Michelson interferometer - high resolution (Item No.: P2220910)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physics	University	Light and Optics	Diffraction and Interference	Michelson interferometer - high resolution

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	2 Hours	1 Hour	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

interference, wavelength, diffraction index, speed of light, phase, virtual light source

Introduction

Overview

Principle

With the aid of two mirrors in a Michelson arrangement, light is brought to interference. While moving one of the mirrors, the alteration in the interference pattern is observed and the wavelength of the laser light determined.

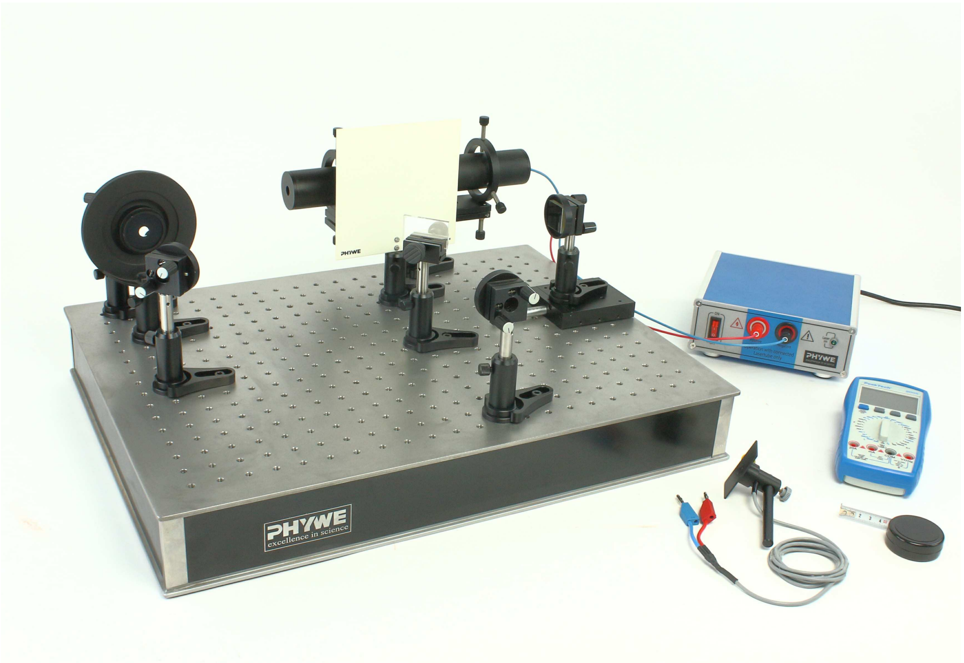

Fig. 1a. Experimental set-up of the Michelson interferometer.

Equipment

Position No.	Material	Order No.	Quantity
1	Optical base plate 450 x 600 mm	08750-00	1
2	He-Ne Laser, 632 nm, 1 mW, linear polarised	08182-93	1
3	Surface mirror 30 x 30 mm	08711-01	4
4	Accesory set for optical base plate	08750-50	1
5	Holder for diaphragms and beam splitters	08719-00	1
6	Beam splitter 1/1, non polarizing	08741-00	1
7	Lens, mounted, f +20 mm	08018-01	1
8	Lensholder for optical base plate	08723-00	1
9	Screen, white, 150x150 mm	09826-00	1
10	linear translation stage, 25 mm	08750-09	1
11	Photoelement	08734-00	1
12	Digital multimeter 2005	07129-00	1
13	Measuring tape, l = 2 m	09936-00	1
14	Adjusting support 35 x 35 mm	08711-00	4

*alternative to Photoelement and Digital multimeter: Digital array camera, Order No. 35612-99 with a connection to a PC/Laptop with at least Windows XP. Take caution during set up due to the size of the camera.

Task

1. Construction of a Michelson interferometer using separate components.
2. The interferometer is used to determine the wavelength of the laser light.
3. The contrast function *K* is qualitatively recorded in order to determine the coherence length with it.

