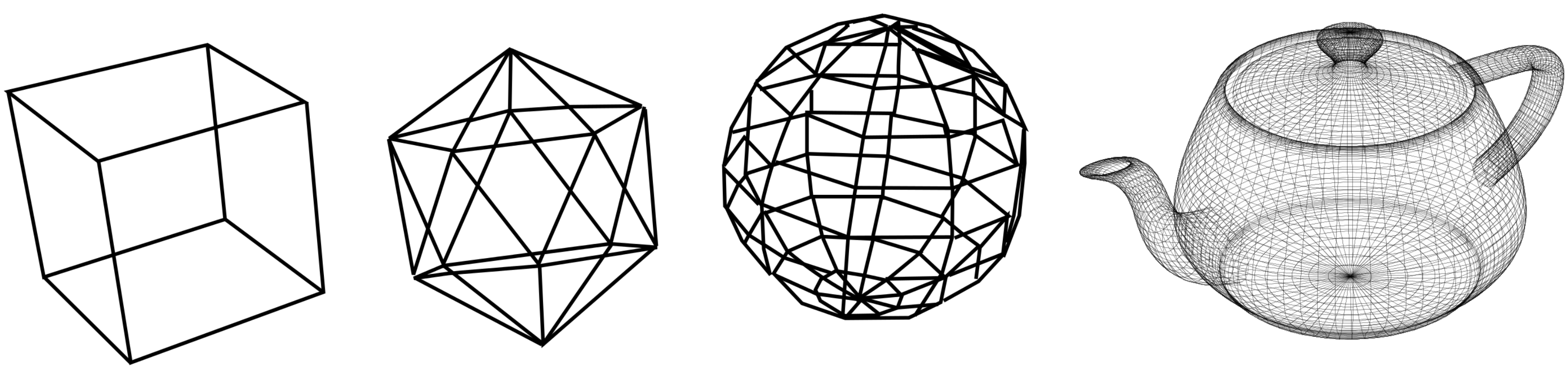
VIETNAM ACADEMY OF SCIENCE AND TECHNOLOGY
UNIVERSITY OF SCIENCE AND TECHNOLOGY OF HANOI

COMPUTER GRAPHICS

Lecture 3: 3D model representation and processing

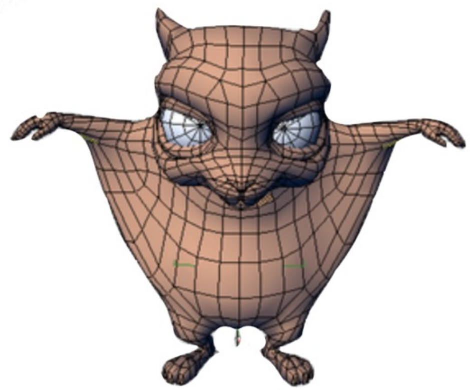
Lecturer: Dr. NGUYEN Hoang Ha



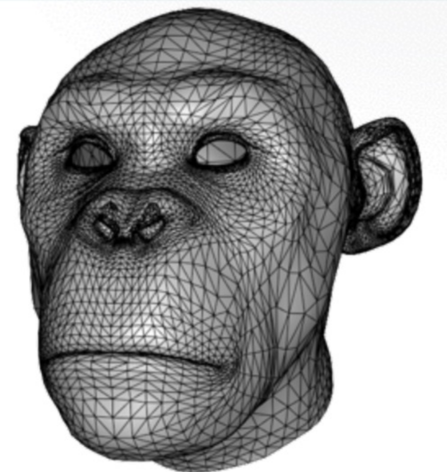
Reference: JungHyun Han. 2011. 3D Graphics for Game Programming (1st ed.), chapter 6

