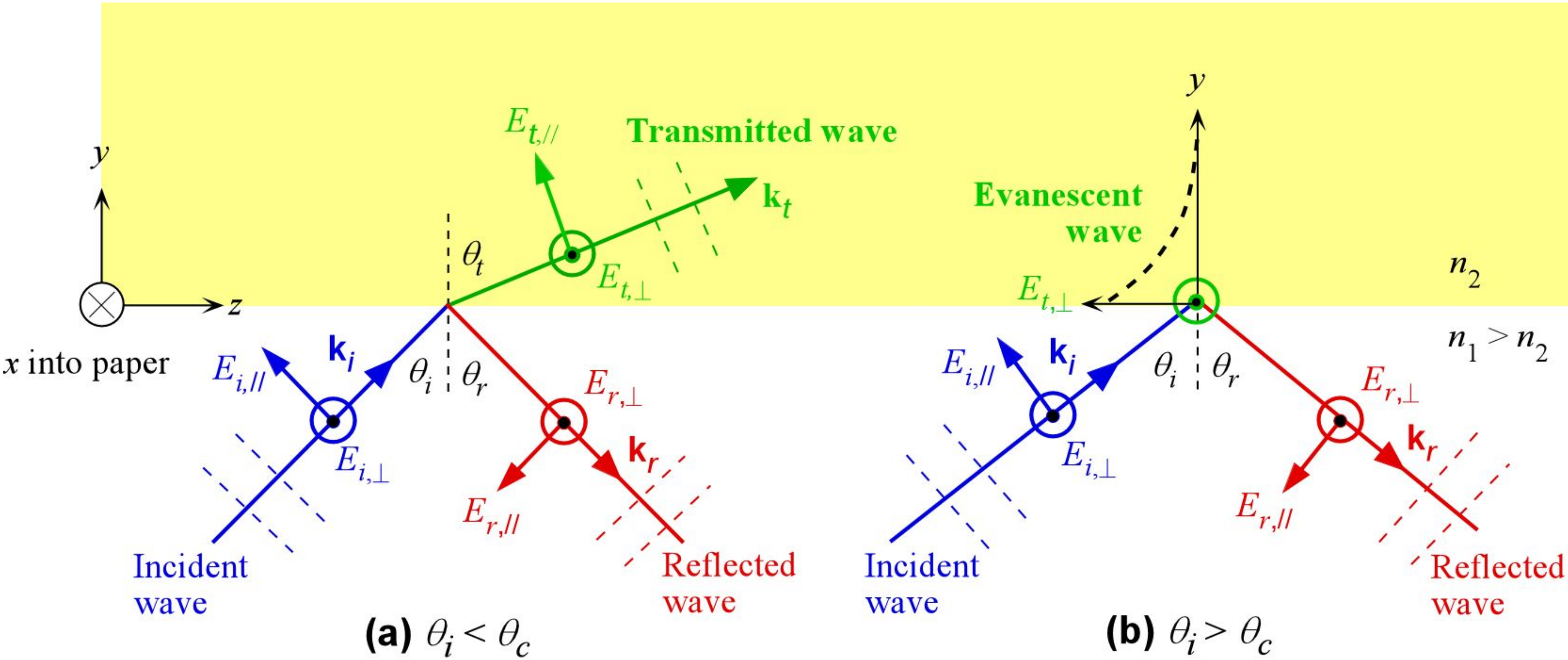
—Sir William Henry Bragg[1]

Augustin Jean Fresnel (1788–1827) was a French physicist and a civil engineer for the French government who was one of the principal proponents of the wave theory of light. He made a number of distinct contributions to optics including the well-known Fresnel lens that was used in lighthouses in the nineteenth century. He fell out with Napoleon in 1815 and was subsequently put under house arrest until the end of Napoleon's reign. During his enforced leisure time he formulated his wave ideas of light into a mathematical theory. *(© INTERFOTO/Alamy.)*

If you cannot saw with a file or file with a saw, then you will be no good as an experimentalist.

—Attributed to Augustin Fresnel

Fresnel's Equations



Light wave traveling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation. It can be resolved into perpendicular and parallel components.

Fresnel's Equations

Describe the incident, reflected and refracted waves by the exponential representation of a traveling plane wave, *i.e.*

$$E_i = E_{io} \exp j(\omega t - \mathbf{k}_i \cdot \mathbf{r})$$ **Incident wave**

$$E_r = E_{ro} \exp j(\omega t - \mathbf{k}_r \cdot \mathbf{r})$$ **Reflected wave**

$$E_t = E_{to} \exp j(\omega t - \mathbf{k}_t \cdot \mathbf{r})$$ **Transmitted wave**

These are **traveling plane waves**

Fresnel's Equations

where r is the position vector, the wave vectors k_i, k_r and k_t describe the directions of the incident, reflected and transmitted waves and E_{io}, E_{ro} and E_{to} are the respective amplitudes.

Any phase changes such as φ_r and φ_t in the reflected and transmitted waves with respect to the phase of the incident wave are incorporated into the complex amplitudes, E_{ro} and E_{to}. Our objective is to find E_{ro} and E_{to} with respect to E_{io}.

Fresnel's Equations

The electric and magnetic fields anywhere on the wave must be perpendicular to each other as a requirement of electromagnetic wave theory. This means that with $E_{//}$ in the EM wave we have a magnetic field $B_{\perp}$ associated with it such that, $B_{\perp} = (n/c)E_{//}$. Similarly $E_{\perp}$ will have a magnetic field $B_{//}$ associated with it such that $B_{//} = (n/c)E_{\perp}$.

We use **boundary conditions**

$$E_{\text{tangential}}(1) = E_{\text{tangential}}(2)$$

Fresnel's Equations

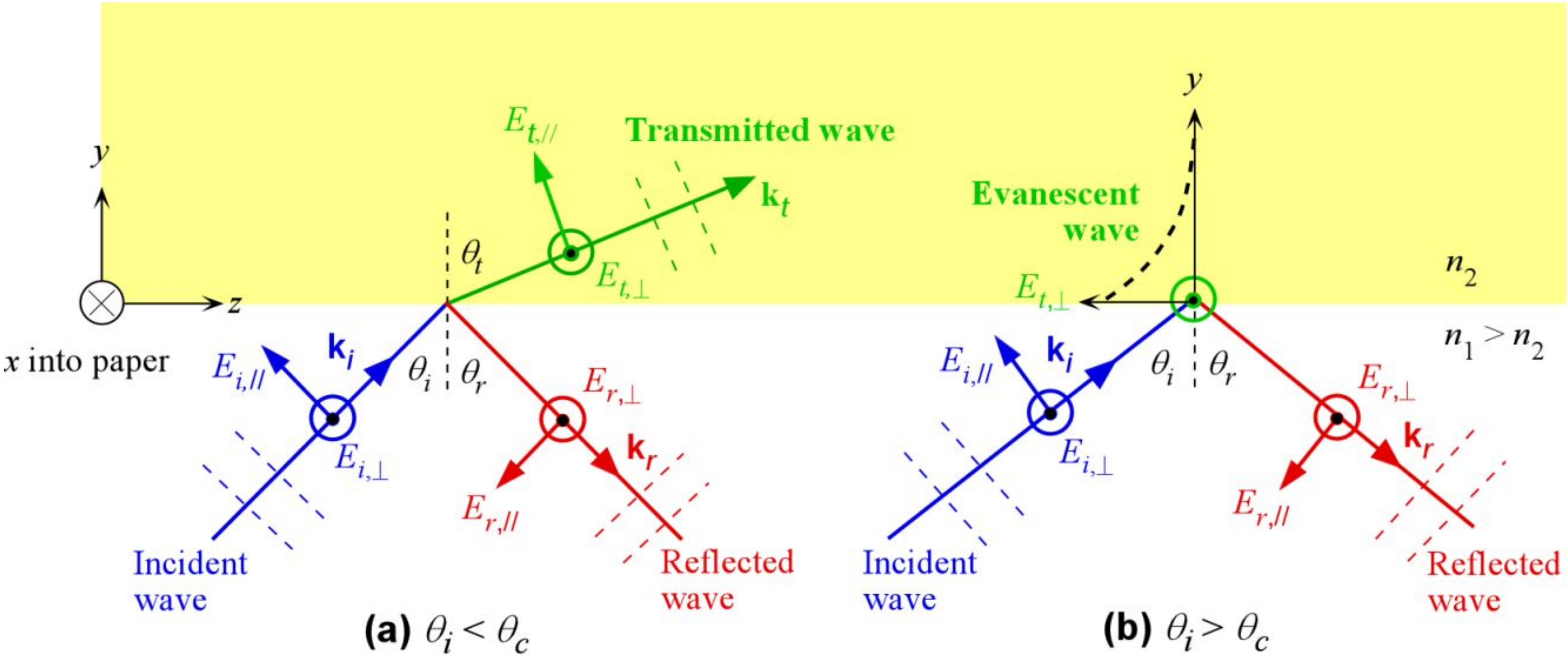
Non-magnetic media (relative permeability, $\mu_r = 1$),

$$B_{\text{tangential}}(1) = B_{\text{tangential}}(2)$$

Using the above boundary conditions for the fields at $y = 0$, and the relationship between the electric and magnetic fields, we can find the reflected and transmitted waves in terms of the incident wave.

The boundary conditions can only be satisfied if the reflection and incidence angles are equal, $\theta_r = \theta_i$ and the angles for the transmitted and incident wave obey Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Fresnel's Equations



Incident wave $\quad E_i = E_{io}\exp j(\omega t - \mathbf{k}_i \cdot \mathbf{r})$

Reflected wave $\quad E_r = E_{ro}\exp j(\omega t - \mathbf{k}_r \cdot \mathbf{r})$

Transmitted wave $\quad E_t = E_{to}\exp j(\omega t - \mathbf{k}_t \cdot \mathbf{r})$

Fresnel's Equations

Applying the boundary conditions to the EM wave going from medium 1 to 2, the amplitudes of the reflected and transmitted waves can be readily obtained in terms of n_1, n_2 and the incidence angle θ_i alone. These relationships are called Fresnel's equations. If we define $n = n_2/n_1$, as the relative refractive index of medium 2 to that of 1, then the reflection and transmission coefficients for $E_\perp$ are,

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

Fresnel's Equations

There are corresponding coefficients for the $E_{//}$ fields with corresponding **reflection and transmission coefficients**, $r_{//}$ and $t_{//}$,

$$r_{//} = \frac{E_{r0,//}}{E_{i0,//}} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

Fresnel's Equations

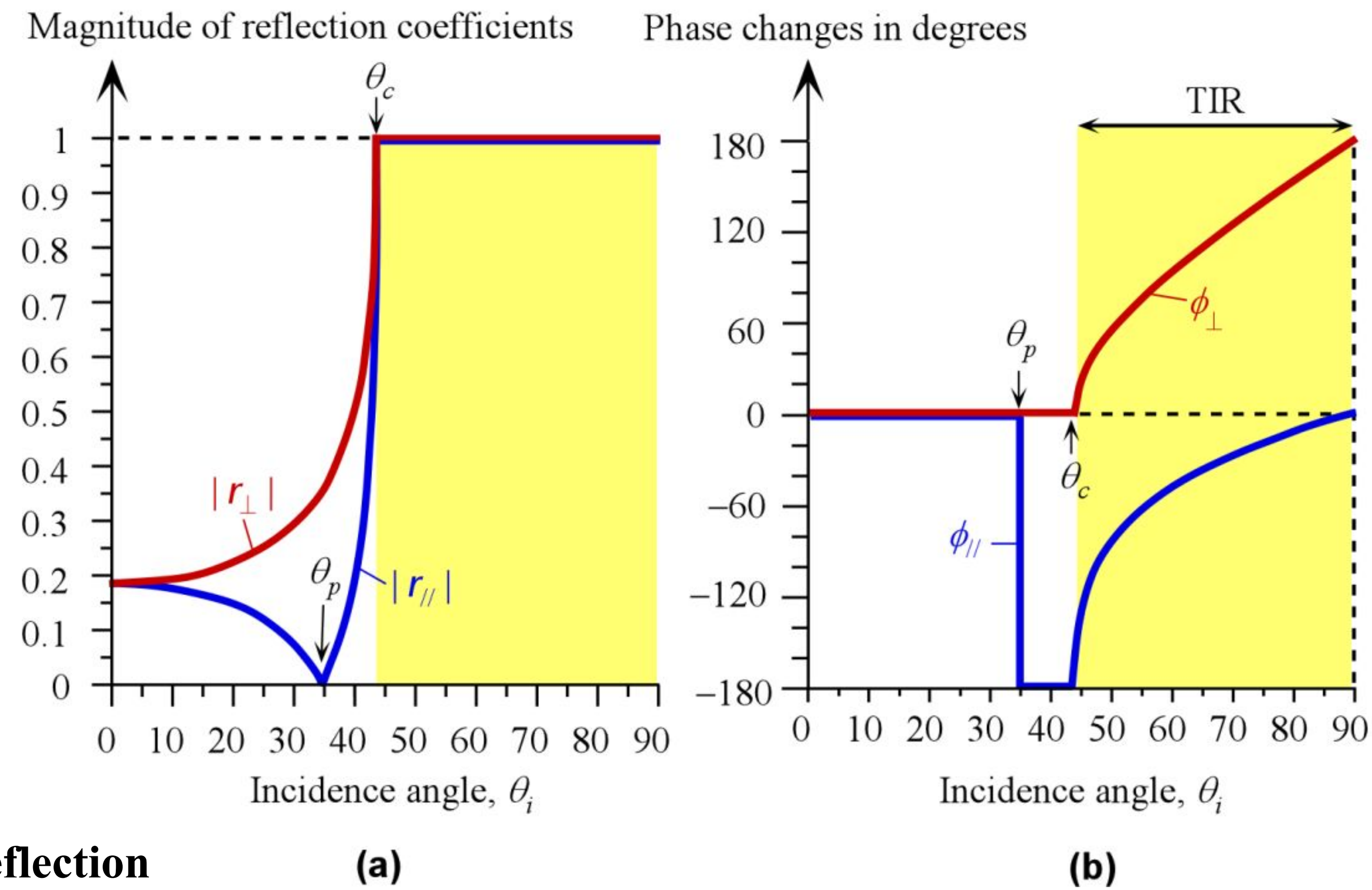
Further, the above coefficients are related by

$$r_{//} + n t_{//} = 1 \quad \text{and} \quad r_{\perp} + 1 = t_{\perp}$$

For convenience we take E_{io} to be a real number so that phase angles of $r_{\perp}$ and $t_{\perp}$ correspond to the phase changes measured with respect to the incident wave.

For normal incidence ($\theta_i = 0$) into Fresnel's equations we find,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$$



Internal reflection

(a) Magnitude of the reflection coefficients $r_{//}$ and $r_{\perp}$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle is 44°.

(b) The corresponding changes $\phi_{//}$ and $\phi_{\perp}$ vs. incidence angle.

Reflection and Polarization Angle

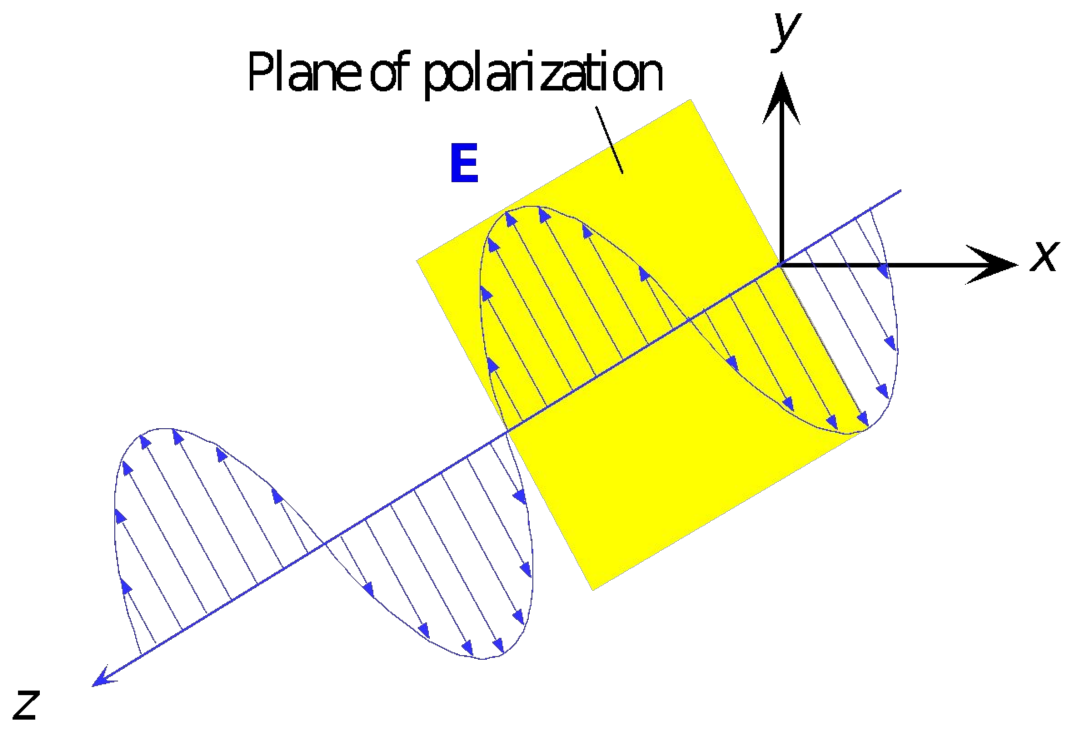
We find a special incidence angle, labeled as θ_p, by solving the Fresnel equation for $r_{//} = 0$. The field in the reflected wave is then always perpendicular to the plane of incidence and hence well-defined. This special angle is called the polarization angle or Brewster's angle,

$$r_{//} = \frac{E_{r0,//}}{E_{i0,//}} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i} = 0$$

$$\tan\theta_p = \frac{n_2}{n_1}$$

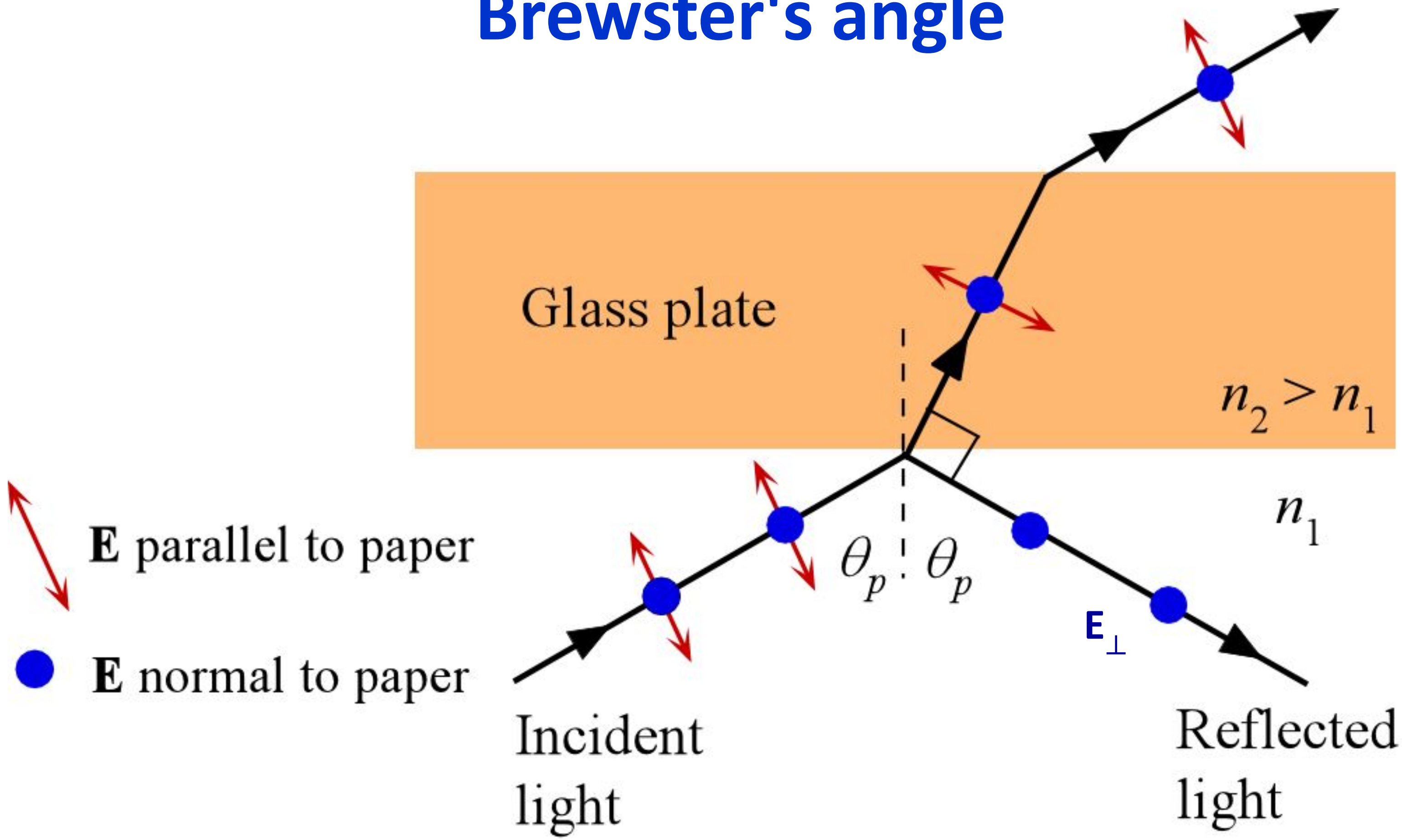
For both $n_1 > n_2$
or $n_1 < n_2$.

Polarized Light



A **linearly polarized** wave has its electric field oscillations defined along a line perpendicular to the direction of propagation, z. The field vector **E** and z define a **plane of polarization**.

Brewster's angle



Reflected light at $\theta_i = \theta_p$ has only $\mathbf{E}_\perp$

for both $n_1 > n_2$ or $n_1 < n_2$.