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Set-up and procedure

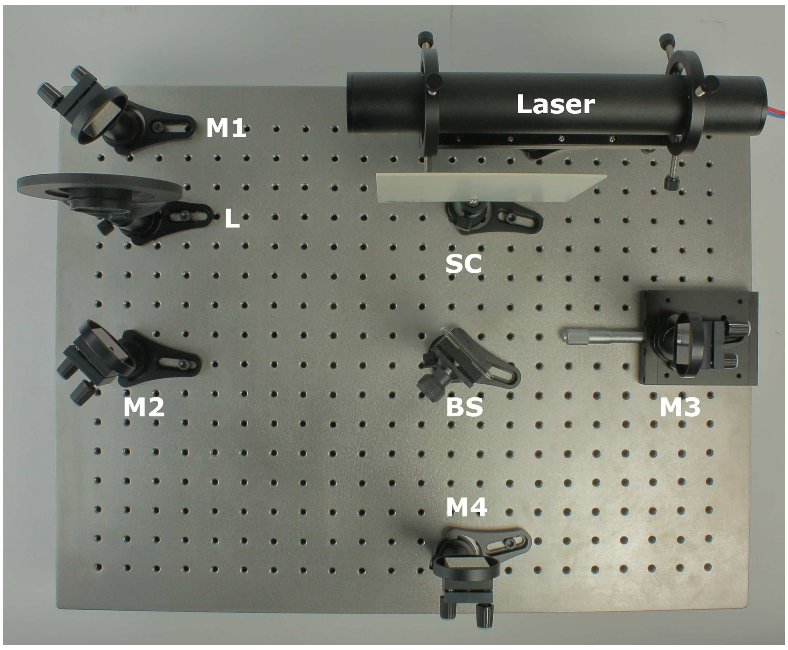

Fig. 1b: Final set-up, top view.

The experimental set-up is shown in Figs. 1a and 1b, which serve as a rough guideline. The recommended set-up height (beam path height) is 120 mm. A ruler may be used to help measure the beam height and maintain such a height after placing each subsequent optical component on the base plate.

Note: Once the opitcal component is mounted on a foot and correctly positioned on the optical base plate, tightly clamp the foot to avoid unwanted movement.

Caution: Never look directly into a non attenuated laser beam.

- The lens **L** must not be in position when making the initial adjustments.
- When adjusting the beam path with the adjustable mirrors $\mathbf{M_1}$ and $\mathbf{M_2}$, the beam path is aligned straight along the base plate until it reaches $\mathbf{M_3}$ (done initially without the beam splitter **BS**).
  $\mathbf{M_3}$is mounted on the linear translation stage as seen in Figs. 1a and 1b.
- Adjust the mirror $\mathbf{M_3}$, again without the beam splitter **BS**, such that the reflected beam strikes the same point on mirror $\mathbf{M_2}$ from which it previously originated.
- Now, place the beam splitter **BS** with its metallized side facing mirror $\mathbf{M_2}$ in the beam path in such a manner that a partial beam strikes mirror $\mathbf{M_3}$ unchanged and the other partial beam strikes mirror $\mathbf{M_4}$ perpendicularly along the base plate.
- The beam which is reflected by mirror $\mathbf{M_4}$ must now be adjusted with the adjusting screws such that it strikes the same point on screen **SC** as the partial beam that originated at mirror $\mathbf{M_3}$ and was subsequently reflected by the beam splitter **BS**. A slight flickering of the luminous points which have been made to coincide indicates nearly exact adjustment.
- By placing the lens **L** in the beam path the luminous points are expanded.
- Now observe the interference patterns on screen **SC** (stripes, circles).
- By meticulously readjusting the mirrors $\mathbf{M_3}$ and $\mathbf{M_4}$ using the adjusting screws, one obtains concentric circles.

On determining the wavelength of the laser light:

- To perform this measurement, the path distance between the mirror $\mathbf{M_3}$ and the beam splitter **BS** must be changed. In the process, the position of mirror $\mathbf{M_3}$ is altered using a lever arm (lever transmission ratio approx. 20:1) and a micrometer screw (2 turns correspond to 1 mm), and thus the optical path length of the light beam is also changed.
- On changing the optical path lengths, one sees changes in the centre of the interference rings from maxima to minima and vise versa. Whether the path length increases or decreases becomes apparent in the following: for decreasing path length, the centre represents a source of maxima and minima; or for increasing path length it is a sink for the interference maxima and minima.
- According to the theory a change from minimum to minimum occurs when the optical path length $\lambda \cdot d$ is changed by λ, i.e. in the set-up used the distance between the beam splitter **BS** and the mirror $\mathbf{M_3}$ changes by $\lambda/2$.

- To determine the wavelength of laser light, the changes in the distance between $\mathbf{M_3}$ and **BS** are measured (by reading the initial and final values on the micrometer screw) and the number of changes from minimum to minimum (or maximum to maximum) are counted.