3D

Point cloud

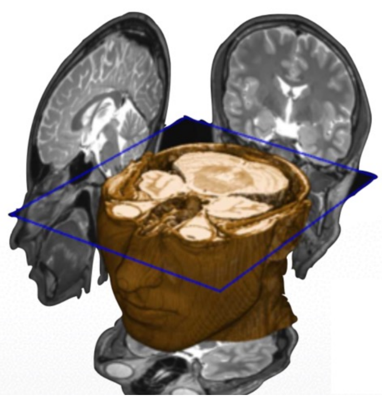


Quad mesh

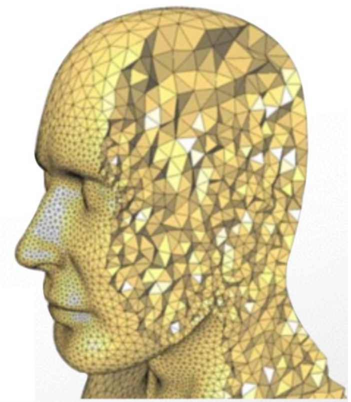


Triangle mesh

Implicit surface/ particles



Volumetric



Tetrahedrons

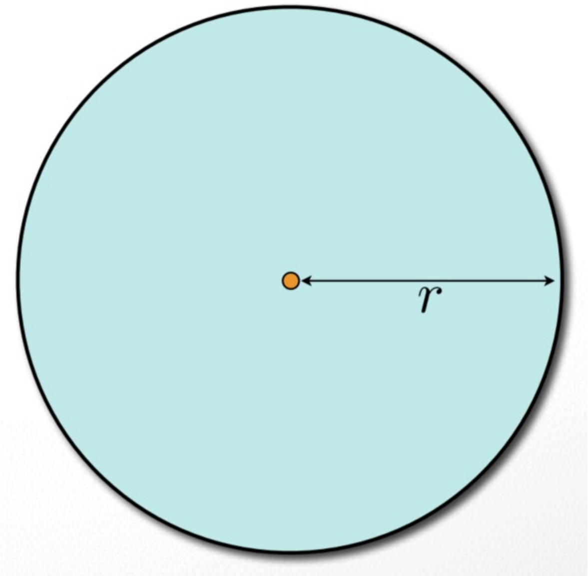
- Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathrm{I\!R}^2 \rightarrow \mathrm{I\!R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