Total Internal Reflection

In linearly polarized light, however, the field oscillations are contained within a well defined plane. Light emitted from many light sources such as a tungsten light bulb or an LED diode is unpolarized and the field is randomly oriented in a direction that is perpendicular to the direction of propagation.

At the critical angle and beyond (past 44° in the figure), *i.e.* when $\theta_i \geq \theta_c$, the magnitudes of both $r_{//}$ and $r_{\perp}$ go to unity so that the reflected wave has the same amplitude as the incident wave. The incident wave has suffered total internal reflection, TIR.

Phase change upon total internal reflection

When $\theta_i > \theta_c$, in the presence of TIR, the reflection coefficients become complex quantities of the type

$$r_\perp = 1 \cdot \exp(-j\varphi_\perp) \text{ and } r_{//} = 1 \cdot \exp(-j\varphi_{//})$$

with the phase angles $\varphi_\perp$ and $\varphi_{//}$ being other than zero or 180°. The reflected wave therefore suffers phase changes, $\varphi_\perp$ and $\varphi_{//}$, in the components $E_\perp$ and $E_{//}$. These phase changes depend on the incidence angle, and on n_1 and n_2.

The phase change $\varphi_\perp$ is given by

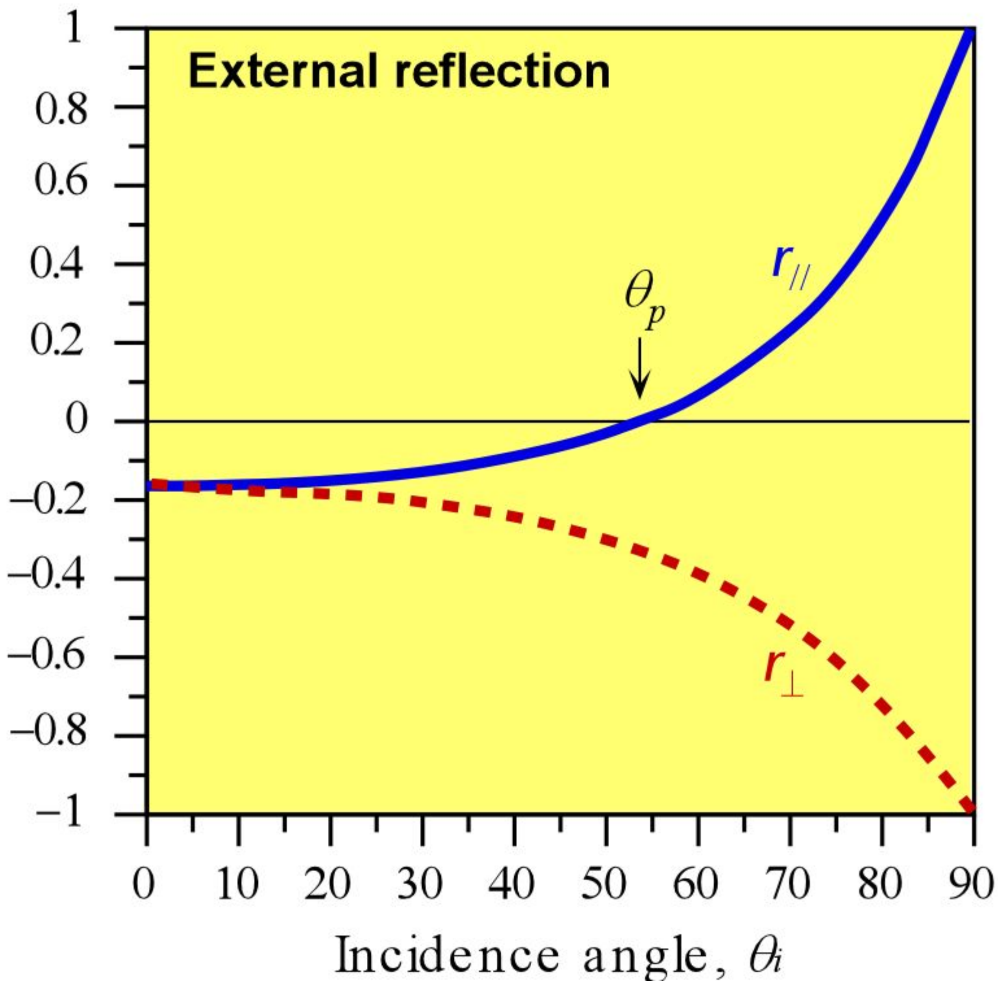
$$\tan\left(\frac{1}{2}\phi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i}$$

Phase change upon total internal reflection

For the $E_{//}$ component, the phase change $\varphi_{//}$ is given by

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i}$$

External Reflection



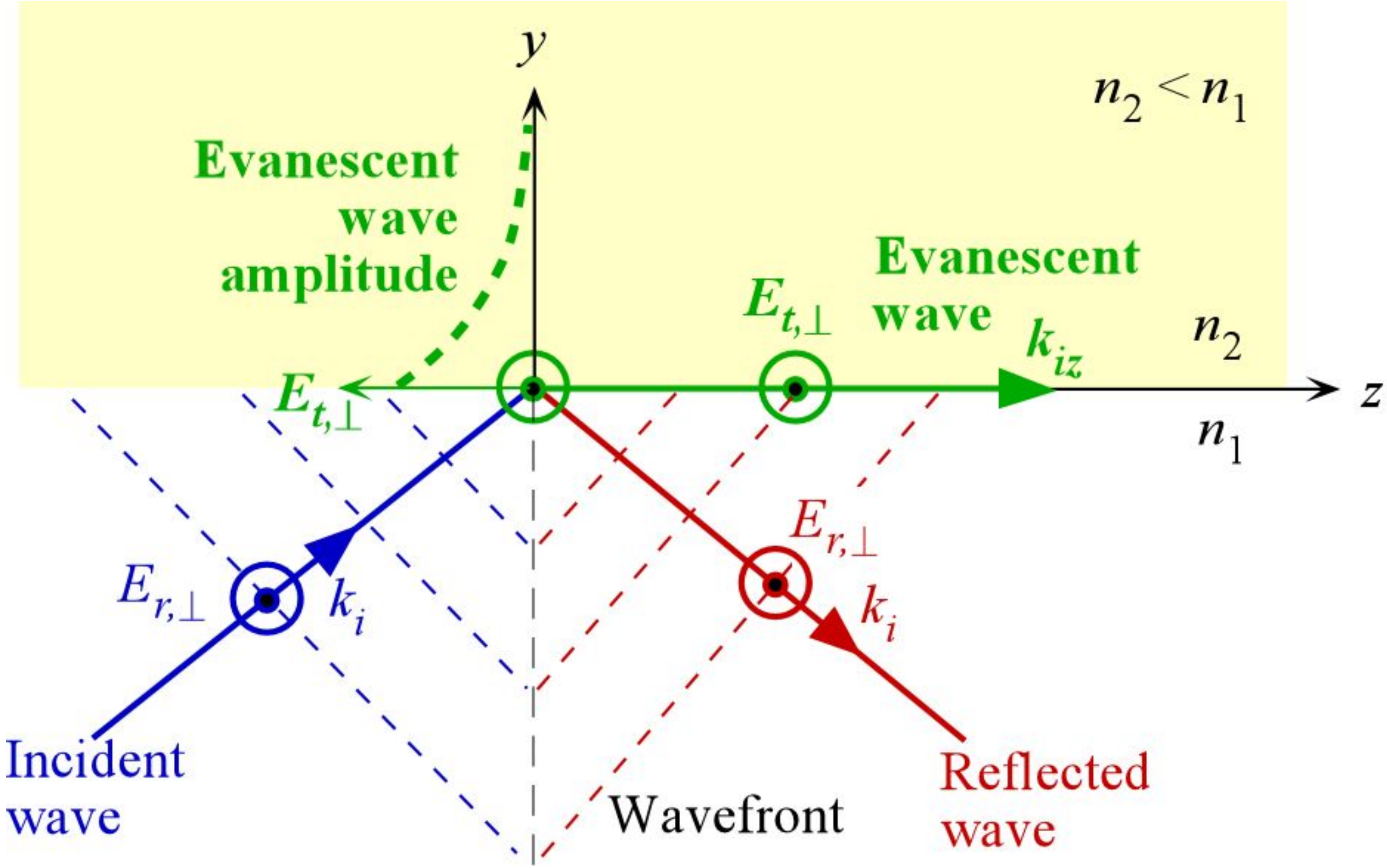
The reflection coefficients $r_{//}$ and $r_{\perp}$ versus angle of incidence θ_i for n_1 = 1.00 and n_2 = 1.44.

Evanescent Wave

In internal reflection ($n_1 > n_2$), the amplitude of the reflected wave from TIR is equal to the amplitude of the incident wave but its phase has shifted.

What happens to the transmitted wave when $\theta_i > \theta_c$?

According to the boundary conditions, there must still be an electric field in medium 2, otherwise, the boundary conditions cannot be satisfied. When $\theta_i > \theta_c$, the field in medium 2 is attenuated (decreases with y, and is called the evanescent wave.



When $\theta_i > \theta_c$, for a plane wave that is reflected, there is an evanescent wave at the boundary propagating along z.

Evanescent wave when plane waves are incident and reflected

$$E_{t,\perp}(y,z,t) \propto e^{-\alpha_2 y} \exp j(\omega t - k_{iz} z)$$

where $k_{iz} = k_i \sin\theta_i$ is the wavevector of the incident wave along the z-axis, and α_2 is an **attenuation coefficient** for the electric field penetrating into medium 2

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

Penetration depth of evanescent wave

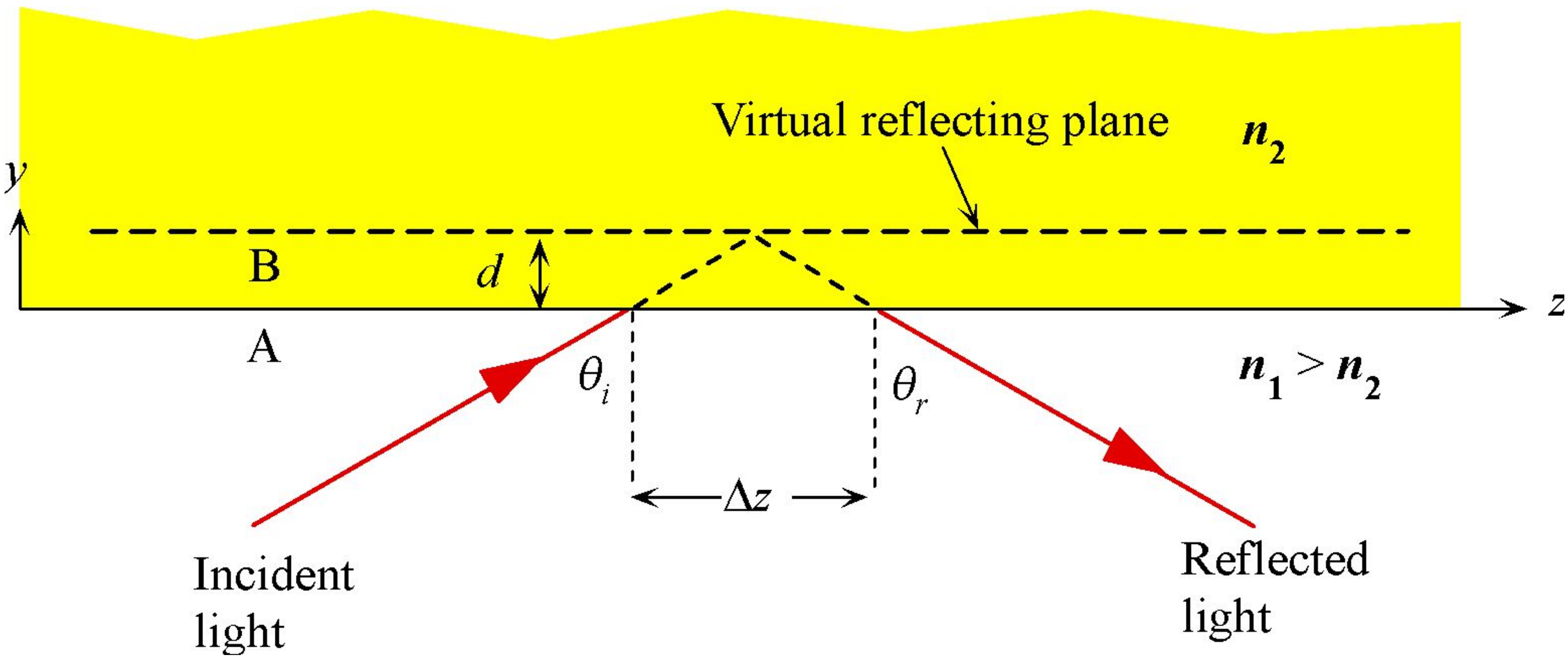
α_2 = **Attenuation coefficient** for the electric field penetrating into medium 2

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

The field of the evanescent wave is e^{-1} in medium 2 when

$y = 1/\alpha_2 = \delta$ = **Penetration depth**

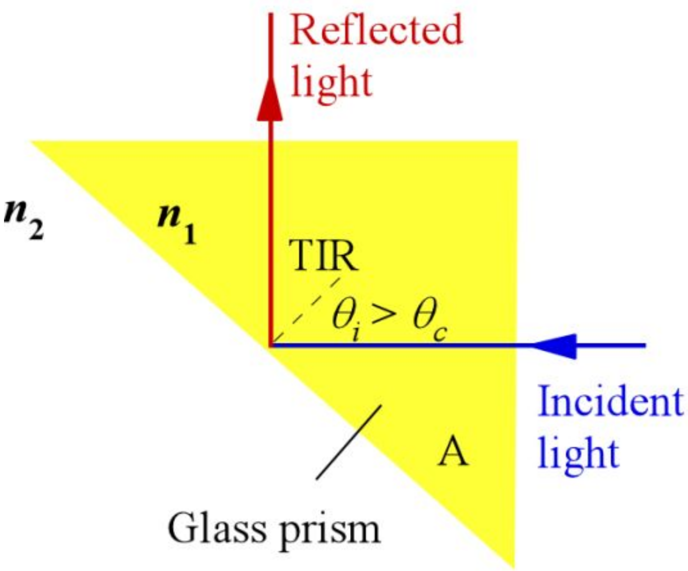
Goos-Hänchen Shift



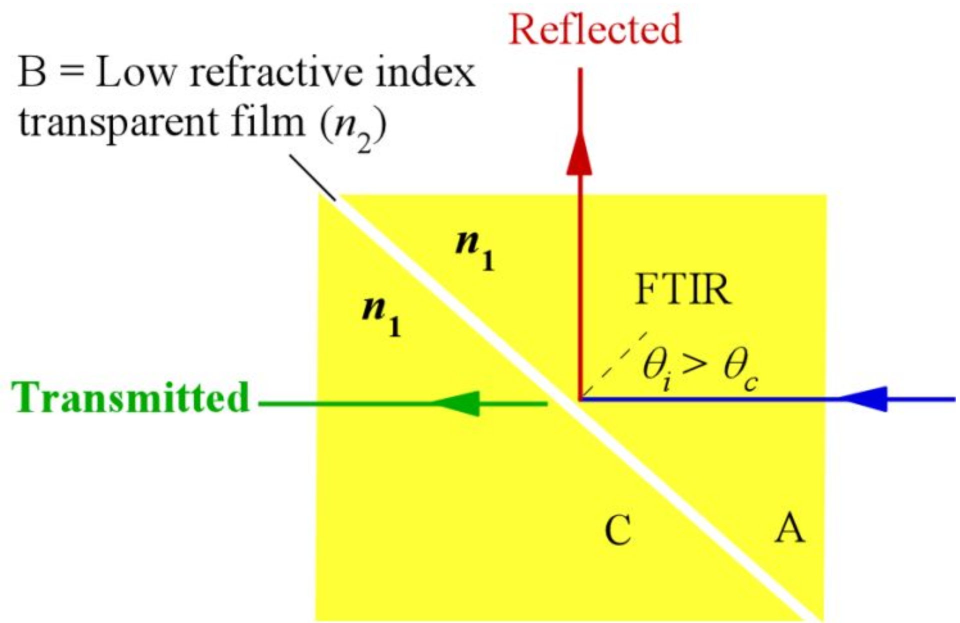
$$\Delta z = 2\delta \tan\theta_i$$

Beam Splitters

Frustrated Total Internal Reflection (FTIR)



(a)

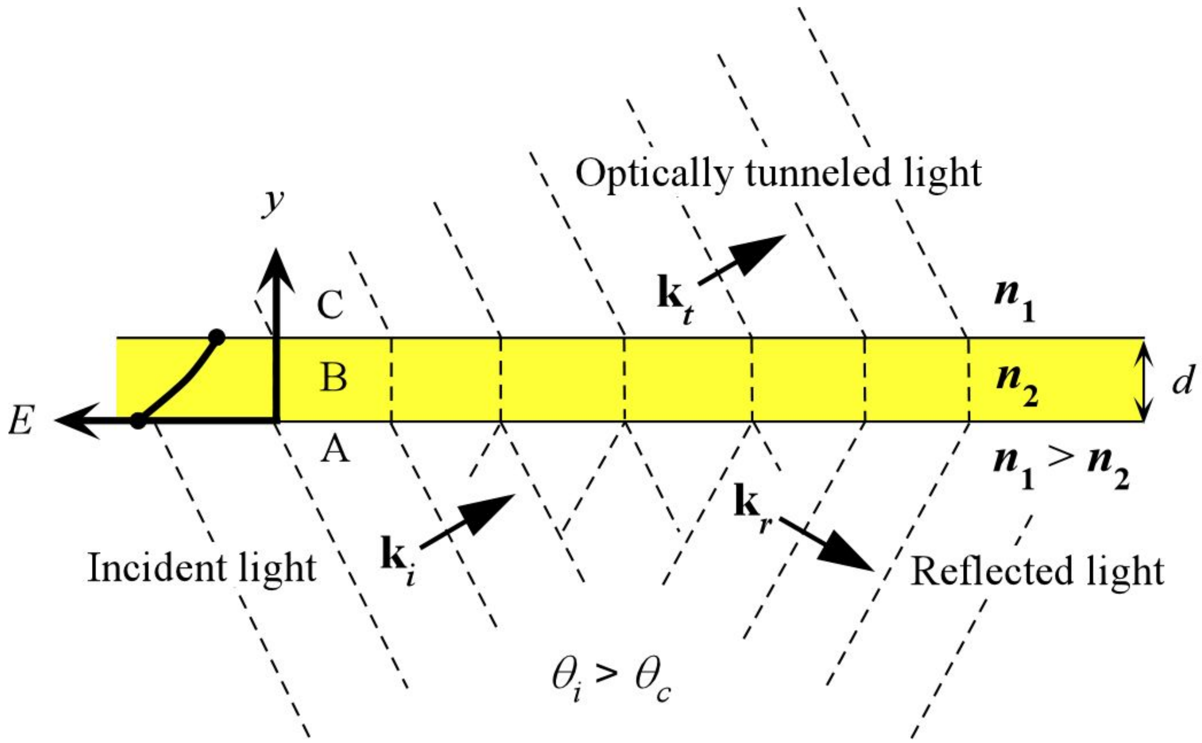


(b)

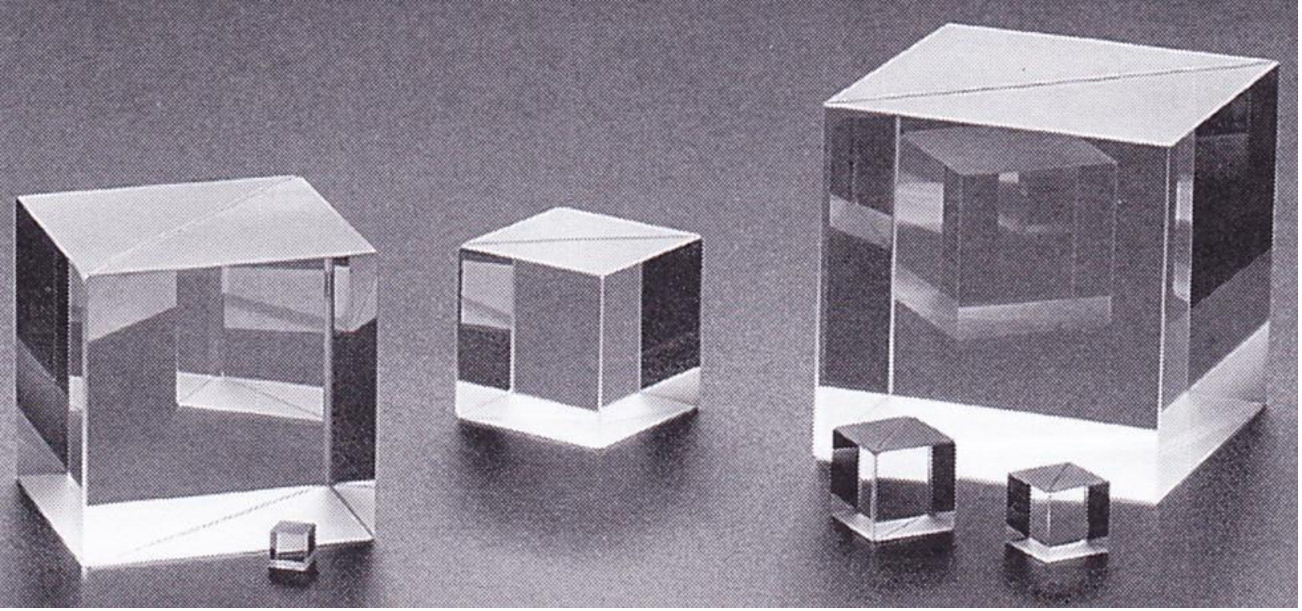
(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.

(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.