On recording the contrast function:

- In this case, the screen **SC** is replaced by a photo cell **PD** for the determination of the contrast function *K.* To ensure that the photocell does not measure the intensity across different maxima and minima of the circular interference fringes, reduce the size of the slotted diaphragm with black tape such that only a small aperture of approximately 1 mm^2 remains in the middle.
- For this part of the experiment, make the room as dark as possible to keep the dark current of the photocell as low as possible.
- To determine the contrast function, measure the intensities of minima and maxima by varying the optical separation of the mirrors. Change the separation using only mirror $\mathbf{M_4}$. This mirror is only to be moved along a straight path.
- Measure the distance between mirrors and beam splitter with a measuring tape. During the repositioning procedure, the mirror must be readjusted at each new position (if necessary, initially without the lens, see above) such that the interference fringes again become visible.
- To measure the intensities of minima and maxima, alter the position of the mirror $\mathbf{M_3}$ slightly using the micrometer screw so that one can see which minimal and maximal voltage values can be measured with the multimeter (measuring range approx. 500 mV).
- The difference in optical path length between the two mirrors and the beam splitter should be varied between 0 and 10 cm: i.e. when the distance from $\mathbf{M_3}$ to the beam splitter is approximately 13 cm, mirror $\mathbf{M_4}$ should be at its minimum distance of approximately 8 cm from the beam splitter **BS** and at its maximum separation of circa 13 cm from it.
- In the process, one must take into consideration that the larger the separation differences are, the smaller the radii of the circular interference fringes are. Consequently, at large separation differences the measurement of the maximum and minimum intensities is uncertain and, as a result of the relatively large diaphragm aperture, subject to large errors.

Theory and evaluation

If two waves having the same frequency ω but different amplitudes and different phases are coincident at one location, they superimpose to:

$$E(t) = A_1 \cdot \sin(\omega t - \varphi_1) + A_2 \cdot \sin(\omega t - \varphi_2).$$

The resulting wave can be described by the following:

$$E(t) = A_1 \cdot \sin(\omega t - \varphi)$$

with the amplitude

$$E^2(t) = A_1^2 + A_2^2 + 2 \cdot A_1 \cdot A_2 \cdot \cos\delta \qquad (1)$$

and the phase difference

$$\delta = \varphi_1 - \varphi_2.$$

In a Michelson interferometer, light is split by a semi-transparent glass plate into two partial beams (amplitude splitting), reflected by two mirrors, again brought to interference behind the glass plate (Fig. 2). Since only extensive luminous spots can exhibit circular interference fringes, the light beam is expanded with lens **L**.

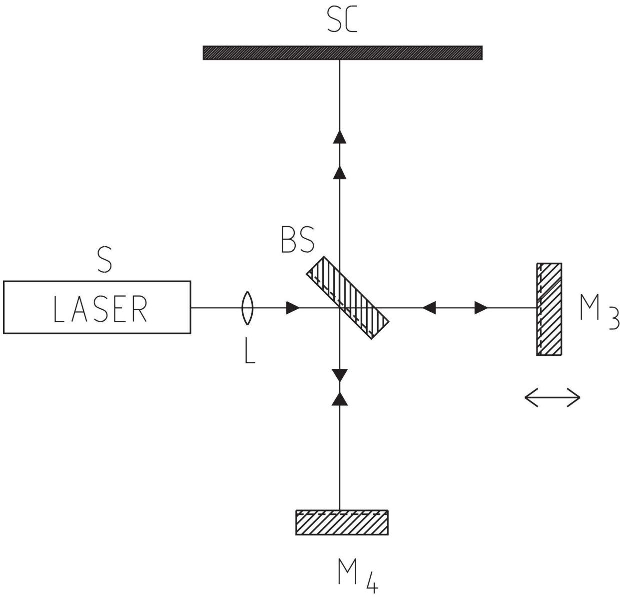

Fig. 2: Michelson arrangement for Interference. S represents the light source; SC the detector (or the position of the screen).

If one replaces the real mirror $\mathbf{M_3}$ between the laser and the glass plate by a virtual image $\mathbf{M_3}$**'**, which is formed by reflection at the glass plate, a point **P** of the light source appears as the points **P'** and **P"** of the virtuallight sources $\mathbf{L_1}$ and $\mathbf{L_2}$.

Due to the different light paths, using the designations in Fig. 3 the phase difference is given by:

$$\delta = \frac{2\pi}{\lambda} \cdot 2 \cdot d \cdot \cos\theta \qquad (2)$$

λ is the wavelength of the laser light used.