- 2D example

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$



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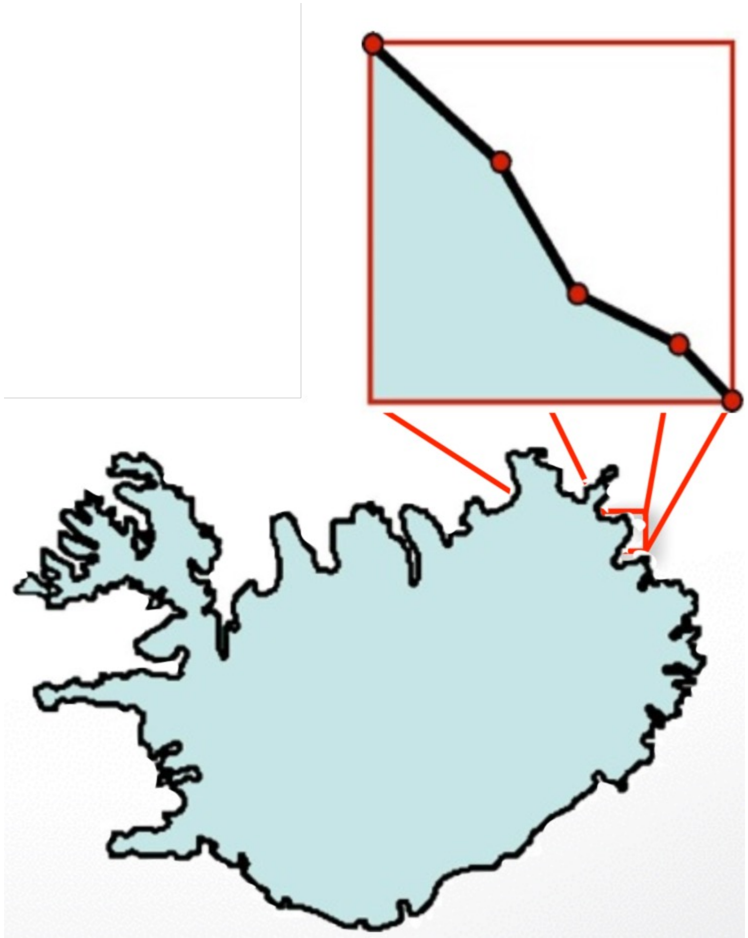
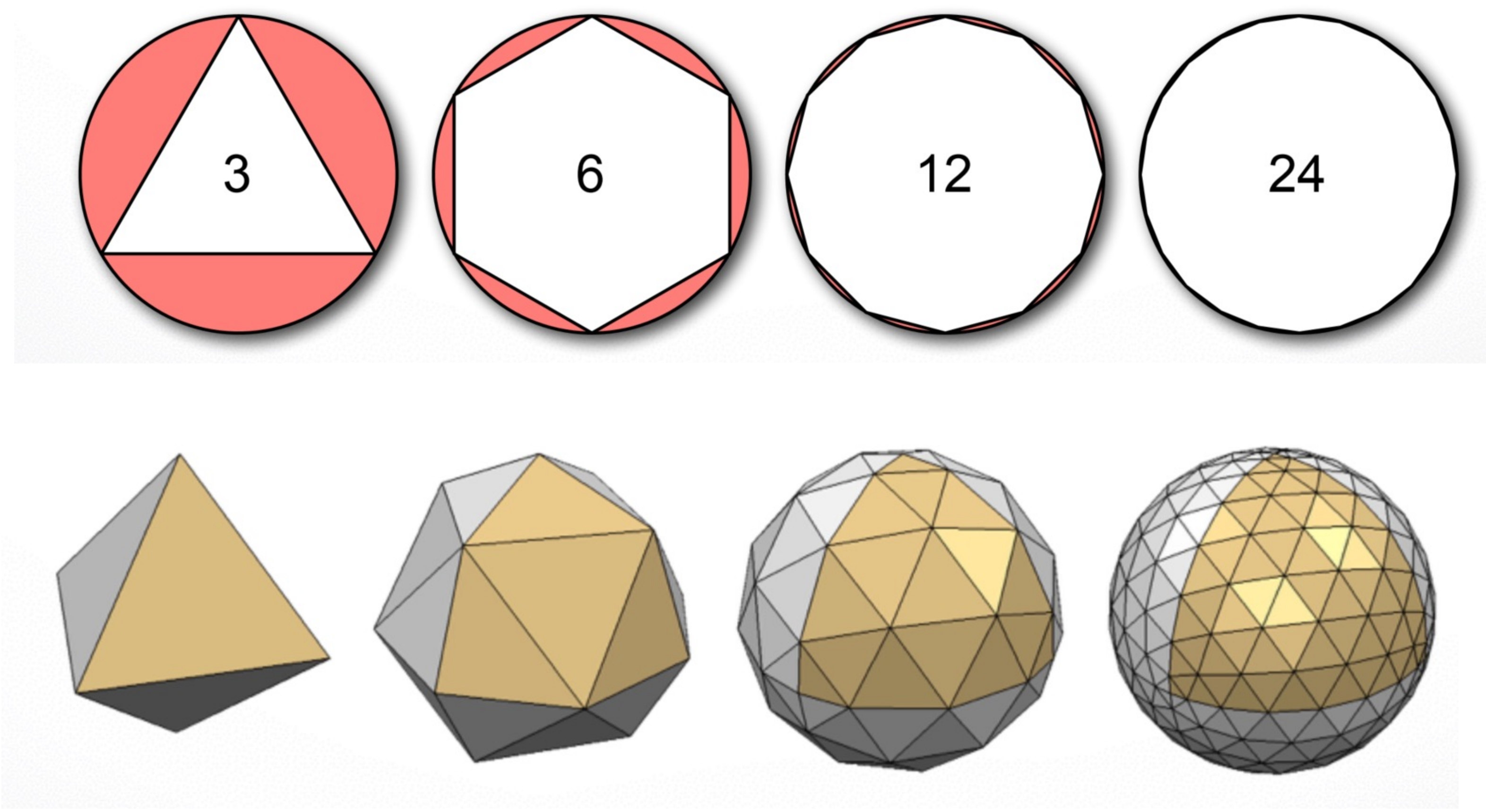
$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ? \\ ? \end{pmatrix}$$