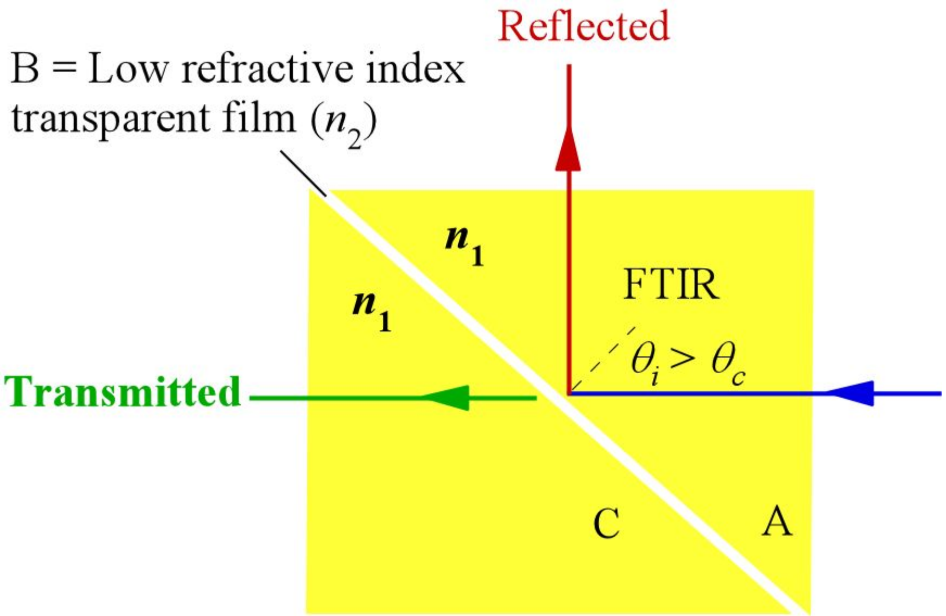
Optical Tunneling



When medium B is thin (thickness d is small), the field penetrates from the AB interface into medium B and reaches BC interface, and gives rise to a transmitted wave in medium C. The effect is the tunneling of the incident beam in A through B to C. The maximum field E_{max} of the evanescent wave in B decays in B along y and but is finite at the BC boundary and excites the transmitted wave.

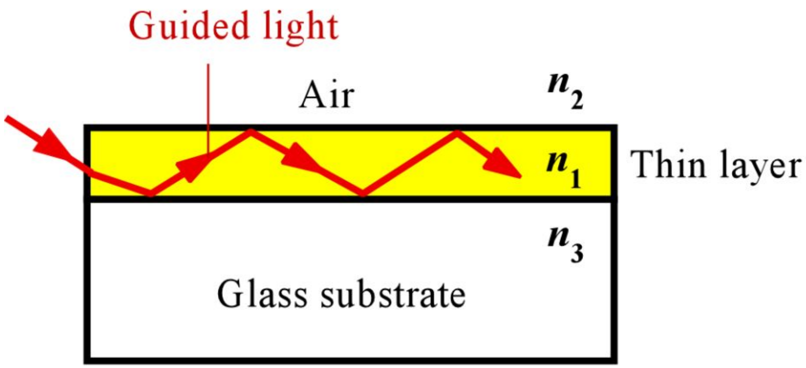


Beam splitter cubes (Courtesy of CVI Melles Griot)

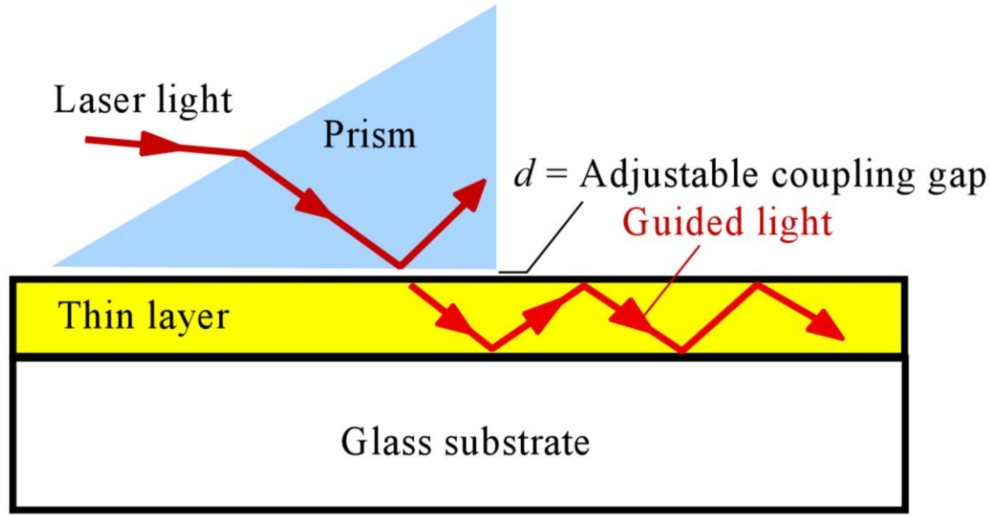


Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.

Optical Tunneling



(a)



(b)

Light propagation along an optical guide by total internal reflections

Coupling of laser light into a thin layer - optical guide - using a prism. The light propagates along the thin layer.

External Reflection

Light approaches the boundary from the lower index side,

$n_1 < n_2$

This is external reflection.
Light becomes reflected by the surface of an optically denser (higher refractive index) medium.

$r_\perp$ and $r_{//}$ depend on the incidence angle θ_i. At normal incidence, $r_\perp$ and $r_{//}$ are negative. In external reflection at normal incidence there is a phase shift of 180°. $r_{//}$ goes through zero at the Brewster angle, θ_p. At θ_p, the reflected wave is polarized in the $E_\perp$ component only.

Transmitted light in both internal reflection (when $\theta_i < \theta_c$) and external reflection does not experience a phase shift.

Intensity, Reflectance and Transmittance

Reflectance R measures the intensity of the reflected light with respect to that of the incident light and can be defined separately for electric field components parallel and perpendicular to the plane of incidence. The reflectances $R_\perp$ and $R_{//}$ are defined by

$$R_\perp = \frac{\left|E_{ro,\perp}\right|^2}{\left|E_{io,\perp}\right|^2} = \left|r_\perp\right|^2$$

and

$$R_{//} = \frac{\left|E_{ro,//}\right|^2}{\left|E_{io,//}\right|^2} = \left|r_{//}\right|^2$$

At normal incidence

$$R = R_{\perp} = R_{//} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

Since a glass medium has a refractive index of around 1.5 this means that typically 4% of the incident radiation on an air-glass surface will be reflected back.

Example: Reflection at normal incidence. Internal and external reflection

Consider the reflection of light at normal incidence on a boundary between a glass medium of refractive index 1.5 and air of refractive index 1.

(a) If light is traveling from air to glass, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(b) If light is traveling from glass to air, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(c) What is the polarization angle in the external reflection in **a** above? How would you make a polaroid from this?

Solution

(a) The light travels in air and becomes partially reflected at the surface of the glass which corresponds to external reflection. Thus n_1 = 1 and n_2 = 1.5. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - 1.5}{1 + 1.5} = -0.2$$

This is negative which means that there is a 180° phase shift. The reflectance (*R*), which gives the fractional reflected power, is

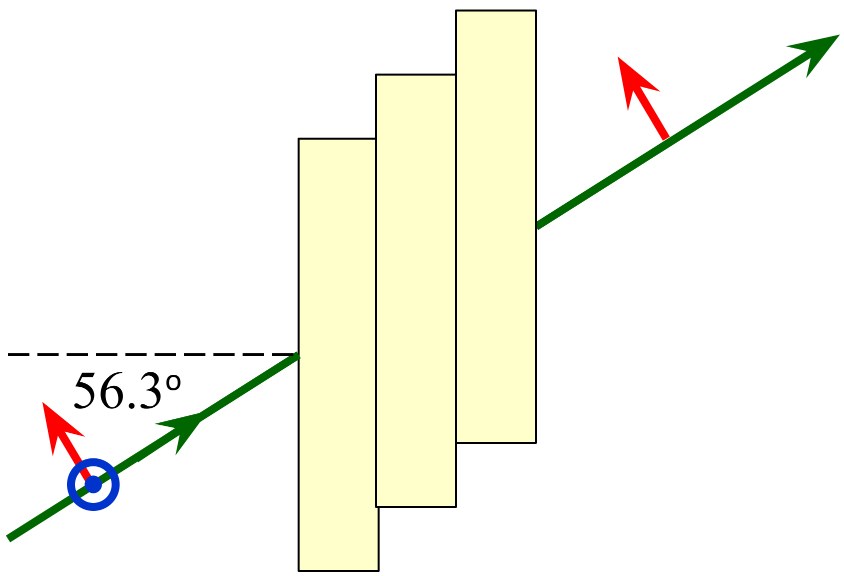
$R = r_{//}^2 = 0.04$ or 4%.

(b) The light travels in glass and becomes partially reflected at the glass-air interface which corresponds to internal reflection. n_1 = 1.5 and n_2 = 1. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

There is no phase shift. The reflectance is again 0.04 or 4%. In both cases **(a)** and **(b)** the amount of reflected light is the same.

(c) Light is traveling in air and is incident on the glass surface at the polarization angle. Here $n_1 = 1$, $n_2 = 1.5$ and $\tan\theta_p = (n_2/n_1) = 1.5$ so that $\theta_p = 56.3°$.



This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812

Transmittance

Transmittance *T* relates the intensity of the transmitted wave to that of the incident wave in a similar fashion to the reflectance.

However the transmitted wave is in a different medium and further its direction with respect to the boundary is also different due to refraction.

For normal incidence, the incident and transmitted beams are normal so that the equations are simple:

Transmittance

$$T_{\perp} = \frac{n_2\left|E_{to,\perp}\right|^2}{n_1\left|E_{io,\perp}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\perp}\right|^2$$

$$T_{//} = \frac{n_2\left|E_{to,//}\right|^2}{n_1\left|E_{io,//}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{//}\right|^2$$

or

$$T = T_{\perp} = T_{//} = \frac{4n_1n_2}{\left(n_1 + n_2\right)^2}$$

Further, the fraction of light reflected and fraction transmitted must add to unity. Thus $R + T = 1$.

Reflection and Transmission – An Example

Question A light beam traveling in air is incident on a glass plate of refractive index 1.50 . What is the Brester or polarization angle? What are the relative intensities of the reflected and transmitted light for the polarization perpendicular and parallel to the plane of incidence at the Brewster angle of incidence?

Solution Light is traveling in air and is incident on the glass surface at the polarization angle θ_p. Here n_1 = 1, n_2 = 1.5 and tanθ_p = (n_2/n_1) = 1.5 so that θ_p = 56.31°. We now have to use Fresnel's equations to find the reflected and transmitted amplitudes. For the perpendicular polarization

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - [n^2 - \sin^2\theta_i]^{1/2}}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$r_\perp = \frac{\cos(56.31^\circ) - [1.5^2 - \sin^2(56.31^\circ)]^{1/2}}{\cos(56.31^\circ) + [1.5^2 - \sin^2(56.31^\circ)]^{1/2}} = -0.385$$

On the other hand, $r_{//}$ = 0. The reflectances $R_\perp$ = | $r_\perp$ |^2 = 0.148 and $R_{//}$ = |$r_{//}$|^2 = 0 so that R = 0.074, and has no parallel polarization in the plane of incidence. Notice the negative sign in $r_\perp$, which indicates a phase change of π.

Reflection and Transmission – An Example

$$t_{\perp} = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{\perp} = \frac{2\cos(56.31^{\circ})}{\cos(56.31^{\circ}) + [1.5^2 - \sin^2(56.31^{\circ})]^{1/2}} = 0.615$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{//} = \frac{2(1.5)\cos(56.31^{\circ})}{(1.5)^2\cos(56.31^{\circ}) + [1.5^2 - \sin^2(56.31^{\circ})]^{1/2}} = 0.667$$

Notice that $r_{//} + nt_{//} = 1$ and $r_{\perp} + 1 = t_{\perp}$, as we expect.

Reflection and Transmission – An Example

To find the transmittance for each polarization, we need the refraction angle θ_t. From Snell's law, $n_1 \sin\theta_i = n_t \sin\theta_t$ *i.e.* $(1)\sin(56.31°) = (1.5)\sin\theta_t$, we find θ_t = **33.69°**.

$$T_{//} = \frac{n_2 \left|E_{to,//}\right|^2}{n_1 \left|E_{io,//}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{//}\right|^2$$

$$T_{\perp} = \frac{n_2 \left|E_{to,\perp}\right|^2}{n_1 \left|E_{io,\perp}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\perp}\right|^2$$

$$T_{//} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.667)^2 = 1$$

$$T_{\perp} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.615)^2 = 0.852$$

Clearly, light with polarization parallel to the plane of incidence has greater intensity.

If we were to reflect light from a glass plate, keeping the angle of incidence at 56.3°, then the reflected light will be polarized with an electric field component perpendicular to the plane of incidence. The transmitted light will have the field greater in the plane of incidence, that is, it will be partially polarized. By using a stack of glass plates one can increase the polarization of the transmitted light. (This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812.)

Example: Reflection of light from a less dense medium (internal reflection)

A ray of light which is traveling in a glass medium of refractive index $n_1 = 1.460$ becomes incident on a less dense glass medium of refractive index $n_2 = 1.440$. The free space wavelength (λ) of the light ray is 1300 nm.

(a) What should be the minimum incidence angle for TIR?
(b) What is the phase change in the reflected wave when $\theta_i = 87°$ and when $\theta_i = 90°$?
(c) What is the penetration depth of the evanescent wave into medium 2 when $\theta_i = 87°$ and when $\theta_i = 90°$?

Solution

(a) The critical angle θ_c for TIR is given by

$\sin\theta_c = n_2/n_1 = 1.440/1.460$ so that $\theta_c = 80.51°$

(b) Since the incidence angle $\theta_i > \theta_c$ there is a phase shift in the reflected wave. The phase change in $E_{r,\perp}$ is given by $\varphi_\perp$.

Using $n_1 = 1.460$, $n_2 = 1.440$ and $\theta_i = 87°$,

$$\tan\left(\tfrac{1}{2}\phi_{\perp}\right)=\frac{\left[\sin^2\theta_i-n^2\right]^{1/2}}{\cos\theta_i}=\frac{\left[\sin^2(87^{\circ})-\left(\dfrac{1.440}{1.460}\right)^2\right]^{1/2}}{\cos(87^{\circ})}$$

$$= 2.989 = \tan[{}^{1}/{}_{2}(143.0°)]$$

so that the phase change $\varphi_{\perp}$ **= 143°.**

For the $E_{r,//}$ component, the phase change is

$$\tan\left(\tfrac{1}{2}\phi_{//}+\tfrac{1}{2}\pi\right)=\frac{\left[\sin^2\theta_i-n^2\right]^{1/2}}{n^2\cos\theta_i}=\frac{1}{n^2}\tan\left(\tfrac{1}{2}\phi_{\perp}\right)$$

so that

$$\tan(\tfrac{1}{2}\varphi_{//} + \tfrac{1}{2}\pi) = (n_1/n_2)^2\tan(\varphi_{\perp}/2) =$$

$$(1.460/1.440)^2\tan(\tfrac{1}{2}143°)$$

which gives $\varphi_{//} = 143.95° - 180°$ or $-36.05°$

Repeat with $\theta_i = 90°$ to find $\varphi_{\perp} = 180°$ and $\varphi_{//} = 0°$.

Note that as long as $\theta_i > \theta_c$, the magnitude of the reflection coefficients are unity. Only the phase changes.

(c) The amplitude of the evanescent wave as it penetrates into medium 2 is

$$E_{t,\perp}(y,t) \propto E_{to,\perp}\exp(-\alpha_2 y)$$

The field strength drops to e^{-1} when $y = 1/\alpha_2 = \delta$, which is called the penetration depth. The attenuation constant α_2 is

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

i.e.

$$\alpha_2 = \frac{2\pi(1.440)}{(1300\times 10^{-9}\ \text{m})}\left[\left(\frac{1.460}{1.440}\right)^2 \sin^2(87^\circ) - 1\right]^{1/2}$$

= 1.10×10^{6} m^{-1}.

The penetration depth is,

$\delta = 1/\alpha_2 = 1/(1.104\times10^{6}\ \text{m}) = 9.06\times10^{-7}$ m, or **0.906 µm**

For 90°, repeating the calculation, $\alpha_2 = 1.164\times10^{6}\ \text{m}^{-1}$, so that

$\delta = 1/\alpha_2 =$ **0.859 µm**

The penetration is greater for smaller incidence angles

Example: Antireflection coatings on solar cells

When light is incident on the surface of a semiconductor it becomes partially reflected. Partial reflection is an important energy loss in solar cells.

The refractive index of Si is about 3.5 at wavelengths around 700 - 800 nm. Reflectance with n_1(air) = 1 and n_2(Si) ≈ 3.5 is

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - 3.5}{1 + 3.5}\right)^2 = 0.309$$

30% of the light is reflected and is not available for conversion to electrical energy; a considerable reduction in the efficiency of the solar cell.

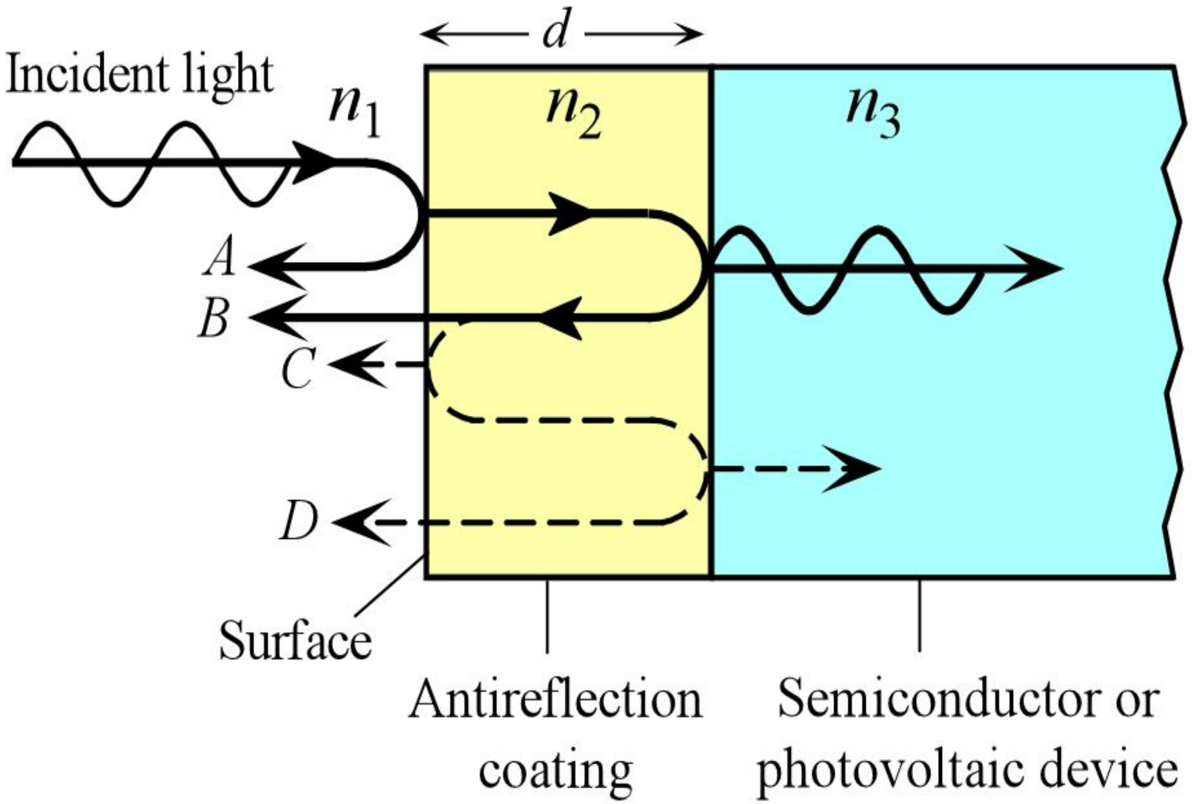
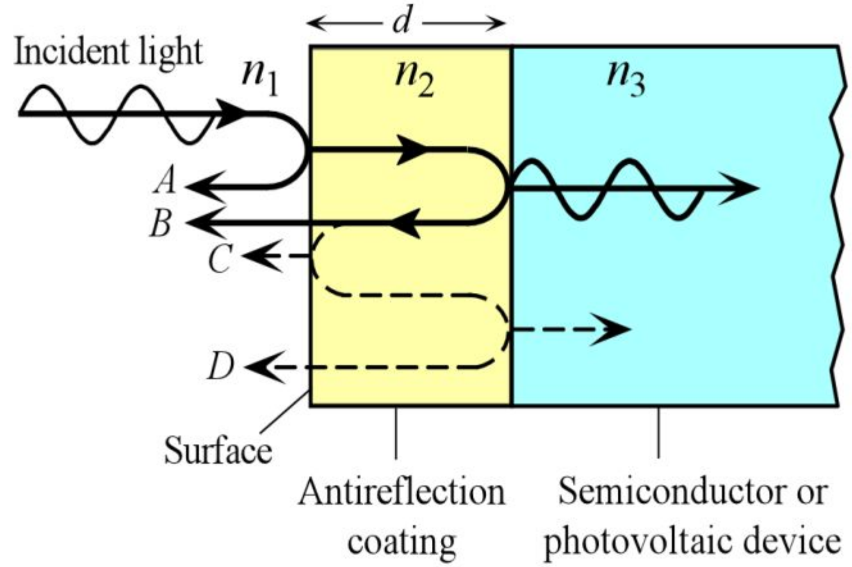


Illustration of how an antireflection coating reduces the reflected light intensity.