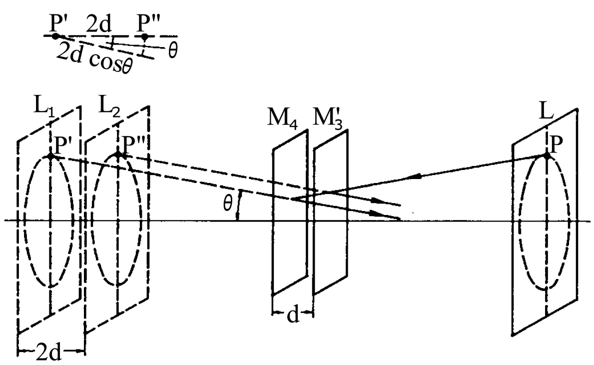

Fig. 3: Formation of circular interference fringes.

According to (1), the intensity distribution for $A_1 = A_2 = A$

$$I = E^2 = 4 \cdot A^2 \cos^2 \frac{\delta}{2} \qquad (3)$$

Maxima thus occur when δ is equal to a multiple of 2π, hence with (2)

$$2 \cdot d \cdot \cos\theta = m \cdot \lambda; m = 1, 2, \ldots. \qquad (4)$$

i.e., there exist circular fringes for selected fixed values of *m,* and *d,* since θ remains constant (see Fig. 3). If one alters the position of the movable mirror $\mathbf{M_3}$ (cf. Fig.1) such that *d,* e.g., decreases, according to (4), the circular fringe diameter would also diminish since *m* is indeed defined for this ring. Thus, a ring disappears each time *d* is reduced by $\lambda/2$. At *d* = 0 the circular fringe pattern disappears. If the reflecting planes of mirrors $\mathbf{M_3}$ and $\mathbf{M_4}$ are not parallel in the sense of Fig. 3, one obtains curved fringes, which change into straight fringes at *d* = 0.

On determining the wavelength:

To measure the wave length of the light, count the circular fringe changes while moving the mirror with the micrometer screw (transmission ratio approx. 20:1).

In the process, a shift of the mirror by 43.157 µm is measured and *N* = 135(1) circular fringe changes are counted.

$$\lambda = \frac{2 \cdot d}{N} = \frac{2 \cdot 43.157}{135}\ \mu\mathrm{m}$$

From these values, the wavelength of light λ = 639(10) nm is obtained.

Temporal coherence and contrast function:

The temporal coherence - the coherence time and length - of a laser can be determined with the aid of a Michelson interferometer.

As a result of the different optical distances which the light traverses in the interferometer, the laser light which has been split into two beams undergoes a temporal retardation τ and is then caused to interfere with itself. The coherence time τ_c is the lag time at which the wave trains are still capable of interference, thus

$$\tau < \tau_c. \qquad (5)$$

The coherence length is therefore:

$$l_c = c \cdot \tau_c \qquad (6)$$

where c is the speed of light.

The laser's resonator determines the possible oscillation modes via its resonance condition $L_R = n \cdot \lambda/2 (n = 1, 2, 3...)$. In the process, only those frequencies which lie within the natural emission spectrum of the amplification medium and above the threshold of resonator losses occur (see Fig.4).

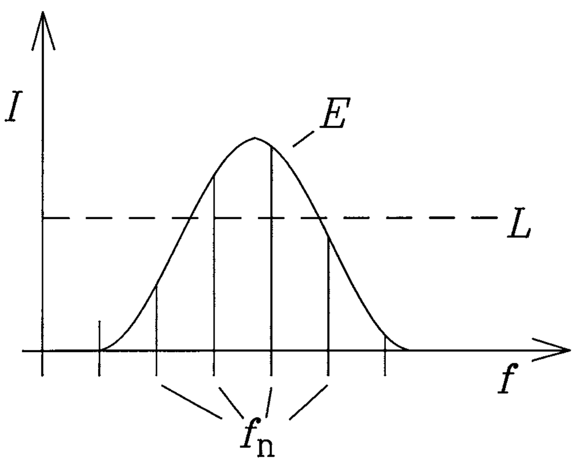

Fig. 4: Frequency spectrum of a laser. With Emission spectrum E, the resonator modes f_n and the laser threshold L.

The contrast between the bright and dark circular fringes of the interference patterns is a measure of the interference capability of light. This can be determined by applying the autocorrelation function of light.