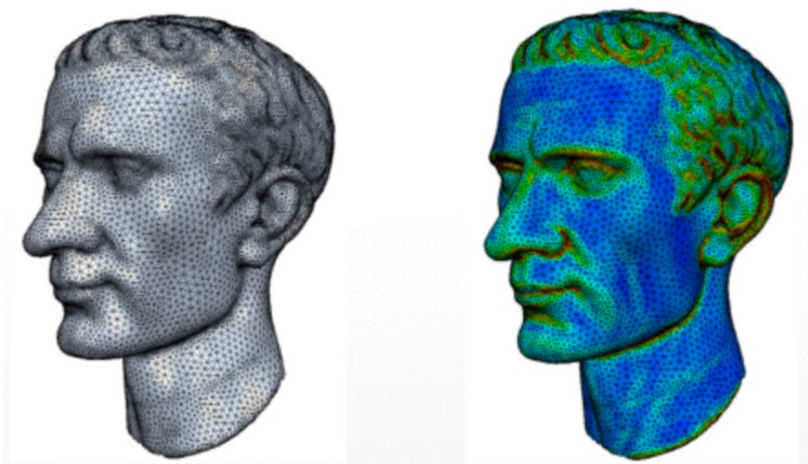
A [illegible]

- Error inversely proportional to face number



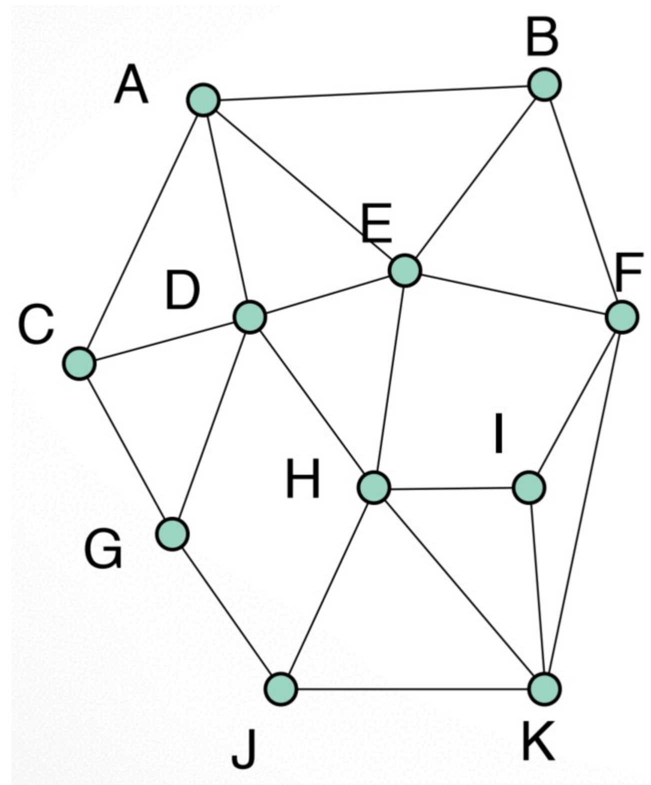
A [illegible]

- Error inversely proportional to face number
- Arbitrary topology surface
- Piecewise smooth surface
- Adaptive sampling



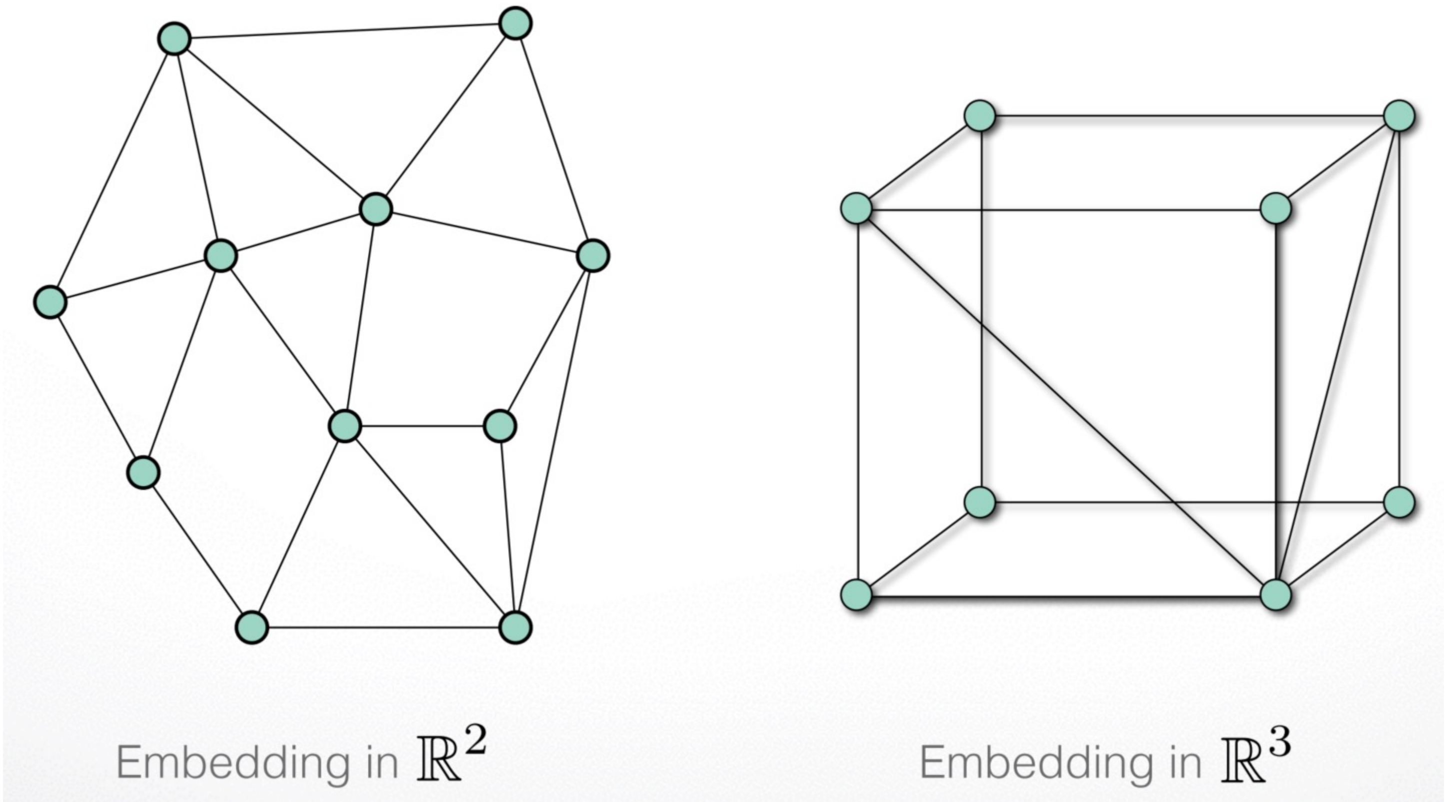
[illegible]: [illegible]