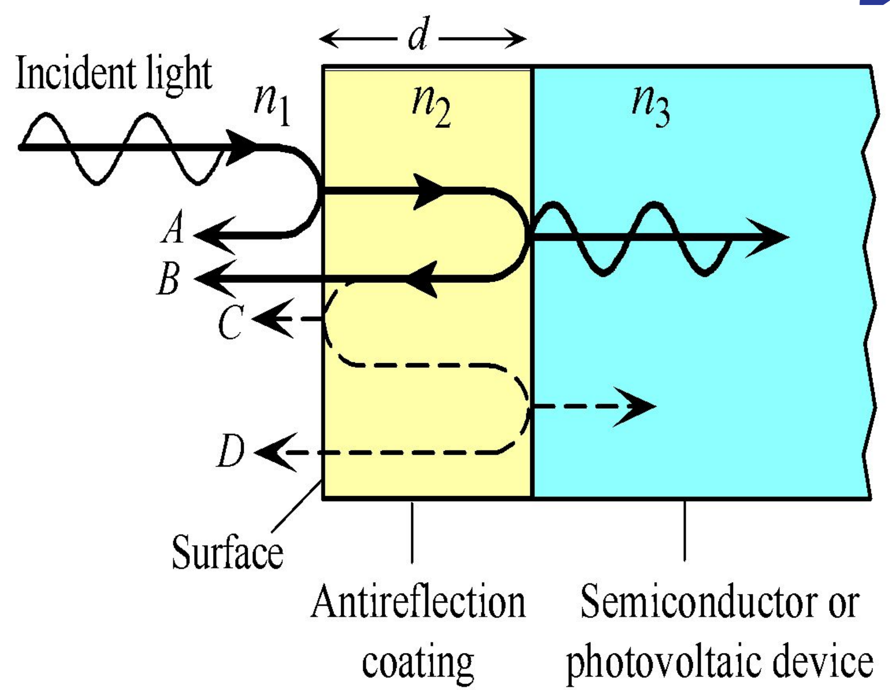
We can coat the surface of the semiconductor device with a thin layer of a dielectric material, *e.g.* Si_3N_4 (silicon nitride) that has an intermediate refractive index.



n_1(air) = 1, n_2(coating) ≈ 1.9 and n_3(Si) = 3.5

Light is first incident on the air/coating surface. Some of it becomes reflected as *A* in the figure. Wave *A* has experienced a 180° phase change on reflection because this is an external reflection. The wave that enters and travels in the coating then becomes reflected at the coating/semiconductor surface.

This reflected wave *B*, also suffers a 180° phase change since $n_3 > n_2$.



When *B* reaches *A*, it has suffered a total delay of traversing the thickness *d* of the coating twice. The **phase difference** is equivalent to $k_c(2d)$ where $k_c = 2\pi/\lambda_c$ is the propagation constant in the coating, *i.e.* $k_c = 2\pi/\lambda_c$ where λ_c is the wavelength in the coating.

Since $\lambda_c = \lambda / n_2$, where λ is the free-space wavelength, the phase difference $\Delta\varphi$ between *A* and *B* is $(2\pi n_2/\lambda)(2d)$. To reduce the reflected light, *A* and *B* **must interfere destructively.** This requires the **phase difference to be π or odd-multiples of π, $m\pi$ where m = 1,3,5**,… is an odd-integer. Thus

$$\left(\frac{2\pi n_2}{\lambda}\right)2d = m\pi$$

or

$$d = m\left(\frac{\lambda}{4n_2}\right)$$

The thickness of the coating must be <u>odd-multiples</u> of the quarter wavelength in the coating and depends on the wavelength.

$$R_{\min} = \left(\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3}\right)^2$$

$$d = m\left(\frac{\lambda}{4n_2}\right)$$

To obtain good destructive interference between waves *A* and *B*, the two amplitudes must be comparable. We need (proved later) $n_2 = \sqrt{(n_1 n_3)}$. When $n_2 = \sqrt{(n_1 n_3)}$ then the reflection coefficient between the air and coating is equal to that between the coating and the semiconductor. For a Si solar cell, √(3.5) or 1.87. Thus, Si_3N_4 is a good choice as an antireflection coating material on Si solar cells.

Taking the wavelength to be 700 nm,

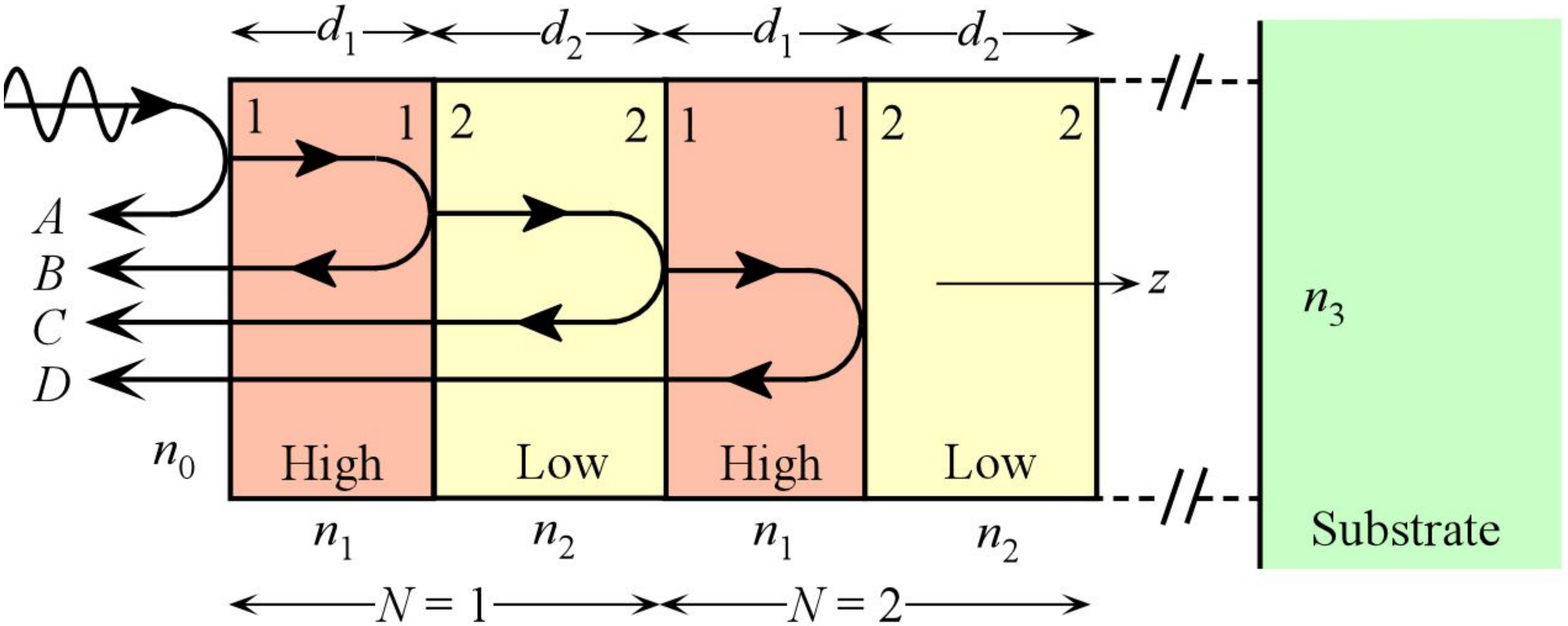
d = (700 nm)/[4 (1.9)] = 92.1 nm or odd-multiples of d.

$$R_{\min} = \left(\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3} \right)^2$$

$$R_{\min} = \left(\frac{1.9^2 - (1)(3.5)}{1.9^2 + (1)(3.5)} \right)^2 = 0.00024 \; or \; 0.24\%$$

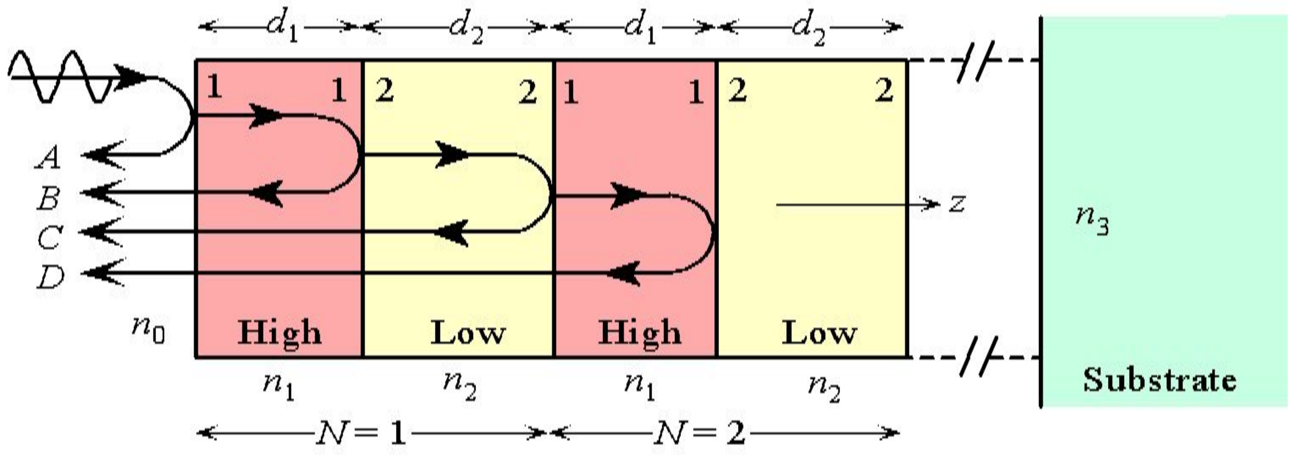
Reflection is almost entirely extinguished
However, only at 700 nm.

Dielectric Mirror or Bragg Reflector

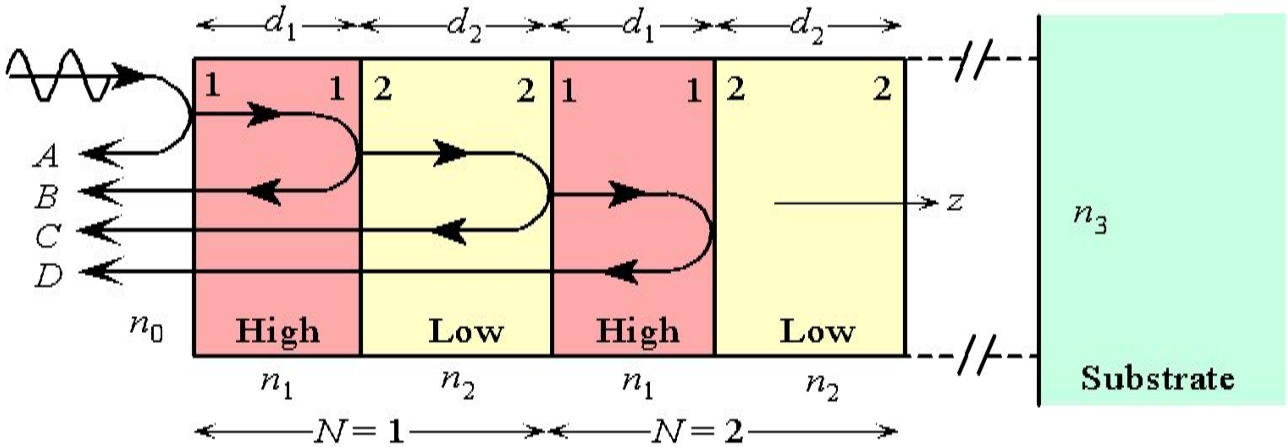


Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers

Dielectric mirrors



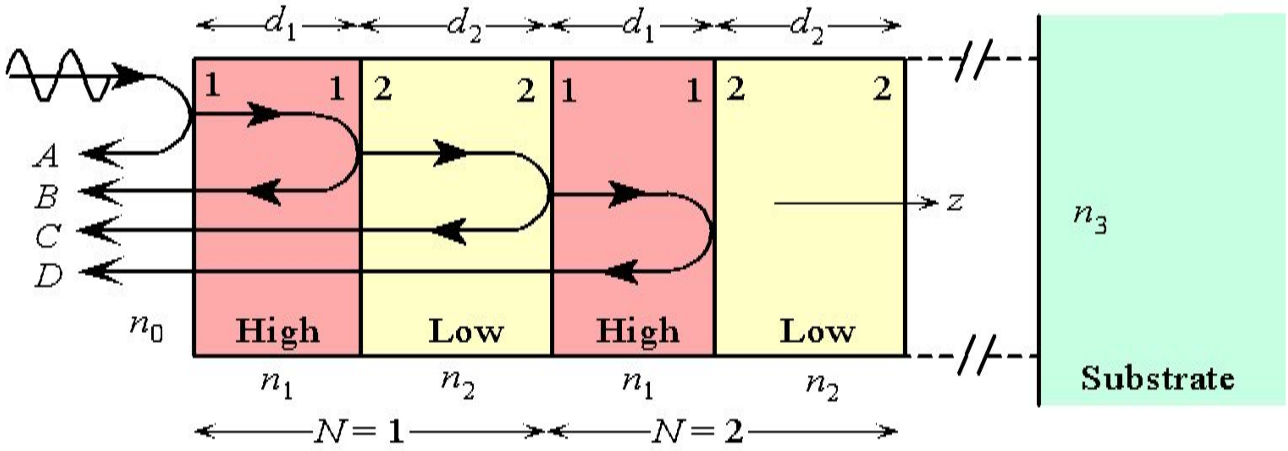
Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers



A dielectric mirror has a stack of dielectric layers of alternating refractive indices. Let n_1 (= n_H) > n_2 (= n_L)

Layer thickness d = Quarter of wavelength or $\lambda_{layer}/4$
$\lambda_{layer} = \lambda_o/n$; λ_o is the free space wavelength at which the mirror is required to reflect the incident light, n = refractive index of layer.

Reflected waves from the interfaces interfere constructively and give rise to a substantial reflected light. If there are sufficient number of layers, the reflectance can approach unity at λ_o.



r_{12} for light in layer 1 being reflected at the 1-2 boundary is $r_{12} = (n_1 - n_2)/(n_1 + n_2)$ and is a positive number indicating no phase change.

r_{21} for light in layer 2 being reflected at the 2-1 boundary is $r_{21} = (n_2 - n_1)/(n_2 + n_1)$ which is $-r_{12}$ (negative) indicating a π phase change.

The reflection coefficient alternates in sign through the mirror

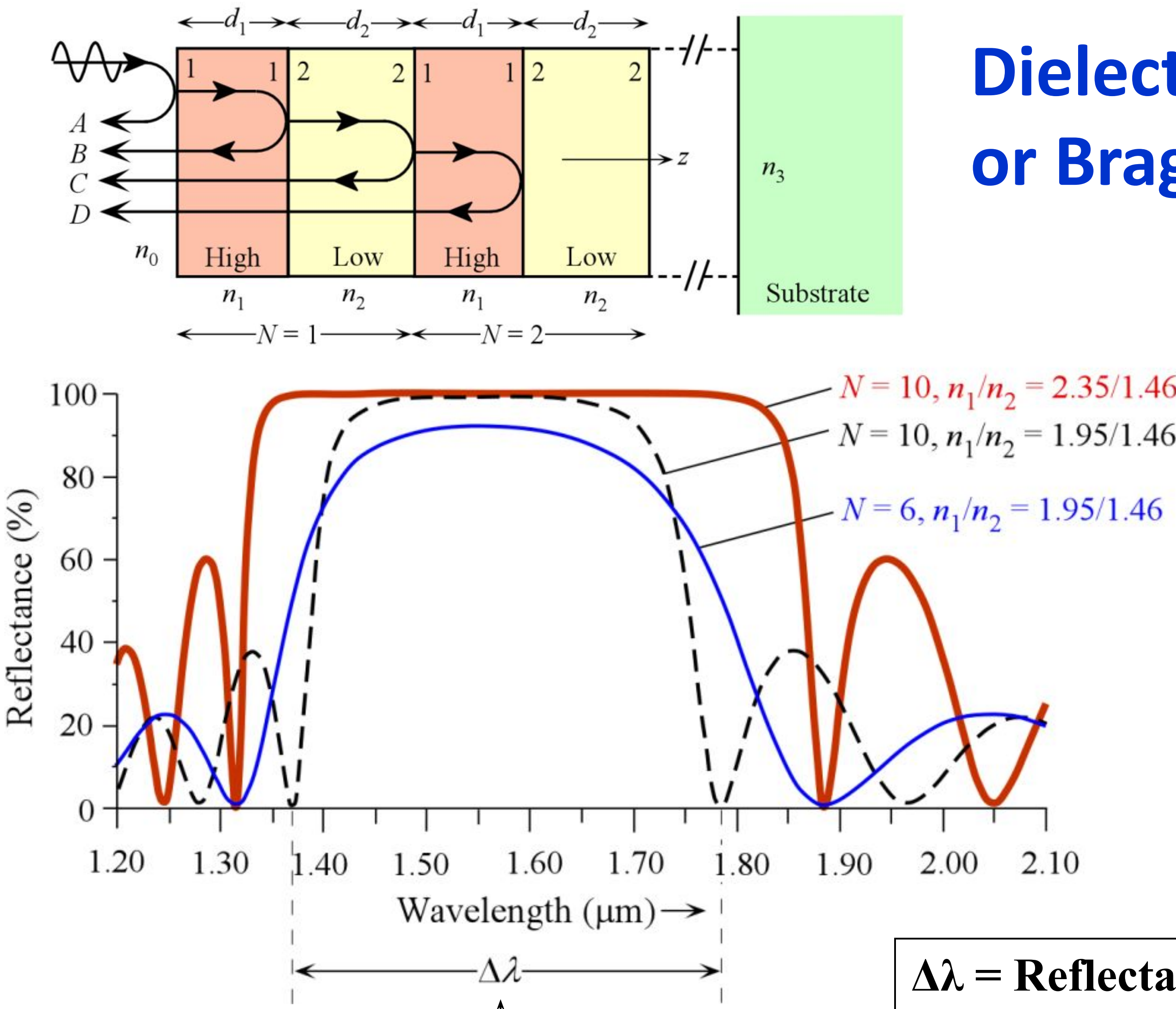
The phase difference between *A* and *B* is

$$0 + \pi + 2k_1 d_1 = 0 + \pi + 2(2\pi n_1/\lambda_o)(\lambda_o/4n_1) = 2\pi.$$

Thus, waves *A* and *B* are in phase and interfere constructively.

Dielectric mirrors are widely used in modern vertical cavity surface emitting semiconductor lasers.

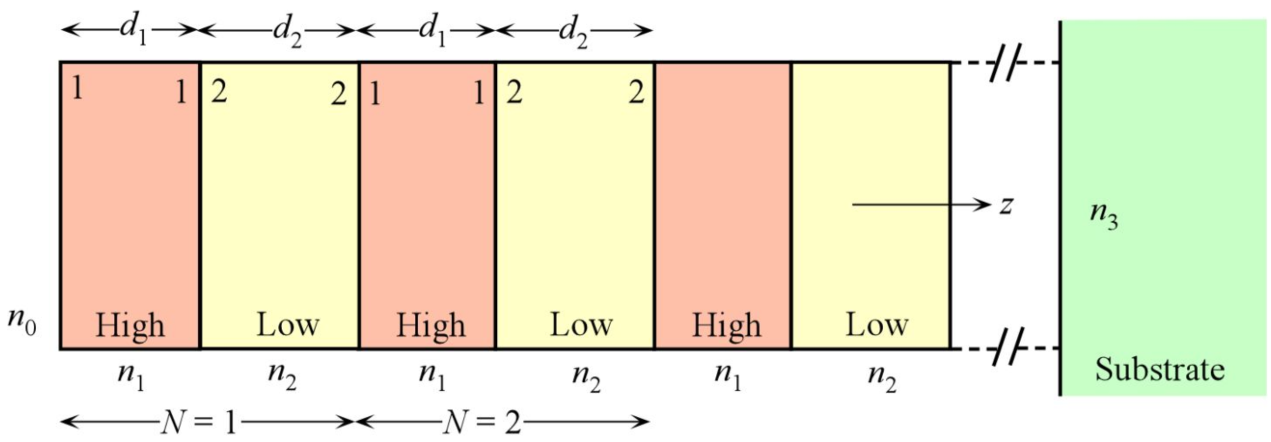
Dielectric Mirror or Bragg Reflector

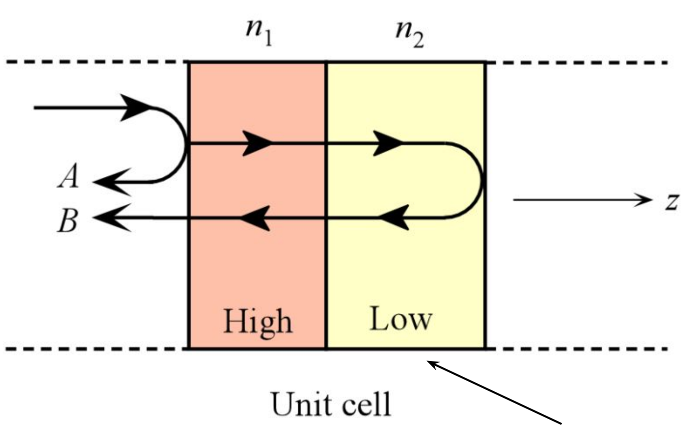


Δλ = Reflectance bandwidth
(**Stop-band** for transmittance)

Dielectric Mirror or Bragg Reflector

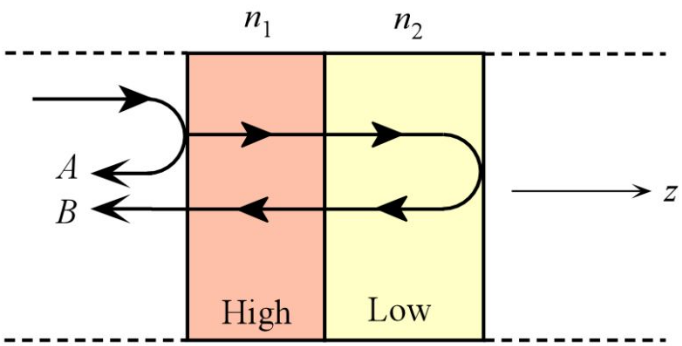
Consider an “infinite stack”





This is a “unit cell”