If $E(r,t) = A \cdot e^{\imath(kr-\omega t+\varphi)}$ is the complex electrical field vector of the light wave at location r and at time *t*, it follows that the intensity *I* (except for a constant factor) is:

$$I = (E \cdot E^*) \quad (7)$$

where *E** is the conjugated complex vector of *E* and < > is a temporal average.

In our case the following results for the two waves E_1 and E_2 in the Michelson interferometer:

$$I_{\text{res}} = ((E_1 + E_2) \cdot (E_1 + E_2)^*) = I_1 + I_2 + 2 \cdot Re(E_1 \cdot E_2^*) \quad (8)$$

In our experiment E_1 and E_2 are identical in the ideal case, except for the temporal shift τ, therefore:

$$E_2(t) = E_1(t+\tau) \quad (9)$$

$$\Gamma(\tau) = (E(t) \cdot E^*(t+\tau)) \quad (10)$$

$\Gamma(\tau)$ is the autocorrelation function or also the Self-coherence function of light in this case.

For the resulting intensity, the following is thus obtained:

$$I_{\text{res}} = 2I + 2Re(\Gamma(\tau)) \quad (11)$$

The contrast in the interference pattern is given by:

$$K = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (12)$$

The standardised self-coherence function is the complex degree of self-coherence:

$$\gamma(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)} \quad (13)$$

so that with (9) and (10) the following results:

$$E_1 = A_1 \cdot e^{i(k_1 x - \omega_1 t)}$$

$$E_2 = A_2 \cdot e^{i(k_2 x - \omega_2 t)}$$

and therefore

$$K = |\gamma(\tau)| \quad (14)$$

For an ideal planar monochromatic wave in the x direction, the following contrast function would result:

$$K = |\gamma(\tau)| = 1$$

for $E = A \cdot e^{i(kx-\omega t)}$ *with* $\gamma(\tau) = e^{-i\omega\tau}$. This means that the coherence time and length would be infinitely long in the ideal case for a single frequency. However, in reality, the coherence length is limited by the natural line width (in gas lasers primarily additionally by Doppler broadening) of the spectral lines.

If a laser oscillates in two modes having the frequencies ω_1 and ω_2 with

$$I_{\max} = 2 \cdot | + 2| \cdot |\gamma(\tau)|$$

$$I_{\min} = 2 \cdot | - 2| \cdot |\gamma(\tau)|$$

the coherence function is the following:

$$\Gamma(\tau) = ((E_1(t) + E_2(t)) \cdot (E_1(t+\tau) + E_2(t+\tau))*)$$

$$= \Gamma_1(\tau) + \Gamma_2(\tau)$$

$$= |A_1|^2 e^{-i\omega_1\tau} + |A_2|^2 e^{-i\omega_2\tau} \quad (15)$$

and the degree of self-coherence is given by:

$$\gamma(\tau) = \frac{|A_1|^2}{|A_1|^2 + |A_2|^2} e^{-i\omega_1\tau} + \frac{|A_2|^2}{|A_1|^2 + |A_2|^2} e^{i\omega_2\tau} \quad (16)$$

For simplification's sake A_1 and A_2 are equal. With $\omega_2 = \omega_1 + \Delta\omega$ we obtain the following results for the contrast of the interference grating:

$$K = |\gamma(\tau)|$$

$$= \sqrt{\tfrac{1}{2}(1 + \cos(\Delta\omega\tau))} \quad (17)$$

$$= |\cos(\tfrac{\Delta\omega \cdot \tau}{2})|$$

If a 5 mW laser (for example) is used, the following frequency separation of the axial modes results for a resonator length *L* of approximately 30 cm:

$\Delta\omega = 2\pi \cdot \Delta f = 2\pi \cdot \frac{c}{2L}(= 3.1\ \mathrm{GHz})$ (18)

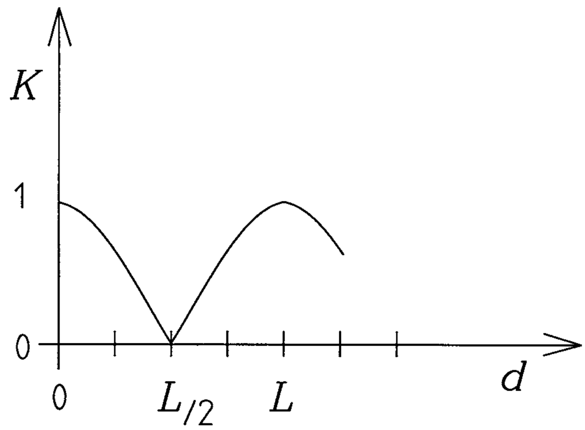

Fig. 5: Theoretical contrast function K of a 2-mode laser.