Dielectric Mirror or Bragg Reflector



Unit cell

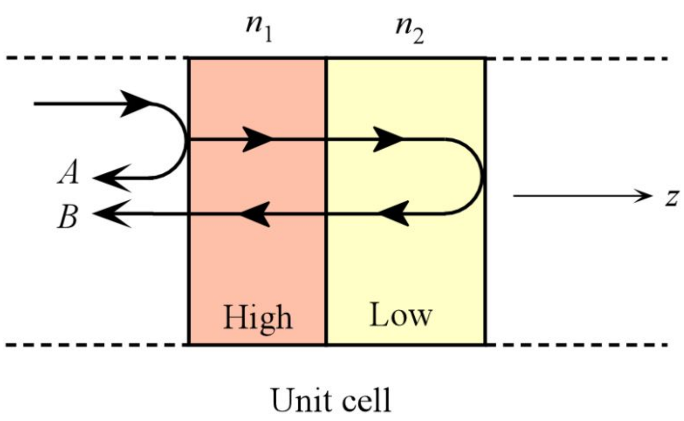
For reflection, the **phase difference between *A* and *B*** must be

$$2k_1d_1 + 2k_2d_2 = m(2\pi)$$

$$2(2\pi n_1/\lambda)d_1 + 2(2\pi n_2/\lambda)d_2 = m(2\pi)$$

$$n_1d_1 + n_2d_2 = m\lambda/2$$

Dielectric Mirror or Bragg Reflector



$$n_1 d_1 + n_2 d_2 = \lambda/2$$

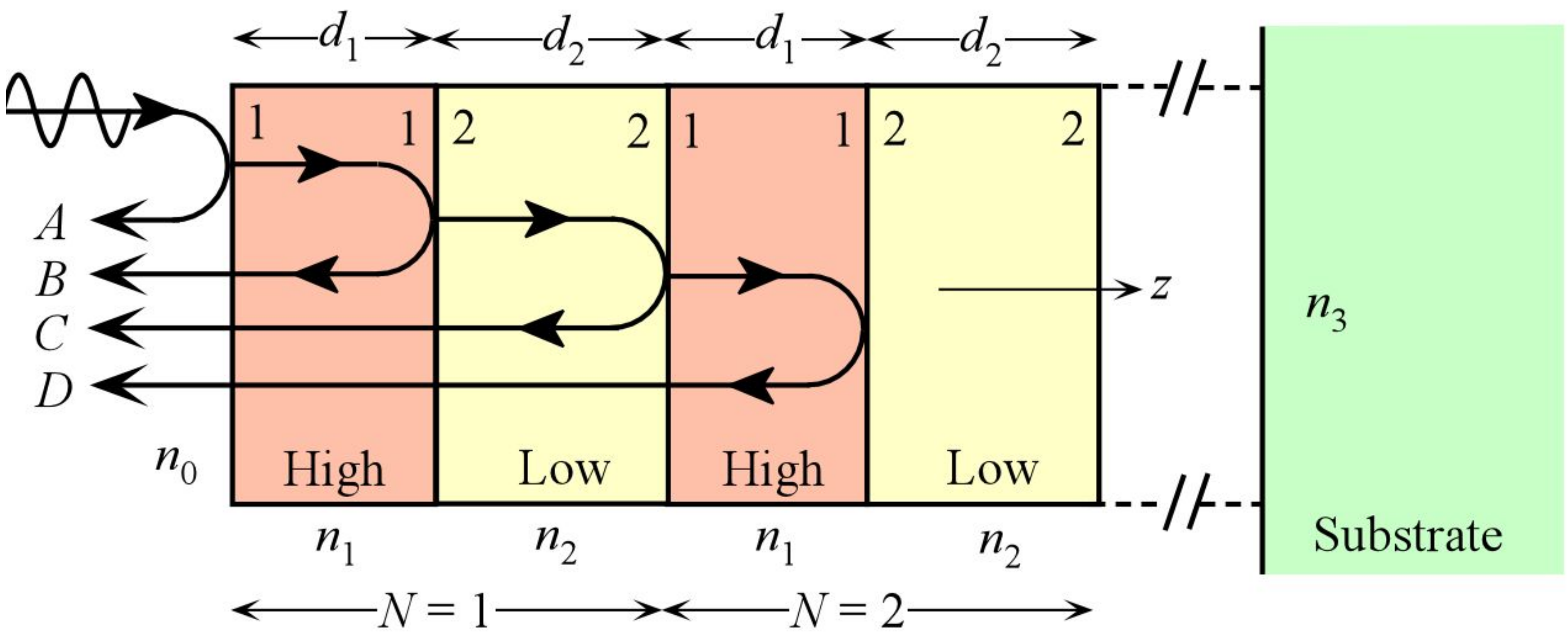
$$d_1 = \lambda/4n_1$$

$$d_2 = \lambda/4n_2$$

Quarter-Wave Stack

$d_1 = \lambda/4n_1$ and $d_2 = \lambda/4n_2$

Dielectric Mirror or Bragg Reflector



$$R_N = \left[\frac{n_1^{2N} - (n_0 / n_3) n_2^{2N}}{n_1^{2N} + (n_0 / n_3) n_2^{2N}} \right]^2$$

$$\frac{\Delta\lambda}{\lambda_o} \approx (4/\pi) \arcsin\left(\frac{n_1 - n_2}{n_1 + n_2} \right)$$

Example: Dielectric Mirror

A dielectric mirror has quarter wave layers consisting of Ta_2O_5 with n_H = 1.78 and SiO_2 with n_L = 1.55 both at 850 nm, the central wavelength at which the mirror reflects light. The substrate is Pyrex glass with an index n_s = 1.47 and the outside medium is air with n_0 = 1. Calculate the maximum reflectance of the mirror when the number N of double layers is 4 and 12. What would happen if you use TiO_2 with n_H = 2.49, instead of Ta_2O_5? Consider the N = 12 mirror. What is the bandwidth and what happens to the reflectance if you interchange the high and low index layers? Suppose we use a Si wafer as the substrate, what happens to the maximum reflectance?

Solution

n_0 = 1 for air, $n_1 = n_H$ = 1.78, $n_2 = n_L$ = 1.55, $n_3 = n_s$ = 1.47, N = 4. For 4 pairs of layers, the maximum reflectance R_4 is

$$R_4 = \left[\frac{(1.78)^{2(4)} - (1/1.47)(1.55)^{2(4)}}{(1.78)^{2(4)} + (1/1.47)(1.55)^{2(4)}}\right]^2 = 0.4 \text{ or } 40\%$$

Solution

N = 12. For 12 pairs of layers, the maximum reflectance R_{12} is

$$R_{12} = \left[\frac{(1.78)^{2(12)} - (1/1.47)(1.55)^{2(12)}}{(1.78)^{2(12)} + (1/1.47)(1.55)^{2(12)}}\right]^2 = 0.906 \text{ or } 90.6\%$$

Now use TiO_2 for the high-n layer with $n_1 = n_H = 2.49$,

R_4 = 94.0% and **R_{12} = 100%** (to two decimal places).

The refractive index contrast is important. For the TiO_2-SiO_2 stack we only need 4 double layers to get roughly the same reflectance as from 12 pairs of layers of Ta_2O_5-SiO_2. If we interchange n_H and n_L in the 12-pair stack, *i.e.* $n_1 = n_L$ and $n_2 = n_H$, the Ta_2O_5-SiO_2 reflectance falls to 80.8% but the TiO_2-SiO_2 stack is unaffected since it is already reflecting nearly all the light.

Solution

We can only compare bandwidths $\Delta\lambda$ for "infinite" stacks (those with $R \approx 100\%$)
For the TiO_2-SiO_2 stack

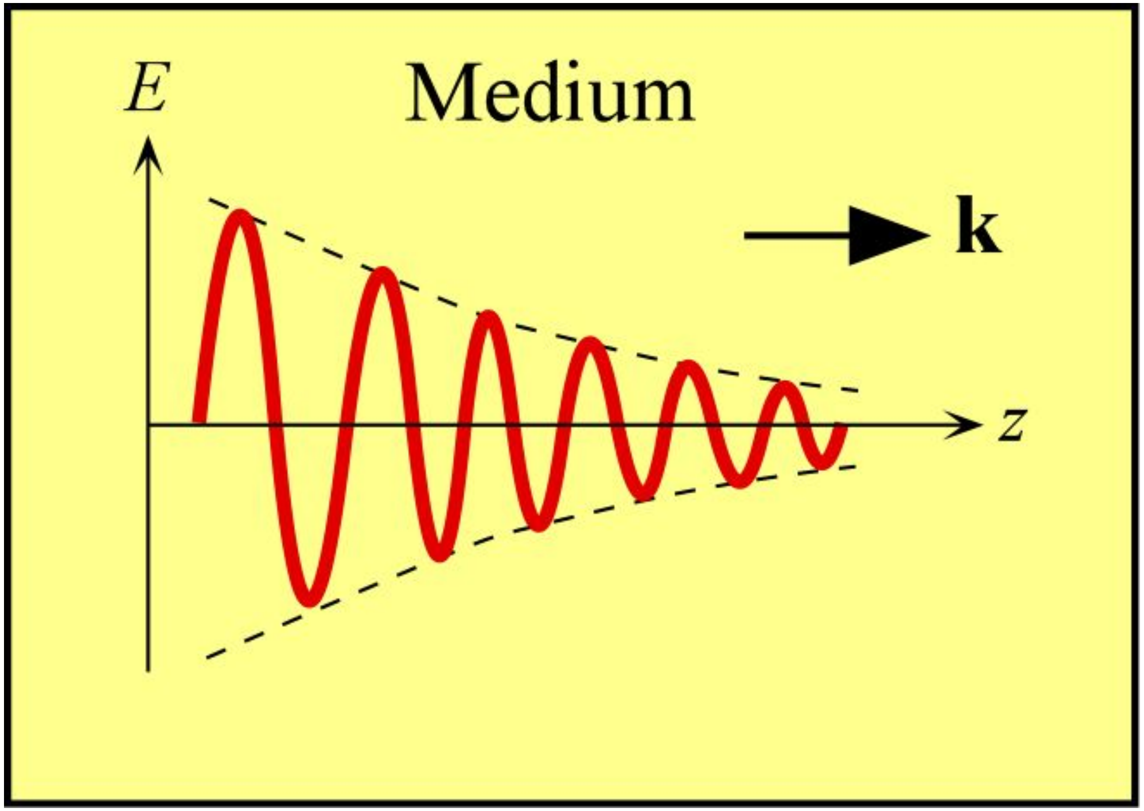
$$\Delta\lambda \approx \lambda_o (4/\pi) \arcsin\left(\frac{n_2 - n_1}{n_2 + n_1}\right)$$

$$\Delta\lambda \approx (850 \text{ nm})(4/\pi) \arcsin\left(\frac{2.49 - 1.55}{2.49 + 1.55}\right) = 254 \text{ nm}$$

For the Ta_2O_5-SiO_2 infinite stack, we get $\Delta\lambda$ =74.8 nm

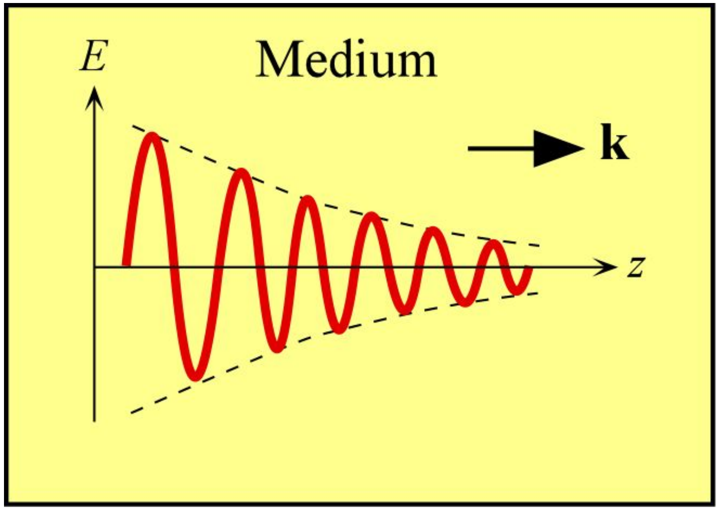
As expected $\Delta\lambda$ is narrower for the smaller contrast stack

Complex Refractive Index



$$\alpha = -\frac{dI}{Idz}$$

Complex Refractive Index



Consider $k = k' - jk''$

$$E = E_o \exp(-k''z)\exp j(\omega t - k'z)$$

$$I \propto |E|^2 \propto \exp(-2k''z)$$

We know from EM wave theory

$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ and $N = \varepsilon_r^{1/2}$

$$N = n - jK = k/k_o = (1/k_o)[k' - jk'']$$

The complex refractive index N with a real part n and imaginary part K is defined as the ratio of the complex propagation constant in a medium to propagation constant in free space.

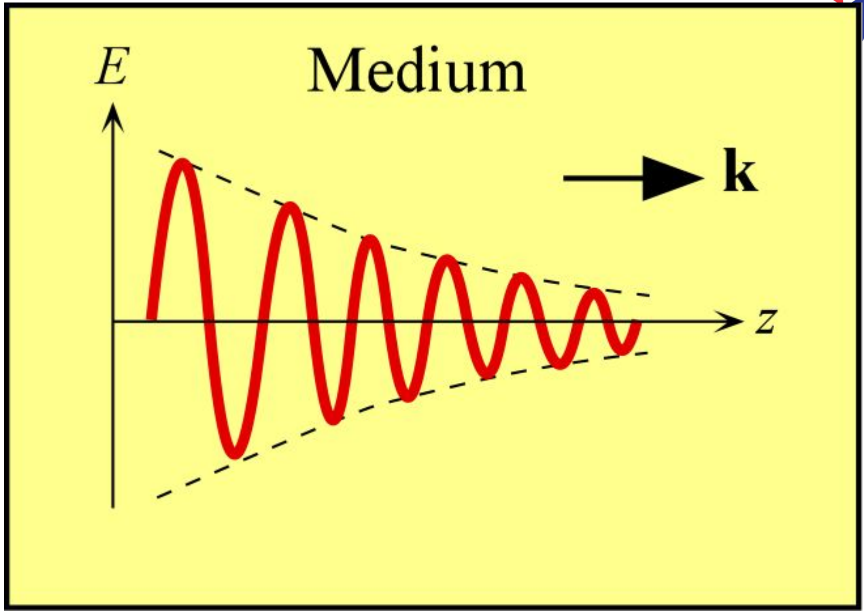
$$N = n - jK = \sqrt{\varepsilon_r} = \sqrt{\varepsilon_r' - j\varepsilon_r''}$$

Reflectance

$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ and $N = \varepsilon_r^{1/2}$

$N = n - jK$

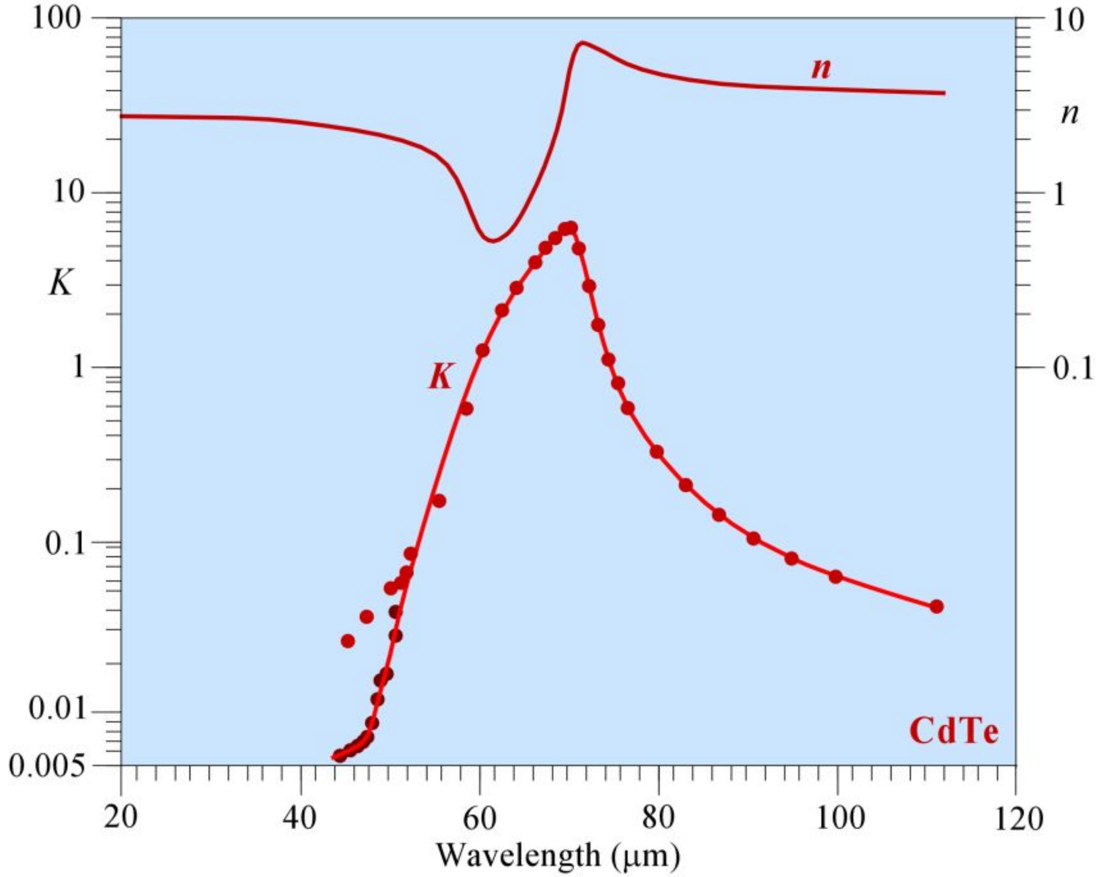
$\boldsymbol{n^2 - K^2 = \varepsilon_r'}$ and $\boldsymbol{2nK = \varepsilon_r''}$



$$R = \left| \frac{n - jK - 1}{n - jK + 1} \right|^2 = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$

The real part n is simply and generally called the refractive index and K is called the extinction coefficient.

Complex Refractive Index for CdTe

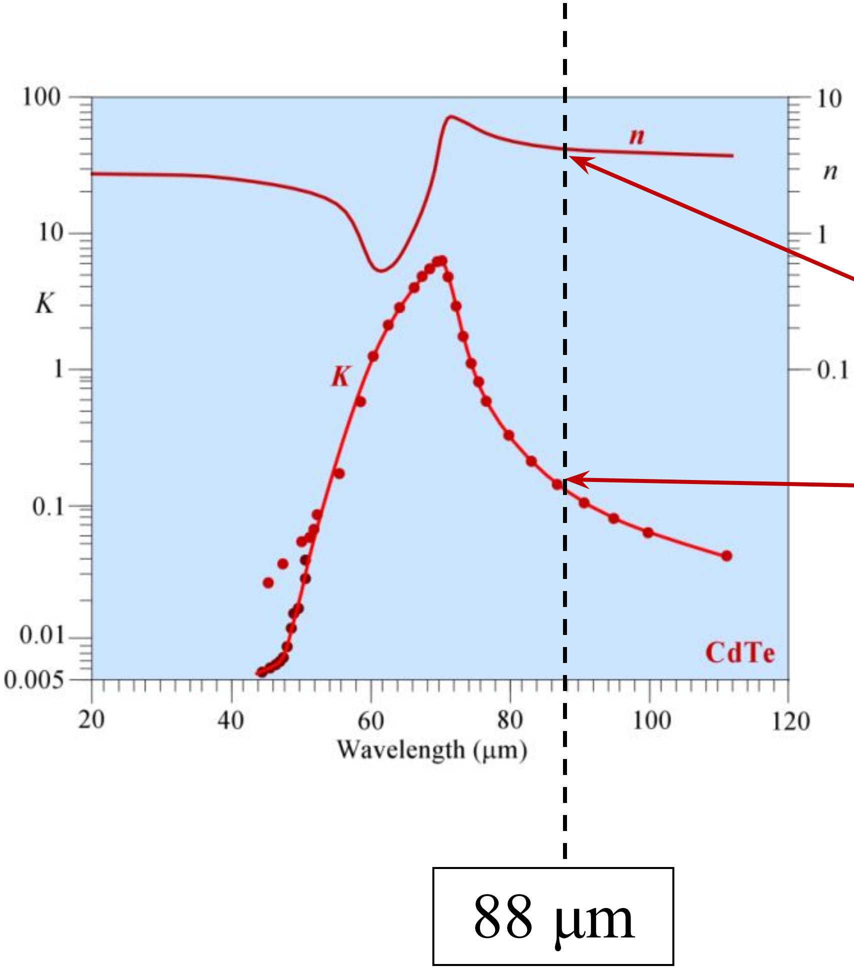


CdTe is used in various applications such as lenses, wedges, prisms, beam splitters, antireflection coatings, windows *etc* operating typically in the infrared region up to 25 μm. It is used as an optical material for low power CO_2 laser applications.

Complex Refractive Index

$$N = n - jK = \sqrt{\varepsilon_r} = \sqrt{\varepsilon_r' - j\varepsilon_r''}$$

$$n^2 - K^2 = \varepsilon_r' \quad \text{and} \quad 2nK = \varepsilon_r''$$



$$R = \left| \frac{n - jK - 1}{n - jK + 1} \right|^2 = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$

Example: Complex Refractive Index for CdTe

Calculate the absorption coefficient α and the reflectance R of CdTe at the Reststrahlen peak, and also at 50 μm. What is your conclusion?

Solution: At the Reststrahlen peak, $\lambda \approx 70$ μm, $K \approx 6$, and $n \approx 4$. The free-space propagation constant is

$$k_o = 2\pi/\lambda = 2\pi/(70\times10^{-6}\text{ m}) = 9.0\times10^{4}\text{ m}^{-1}$$

The absorption coefficient α is $2k$,

$$\alpha = 2k'' = 2k_oK = 2(9.0\times10^{4}\text{ m}^{-1})(6) = \mathbf{1.08\times10^{6}\ m^{-1}}$$

which corresponds to an **absorption depth** $1/\alpha$ of about 0.93 micron.

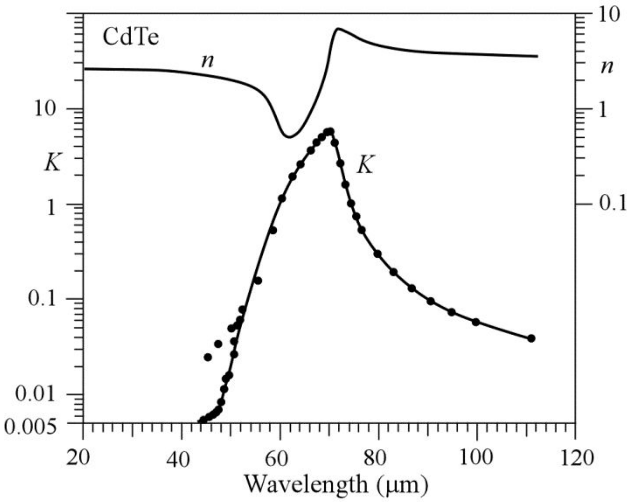
Solution continued: At the Reststrahlen peak, $\lambda \approx 70$ μm, $K \approx 6$, and $n \approx 4$, so that

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} = \frac{(4-1)^2 + 6^2}{(4+1)^2 + 6^2} \approx 0.74 \text{ or } 74\%$$

At $\lambda = 50$ μm, $K \approx 0.02$, and $n \approx 2$. Repeating the above calculations we get

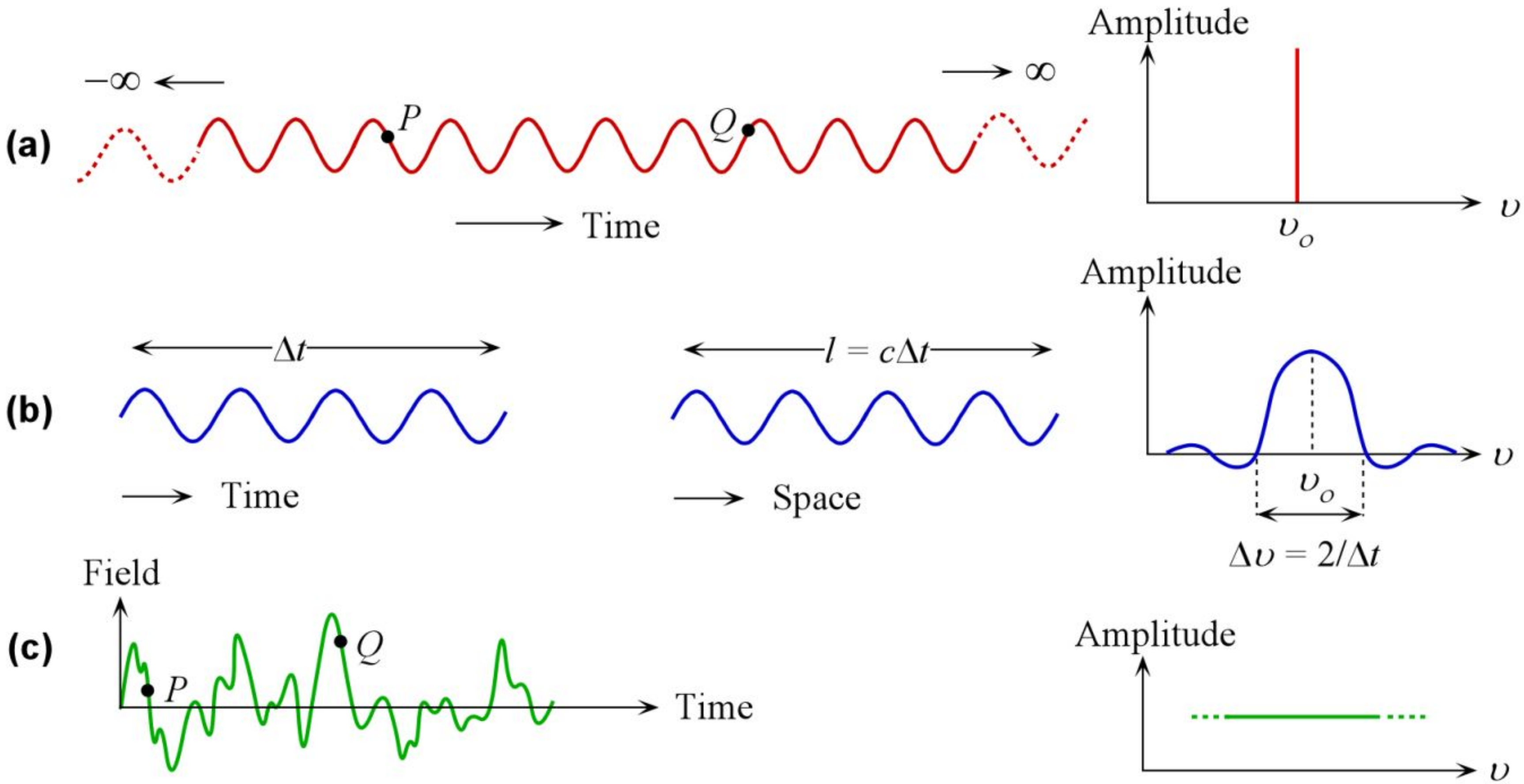
$$\alpha = 5.0 \times 10^3 \text{ m}^{-1}$$

$$R = 0.11 \text{ or } 11\ \%$$



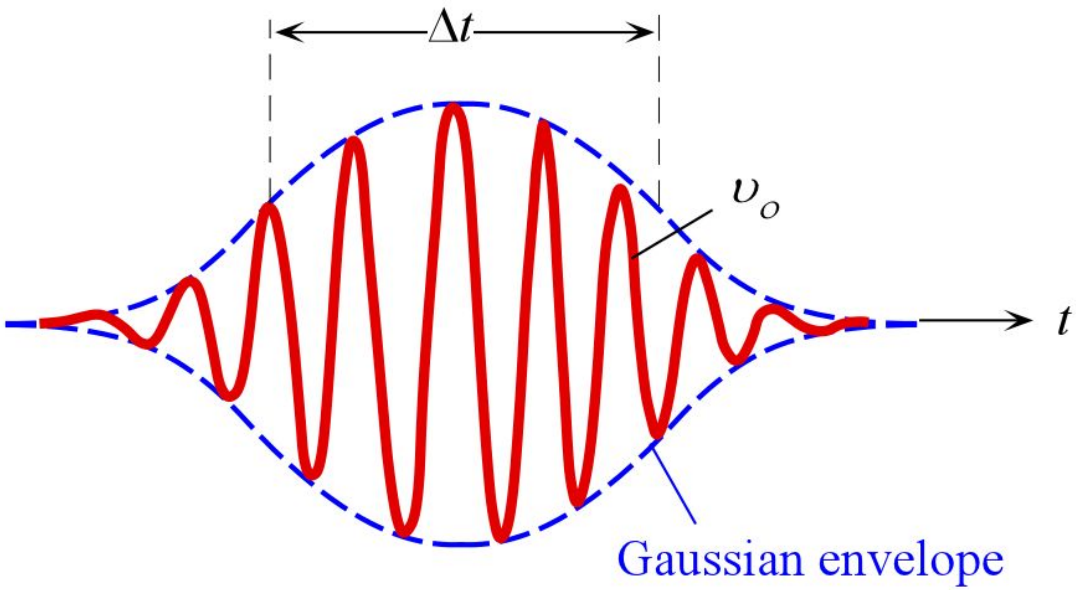
There is a sharp increase in the reflectance from 11 to 72% as we approach the Reststrahlen peak

Temporal and Spatial Coherence

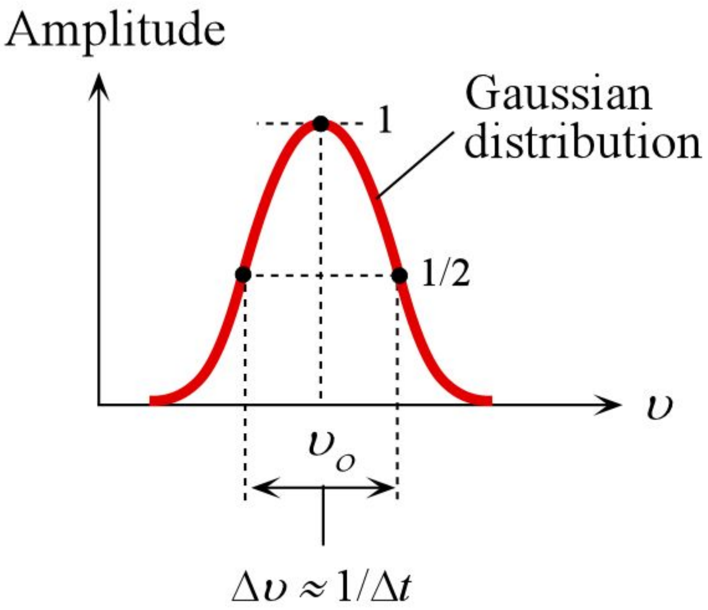


(a) A sine wave is perfectly coherent and contains a well-defined frequency υ_o. (b) A finite wave train lasts for a duration Δt and has a length l. Its frequency spectrum extends over $\Delta\upsilon = 2/\Delta t$. It has a coherence time Δt and a coherence length λ. (c) White light exhibits practically no coherence.

Temporal and Spatial Coherence



Gaussian wave packet

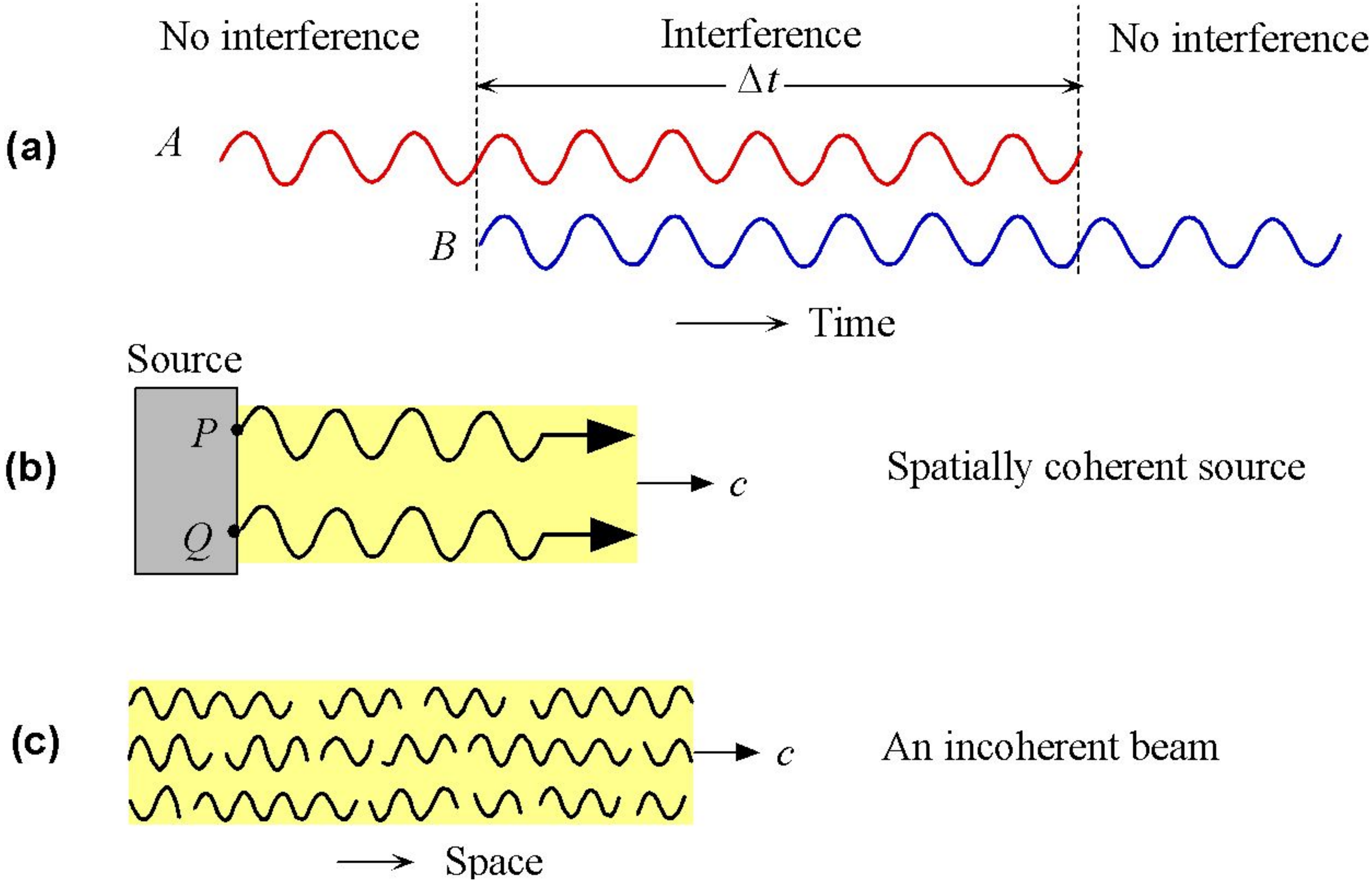


Spectrum

$$\Delta \upsilon \approx \frac{1}{\Delta t}$$

FWHM spreads

Temporal and Spatial Coherence



(a) Two waves can only interfere over the time interval Δt. (b) Spatial coherence involves comparing the coherence of waves emitted from different locations on the source. (c) An incoherent beam

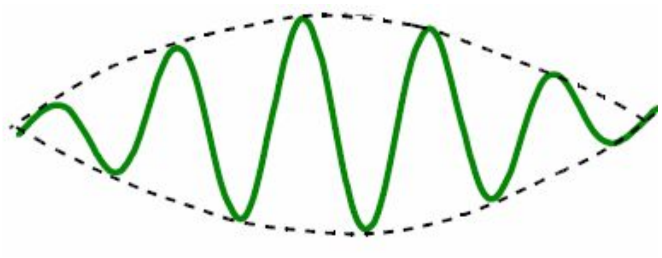
Temporal and Spatial Coherence

Δt = **coherence time**

$l = c\Delta t$ = **coherence length**

For a Gaussian light pulse

$$\Delta \upsilon \approx \frac{1}{\Delta t}$$



Spectral width (→ $\Delta \upsilon$)

Pulse duration (→ Δt)

Temporal and Spatial Coherence

Δt = coherence time

$l = c\Delta t$ = coherence length

$$\Delta \upsilon \approx \frac{1}{\Delta t}$$

Na lamp, orange radiation at 589 nm has spectral width $\Delta \upsilon \approx 5\times10^{11}$ Hz.

$\Delta t \approx 1/\Delta \upsilon = 2\times10^{-12}$ s or 2 ps,

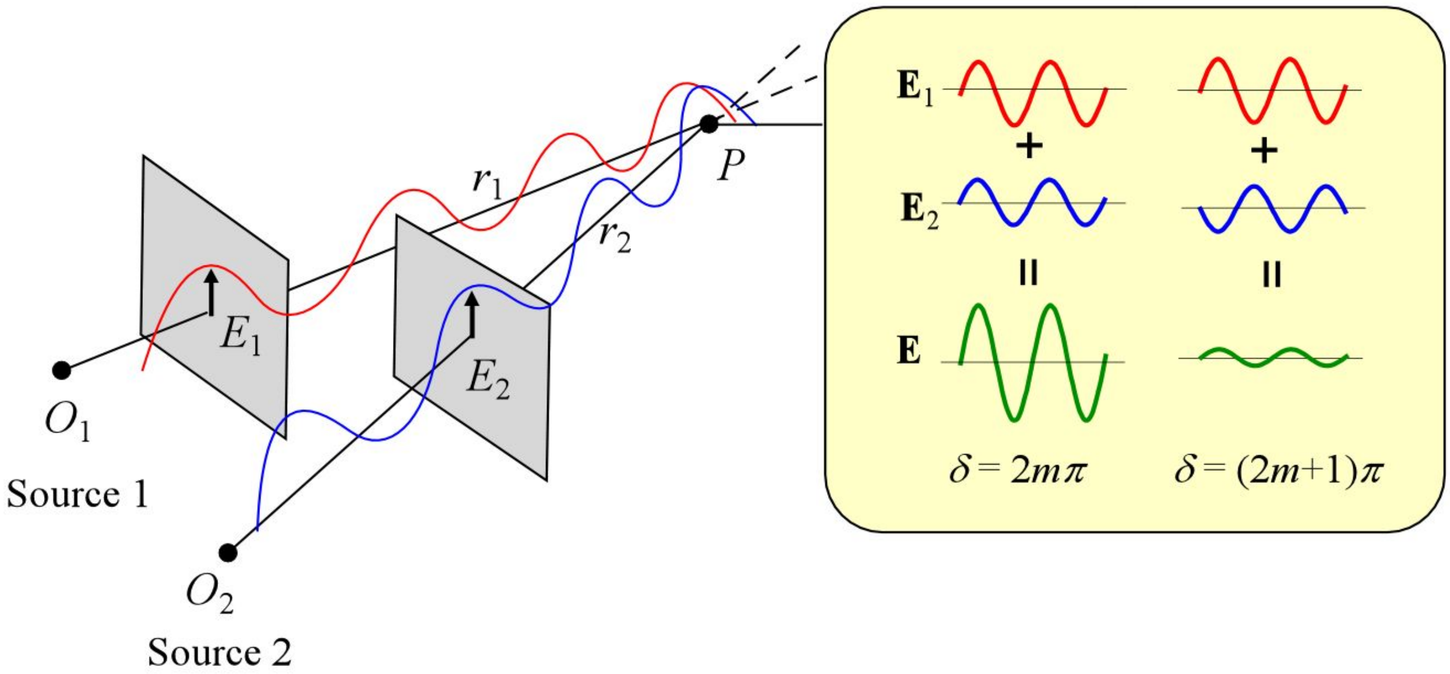
and its coherence length $l = c\Delta t$,

$l = 6\times10^{-4}$ m or 0.60 mm.

He-Ne laser operating in multimode has a spectral width around 1.5×10^{9} Hz, $\Delta t \approx 1/\Delta \upsilon = 1/1.5\times10^{9}$ s or 0.67 ns

$l = c\Delta t = 0.20$ m or 200 mm.

Interference

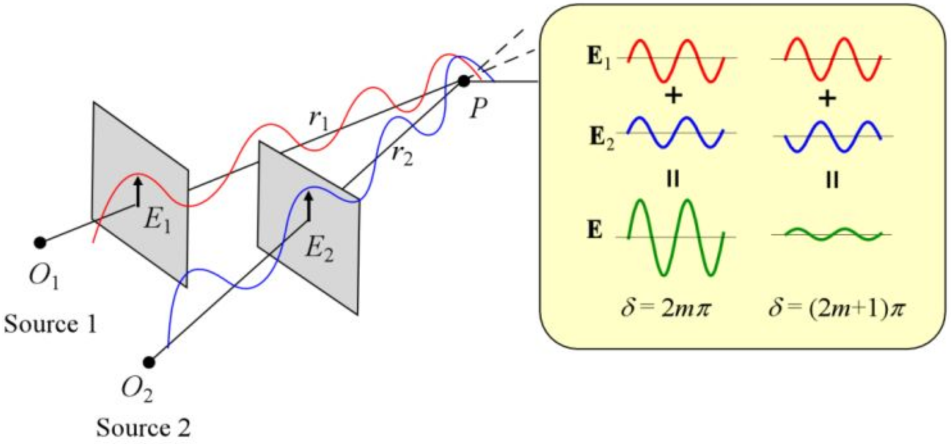


$\mathbf{E}_1 = \mathbf{E}_{o1}\sin(\omega t - kr_1 - \varphi_1)$ and $\mathbf{E}_2 = \mathbf{E}_{o2}\sin(\omega t - kr_2 - \varphi_2)$

Interference results in $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$\overline{\mathbf{E}\cdot\mathbf{E}} = \overline{(\mathbf{E}_1 + \mathbf{E}_2)\cdot(\mathbf{E}_1 + \mathbf{E}_2)} = \overline{\mathbf{E}_1^2} + \overline{\mathbf{E}_2^2} + 2\overline{\mathbf{E}_1\mathbf{E}_2}$$

Interference



Resultant intensity I is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos\delta$$

$$\delta = k(r_2 - r_1) + (\varphi_2 - \varphi_1)$$

Phase difference due to optical path difference

Constructive interference

$$I_{max} = I_1 + I_2 + 2(I_1 I_2)^{1/2}$$

and

Destructive interference

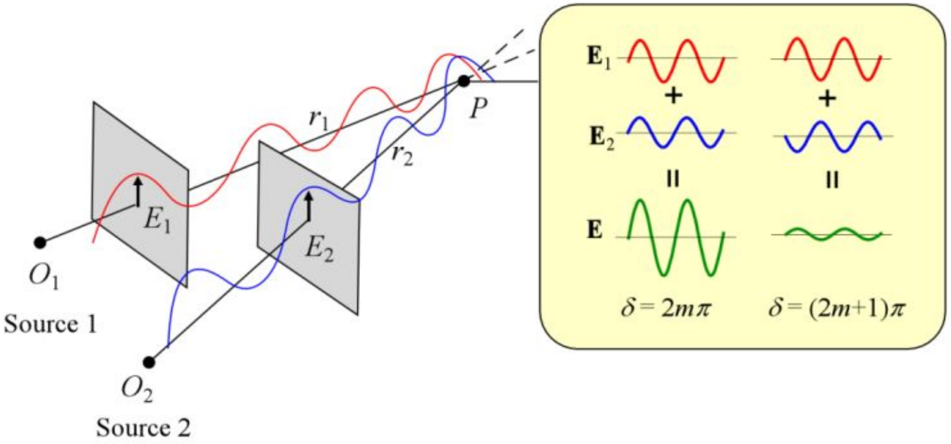
$$I_{min} = I_1 + I_2 - 2(I_1 I_2)^{1/2}$$