The propagation delay time *T* results from the mirror shift *d:*

$\tau = \frac{2d}{c}$ (19)

With (18) and (19) the following results for the contrast function *K:*

$K = |\cos(\frac{2\cdot\pi\cdot d}{2\cdot L})| = |\cos(\frac{\pi\cdot d}{L})|$ (20)

The following is valid using the 5 mW laser example:

--according to (18), the mode separation Δf = 490 MHz is obtained.

--with

$K = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = |\cos(\frac{\Delta\omega\cdot\tau}{2})|$

one obtains for the critical delay time τ_c *:*

$K = 0 = |\cos(\frac{\Delta\omega\cdot\tau_c}{2})|$ *or*

$\frac{\pi}{2} = \frac{2\pi\cdot\Delta f\cdot\tau_c}{2} \rightarrow \tau_c \approx 1\ \mathrm{ns}.$

--This results in a minimum of the contrast function (20) at:

$o = |\cos(\frac{\pi\cdot d}{L})|$ or

$\frac{\pi}{2} = \frac{\pi\cdot d}{L} \rightarrow d = \frac{L}{2} \approx 15.0\ \mathrm{cm}.$

The experimental data for the contrast function *K* as a function of the mirror shift *d* are given in Fig. 6.

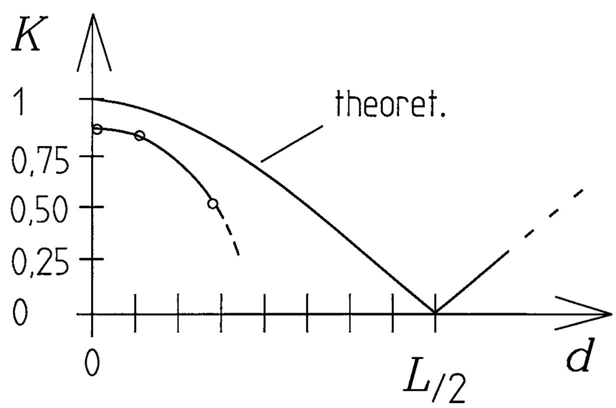

Fig. 6: Experimentally determined contrast function in comparison to the theoretical contrast function K of a 2-mode laser.

It becomes apparent that the theoretical maximum is not reached; this can be due to several factors:

1. If the mirrors are inadequately parallel, the contrast function only reaches a smaller maximum value (thus, does not reach 1) and a larger minimum value (hence, does not drop to zero), see Fig. 7.
2. The division of the beam splitter is not ideal, i.e. 50:50. Therefore, the derivation of the contrast function would have had to have been modified.
3. The 5 mW laser does not only oscillate in 2 modes, but rather the amplification is sufficient to allow the laser to oscillate in three axial modes: this shortens the coherence time. (A spectral analysis has established that a third mode is only possible to a considerably lesser degree than the two primary axial modes!)
4. The aperture in front of the photodiode was not made small enough. Consequently, it covers and averages different intensity regions.

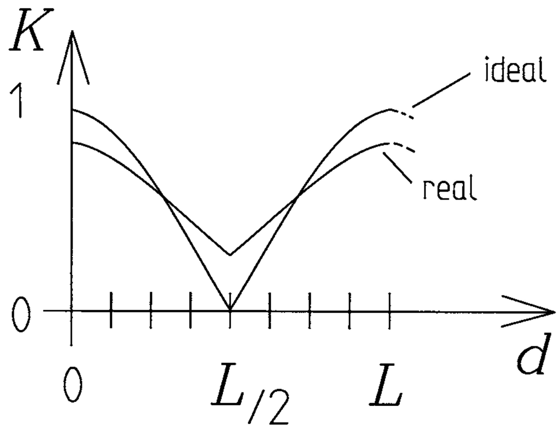

Fig. 7: Theoretical contrast function K of 2-mode laser under ideal and real conditions.

Table 1: Experimental data.

No.	I_{min} in mV	I_{max} in mV	d in cm	$K_{\text{acc.}}$ (12)
1	20	220	0.2	0.835
2	23	214.7	2.0	0.8065
3	67	200.5	5.3	0.4990