If the interfering beams have equal irradiances, then

$$I_{max} = 4I_1$$

$$I_{min} = 0$$

Interference between coherent waves



Resultant intensity I is

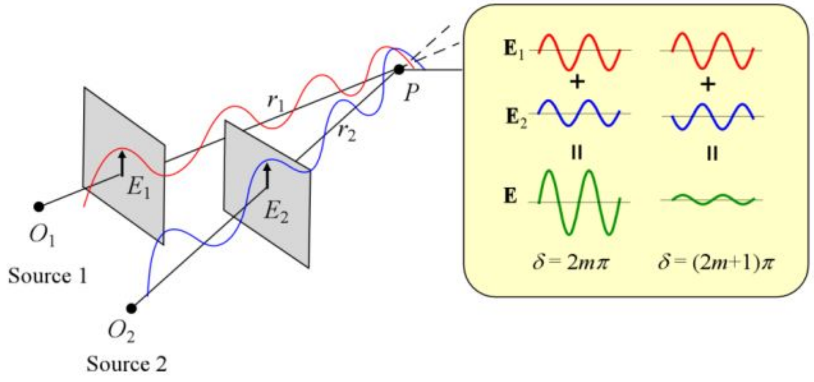
$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos\delta$$

$$\delta = k(r_2 - r_1) + (\varphi_2 - \varphi_1)$$

Interference between incoherent waves

$$I = I_1 + I_2$$

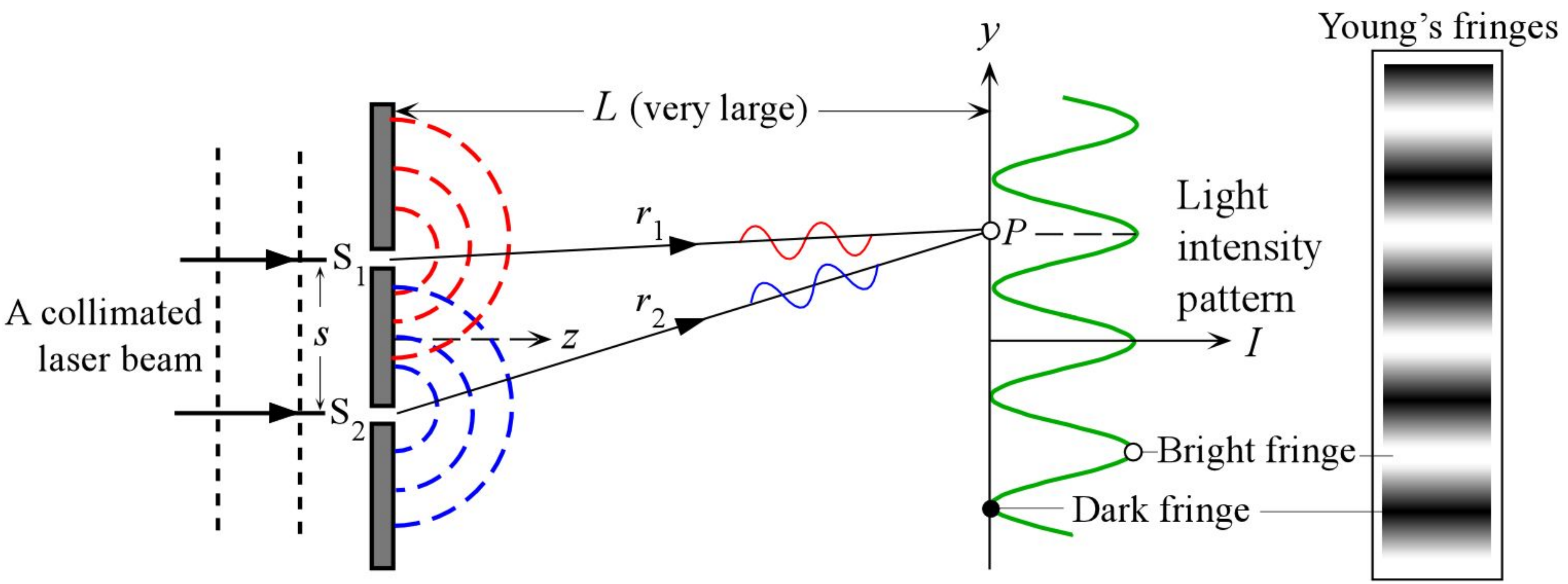
Interference between coherent waves



Resultant intensity *I* is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos\delta$$

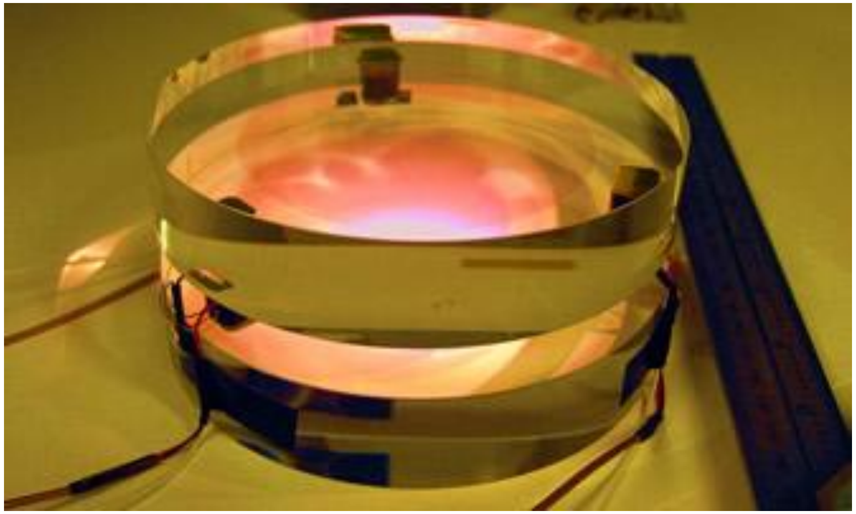
$$\delta = k(r_2 - r_1) + (\varphi_2 - \varphi_1)$$



Optical Resonator

Fabry-Perot Optical Cavity

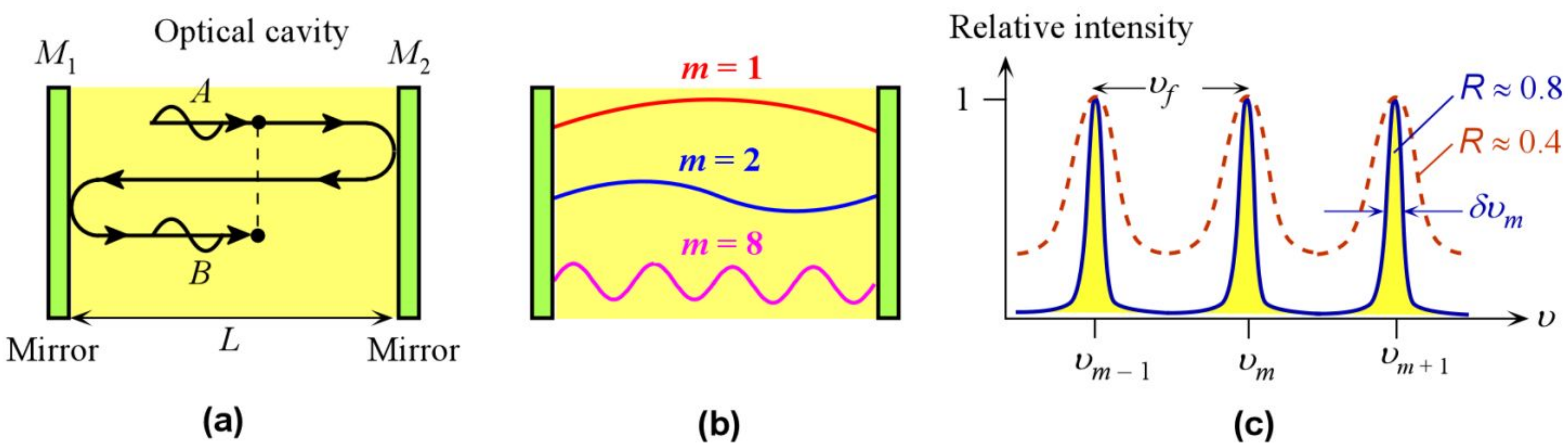
Charles Fabry (1867–1945), left, and Alfred Perot (1863–1925), right, were the first French physicists to construct an optical cavity for interferometry. *(Perot:* The Astrophysical Journal, *Vol. 64, November 1926, p. 208, courtesy of the American Astronomical Society. Fabry: Courtesy of Library of Congress Prints and Photographs Division, Washington, DC 20540, USA.)*



This is a tunable large aperture (80 mm) etalon with two end plates that act as reflectors. The end plates have been machined to be flat to λ/110. There are three piezoelectric transducers that can tilt the end plates and hence obtain perfect alignment. (Courtesy of Light Machinery)

Optical Resonator

Fabry-Perot Optical Cavity

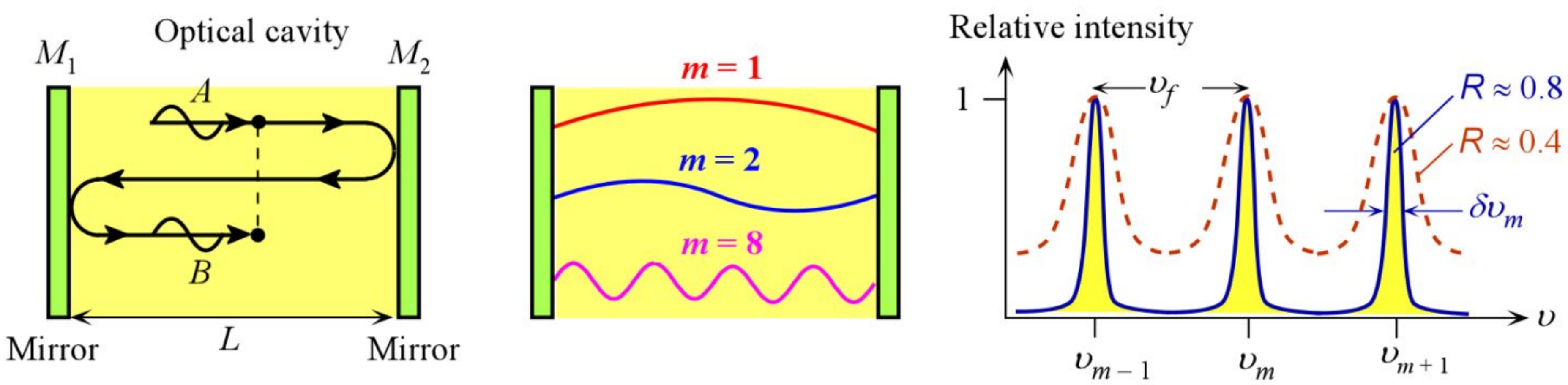


Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, modes, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. R is mirror reflectance and lower R means higher loss from the cavity.

Note: The two curves are sketched so that the maximum intensity is unity

Optical Resonator

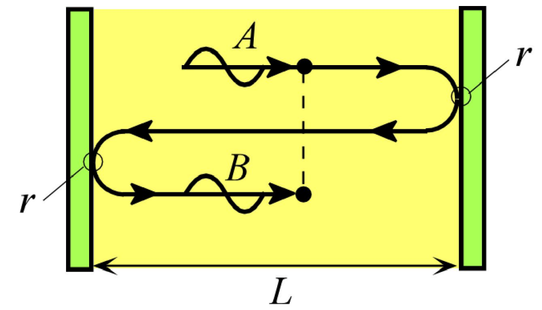
Fabry-Perot Optical Cavity



(a) (b) (c)

Each allowed EM oscillation is a cavity **mode**

Optical Resonator Fabry-Perot
Optical Cavity



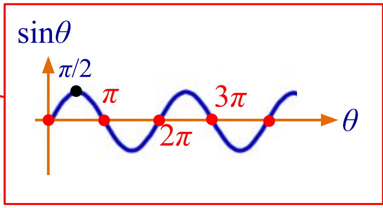
$A + B = A + Ar^2\exp(-j2kL)$

$E_{\text{cavity}} = A + B + \ldots = A + Ar^2\exp(-j2kL) + Ar^4\exp(-j4kL) + Ar^6\exp(-j6kL) + \ldots$

$$E_{\text{cavity}} = \frac{A}{1 - r^2\exp(-j2kL)}$$

$$I_{\text{cavity}} = \frac{I_o}{(1-R)^2 + 4R\sin^2(kL)}$$

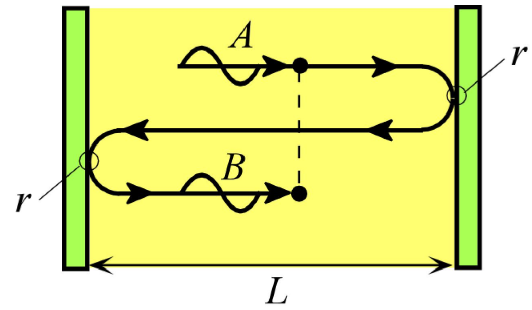
$$I_{\max} = \frac{I_o}{(1-R)^2}$$



$$\text{Maxima at } k_m L = m\pi$$

$$m = 1,2,3,\ldots\text{integer}$$

Optical Resonator Fabry-Perot Optical Cavity



$$I_{cavity} = \frac{I_o}{(1-R)^2 + 4R\sin^2(kL)}$$

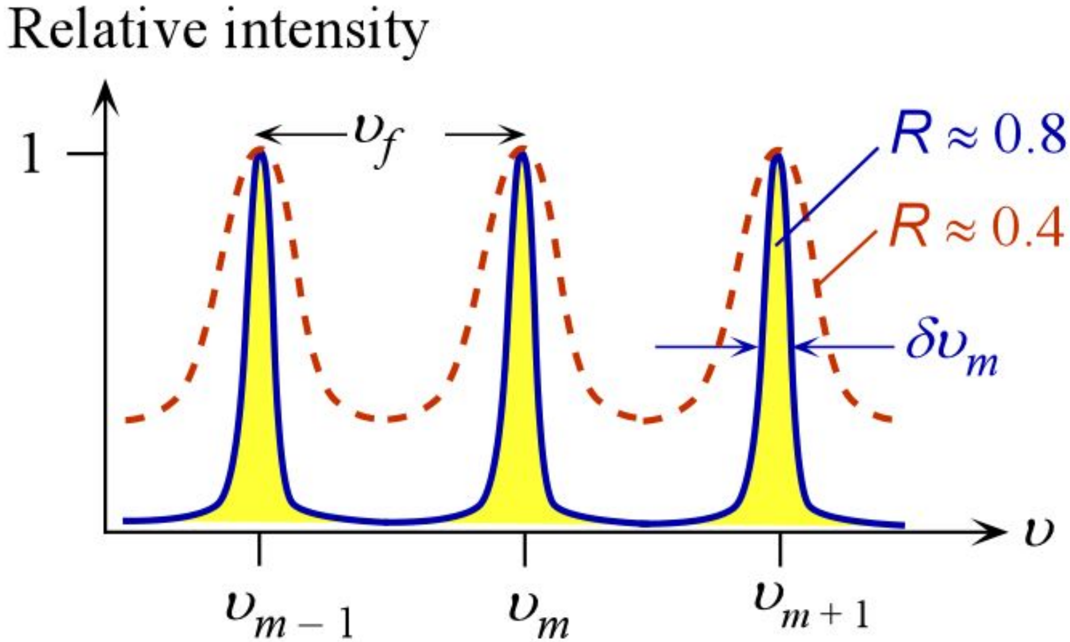
$$I_{max} = \frac{I_o}{(1-R)^2}$$

Maxima at $k_m L = m\pi$

$m = 1,2,3,\ldots$integer

$$(2\pi/\lambda_m)L = m\pi$$

$$m(\lambda_m/2) = L$$



$\upsilon_m = m(c/2L) = m\upsilon_f$ = Mode frequency

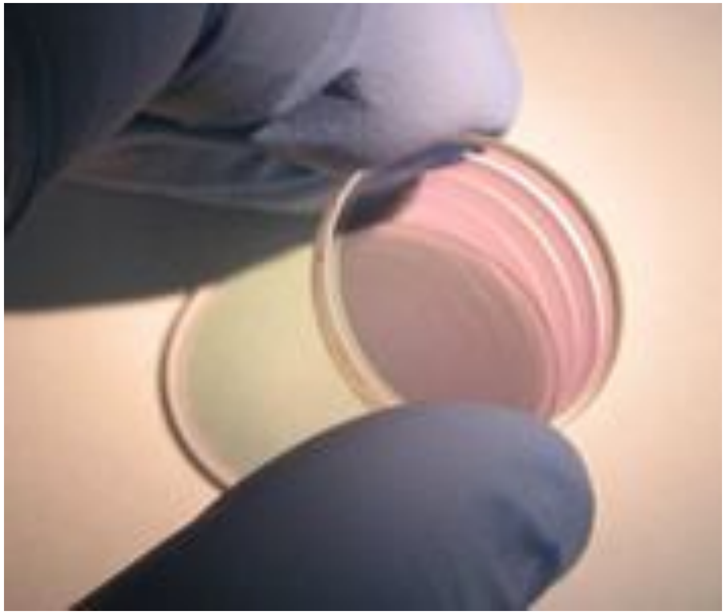
m = integer, 1,2,…

υ_f =free spectral range = $c/2L$ = Separation of modes

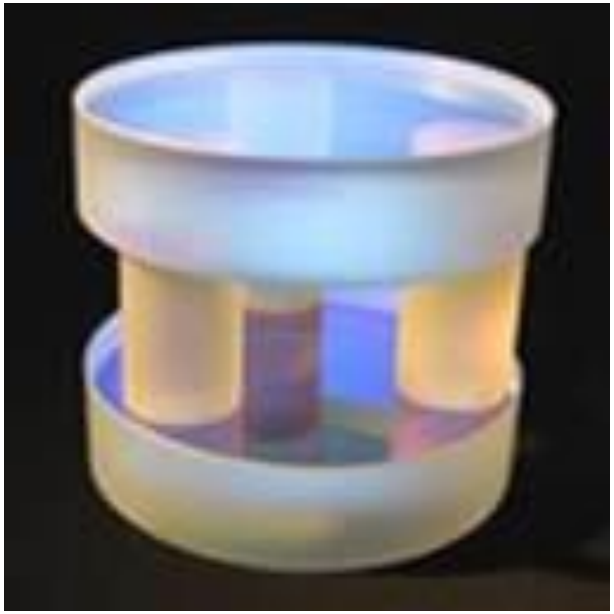
$$\delta\upsilon_m = \frac{\upsilon_f}{F}$$

$$F = \frac{\pi R^{1/2}}{1-R}$$

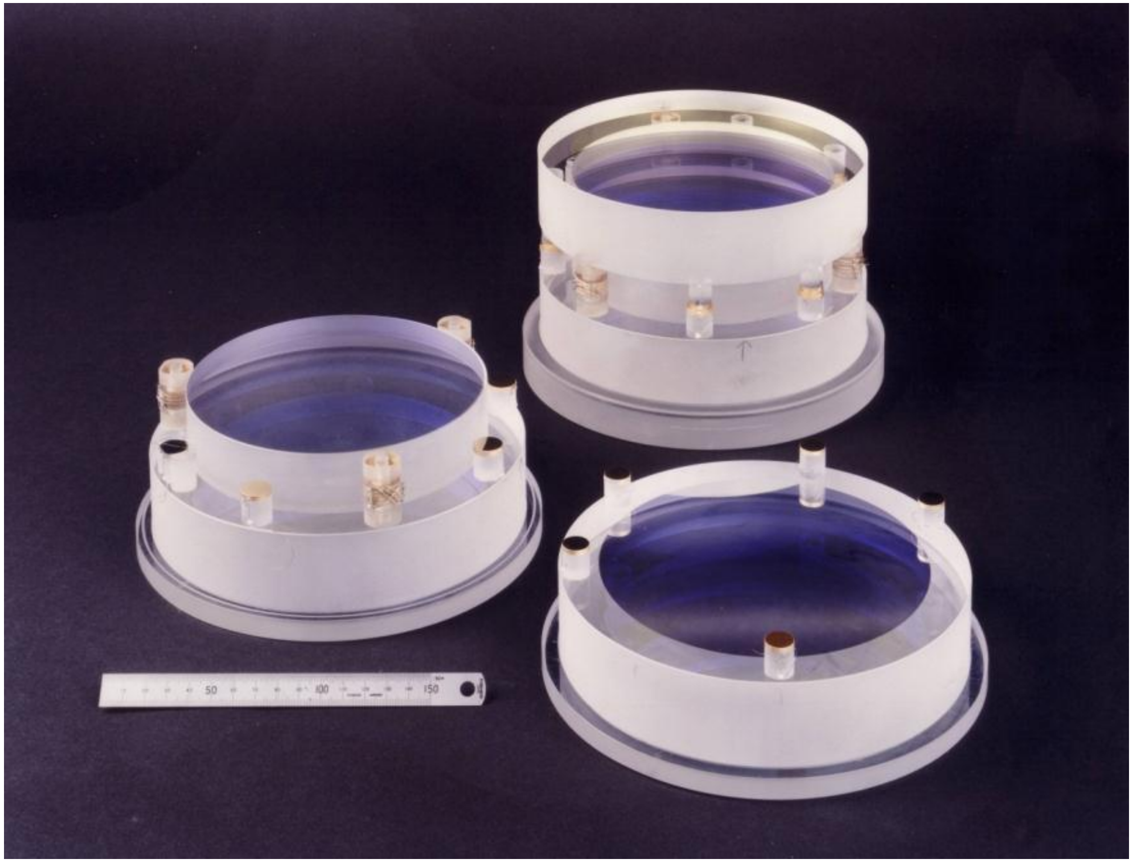
F = Finesse
R = Reflectance ($R > 0.6$)



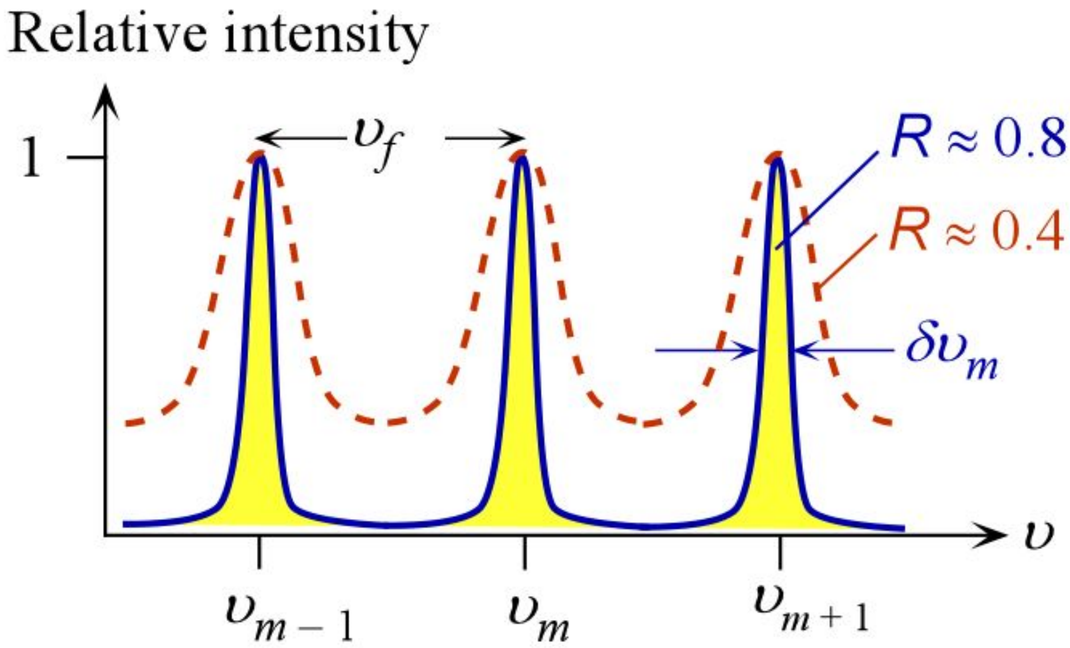
Fused silica etalon
(Courtesy of Light Machinery)



A 10 GHz air spaced etalon
with 3 zerodur spacers.
(Courtesy of Light Machinery)



Fabry-Perot etalons can be made to operate from UV to IR wavelengths with optical cavity spacings from a few microns to many centimeters
(Courtesy of IC Optical Systems Ltd.)

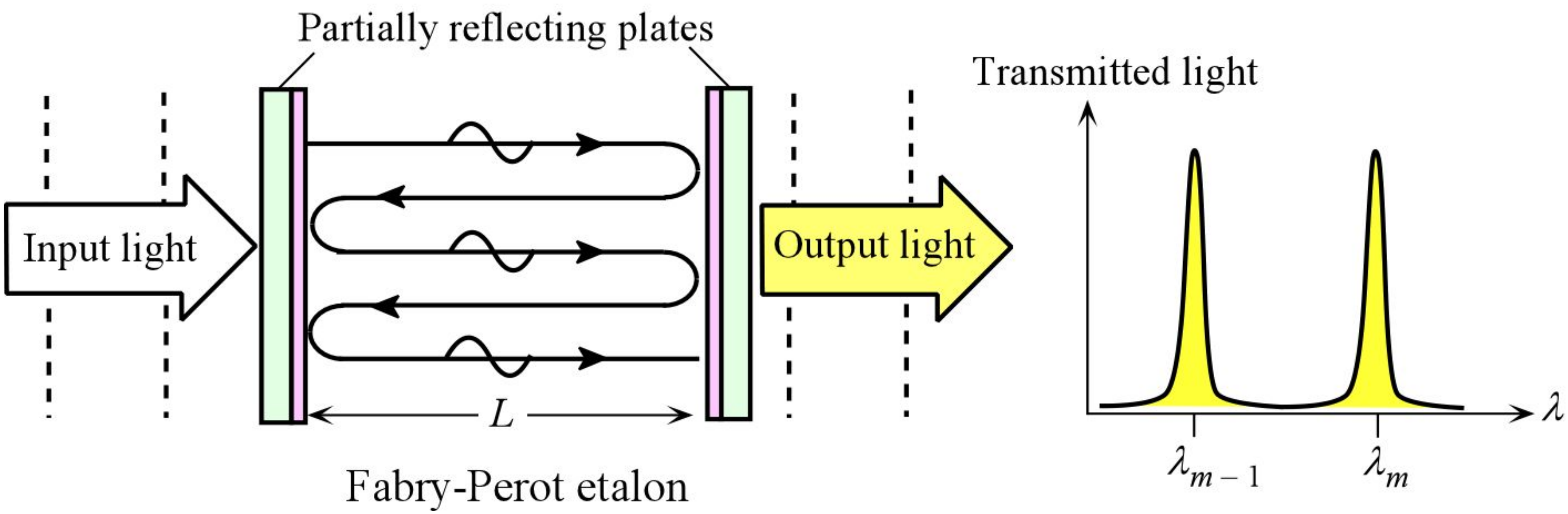


Quality factor Q is similar to the Finesse F

$$Q = \frac{\text{Resonant frequency}}{\text{Spectral width}} = \frac{\upsilon_m}{\delta\upsilon_m} = mF$$

Optical Resonator is also an optical filter

Only certain wavelengths (cavity modes) are transmitted



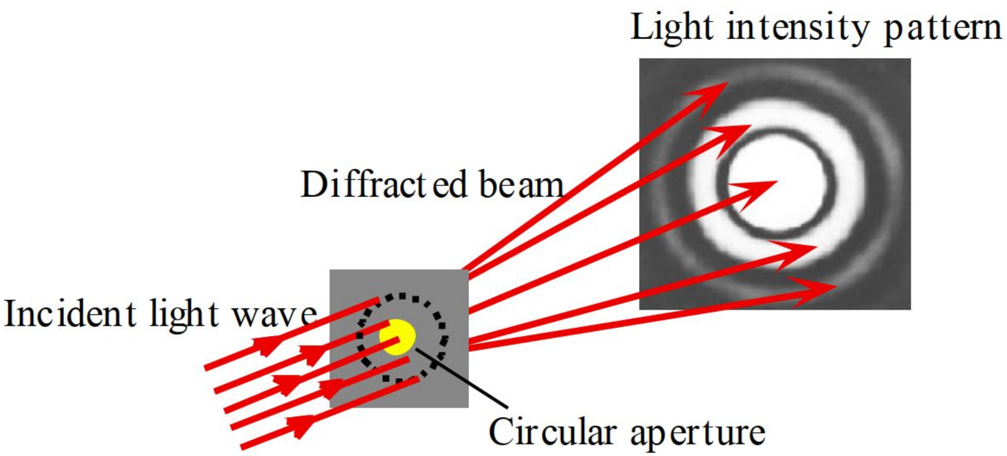
$$I_{\text{transmitted}} = I_{\text{incident}} \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

Introduction to Diffraction

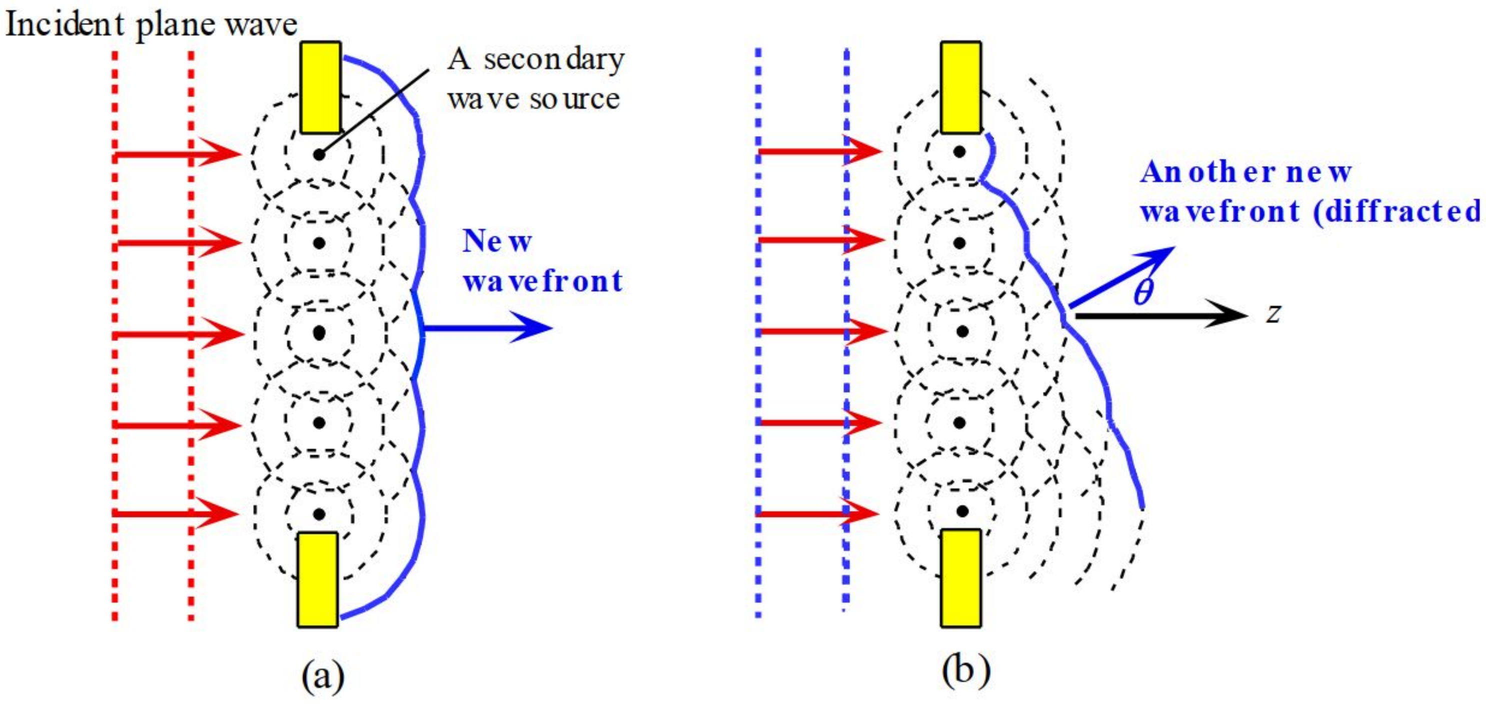
Airy rings are a diffraction pattern clearly visible when light passes through a circular aperture

- The diffracted beam does NOT correspond to the shadow of the aperture
- Instead the light imaged passed the aperture is the result of both light passing through the aperture and light scattered off the edges. The scattered light generates an interference pattern in the image
- Diffracted light from a distance generates the image in a planer wavefront: Fraunhofer Diffraction
- Diffracted light from a near by aperture images the surface with significant wavefront curvature: Fresnel Diffraction

A light beam incident on a small circular aperture becomes diffracted and its light intensity pattern after passing through the aperture is a diffraction pattern with circular bright rings (called Airy rings). If the screen is far away from the aperture, this would be a Fraunhofer diffraction pattern.



Introduction to Diffraction



(a) Huygens-Fresnel principles states that each point in the aperture becomes a source of secondary waves (spherical waves). The spherical wavefronts are separated by λ. The new wavefront is the envelope of the all these spherical wavefronts.
(b) Another possible wavefront occurs at an angle θ to the z-direction which is a diffracted wave.

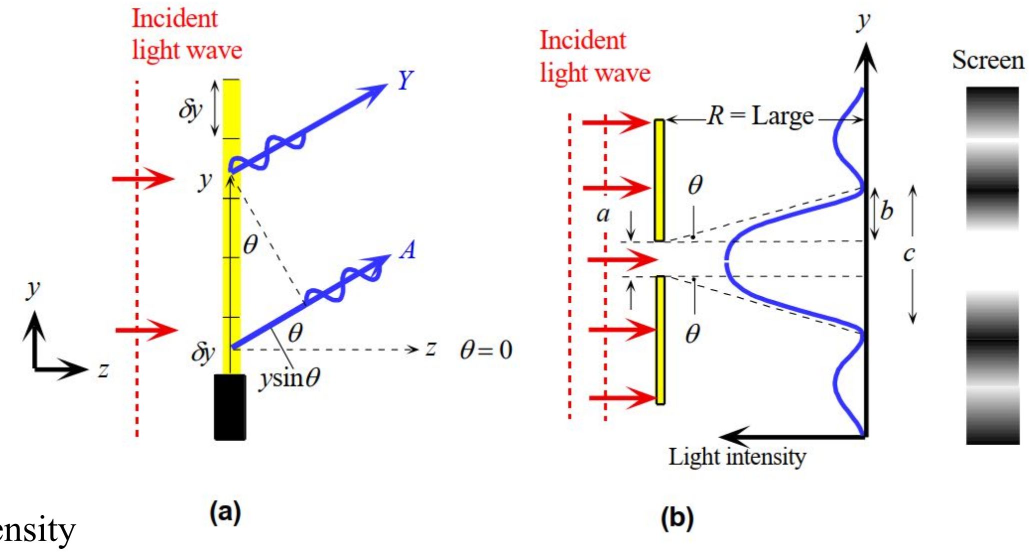
Introduction to Diffraction

Light emitted from a point source

$$E \approx (\delta y)e^{-jk\sin\theta}$$

$$E(\theta) = C\int_{y=0}^{y=a}(\delta y)e^{-jk\sin\theta}$$

$$E(\theta) = \frac{Ce^{-jk\sin\theta}a\sin\left(\frac{1}{2}ka\sin\theta\right)}{\frac{1}{2}ka\sin\theta}$$



The single slit diffraction equation yields an intensity

$$I(\theta) = \left[\frac{C'a\sin\left(\frac{1}{2}ka\sin\theta\right)}{\frac{1}{2}ka\sin\theta}\right]^2 = I(0)\,\mathrm{sinc}^2(\beta); \quad \beta = \tfrac{1}{2}(ka\sin\theta)$$

With zero intensity points at

$$\sin\theta = \frac{m\lambda}{a}$$

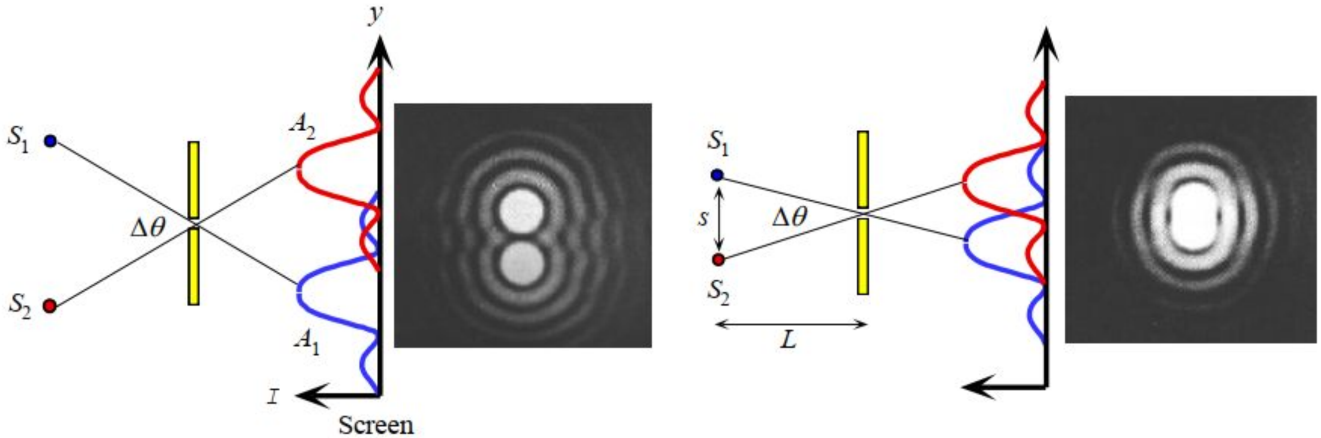
$$m = \pm1, \pm2, \ldots$$

$$\sin\theta = 1.22\frac{\lambda}{D}$$

where D is the diameter of the aperture

(a) The aperture is divided into N number of point sources each occupying δy with amplitude ∝ δy.
(b) The intensity distribution in the received light at the screen far away from the aperture: the diffraction pattern

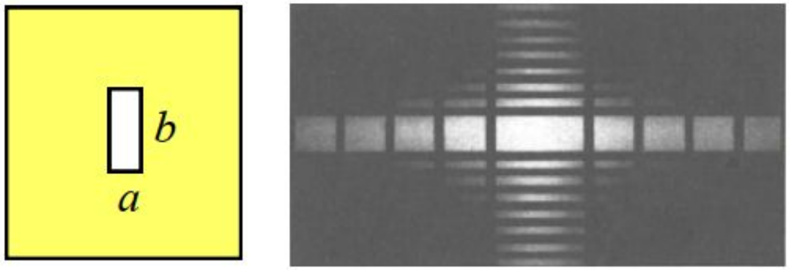
Image Resolution



Resolution of imaging systems is limited by diffraction effects. As points S_1 and S_2 get closer, eventually the Airy disks overlap so much that the resolution is lost.

According to the Rayleigh criterion, the two spots are just observable when the principle maximum of one diffraction pattern coincides with the minimum of another. This minimum is obtained by the angular radius of the Airy disk, with D is the diameter of the aperture:

$$\sin\theta = 1.22\frac{\lambda}{D}$$



The rectangular aperture of dimensions a × b on the left gives the diffraction pattern on the right.

Diffraction Gratings

Bragg Diffraction Condition

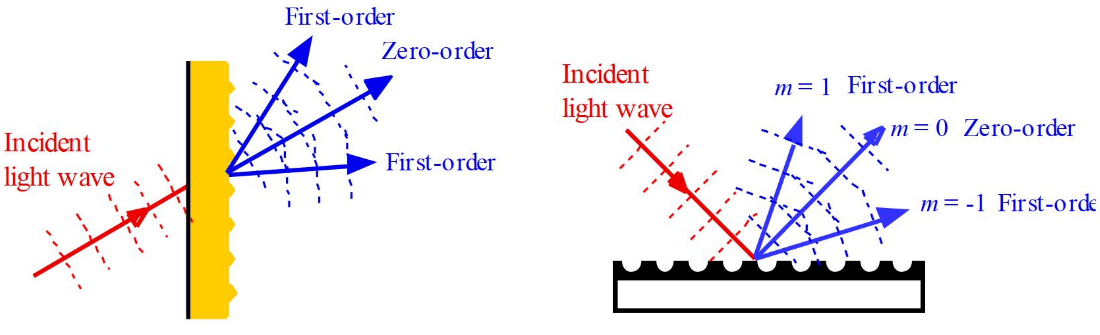
$$d \sin\theta = m\lambda$$

$$m = 0, \pm1, \pm2, \pm3, \ldots$$

For light incident at an angle

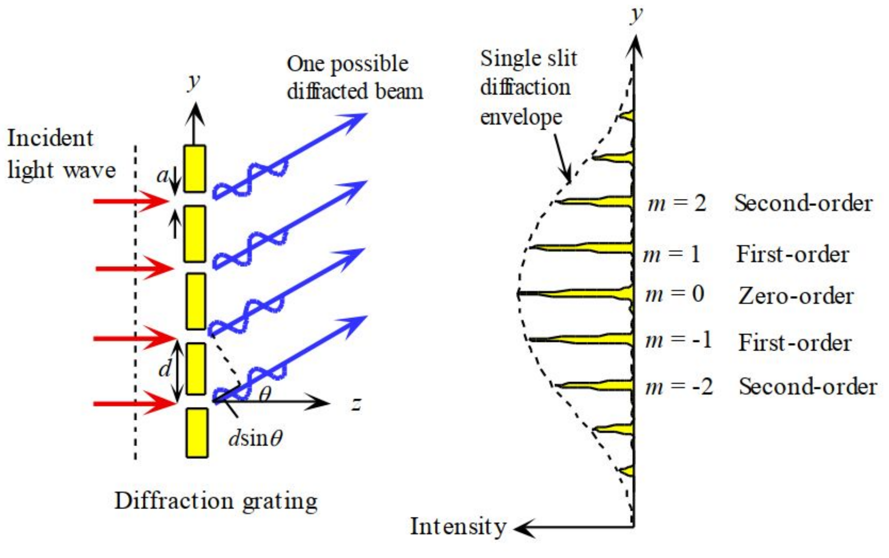
$$d(\sin\theta_m + \sin\theta_i) = m\lambda$$

$$m = 0, \pm1, \pm2, \pm3, \ldots$$



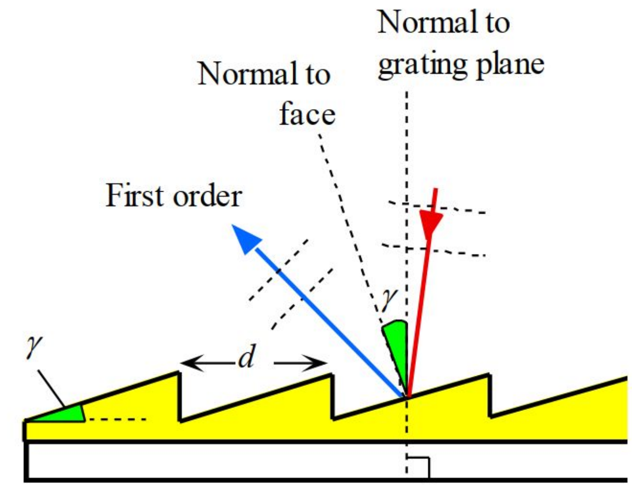
(a) Transmission grating (b) Reflection grating

(a) Ruled periodic parallel scratches on a glass serve as a transmission grating. (b) A reflection grating. An incident light beam results in various "diffracted" beams. The zero-order diffracted beam is the normal reflected beam with an angle of reflection equal to the angle of incidence.



(a) (b)

(a) A diffraction grating with N slits in an opaque scree. (b) The diffracted light pattern. There are distinct beams in certain directions (schematic)



Blazed (echelette) grating.

Thank you!