- Graph {V, E}
- Vertices V = {A, B, C… K}
- Edges E = {(AB), (AE), …}
- Vertex degree or valance: number of incident edges
  - D□□(A) = 4
  - D□□(E) = 5



G[illegible]

- Graph is embedding in R^d if each vertex is assigned a position in R^d



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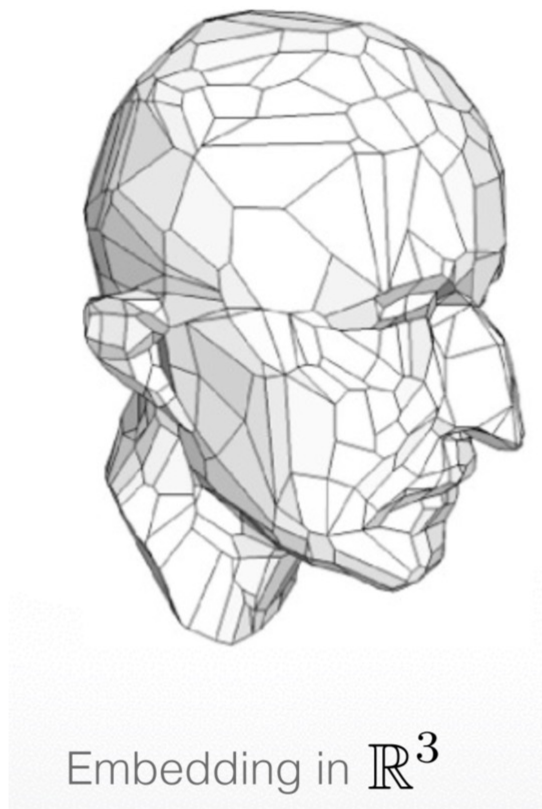
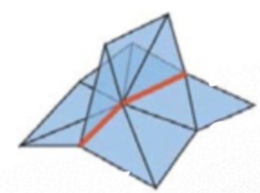
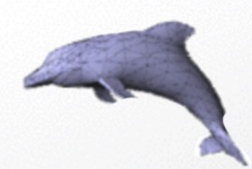
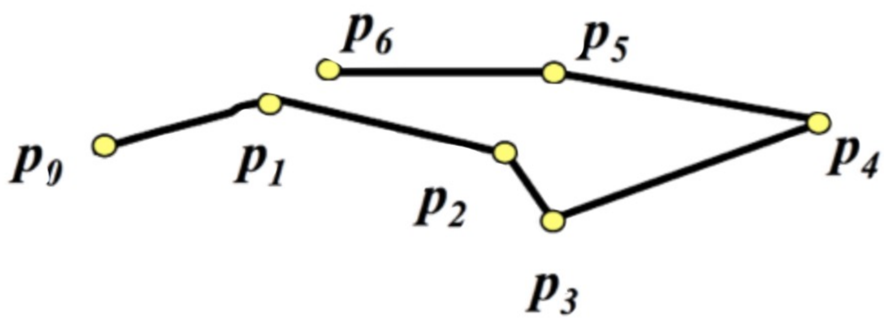


- Mesh: straight-line graph embedded in R^3
- Boundary edge: adjacent to exactly 1 face
- Singular edge: adjacent to more than 2 face
- Regular edge: adjacent to exactly 2 face
- Closed mesh: mesh with no boundary edges





A geometric graph $Q = (V, E)$
with $V = \{\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{n-1}\}$ in $\mathbb{R}^d$, $d \geq 2$
and $E = \{(\mathbf{p}_0, \mathbf{p}_1) \ldots (\mathbf{p}_{n-2}, \mathbf{p}_{n-1})\}$
is called a **polygon**

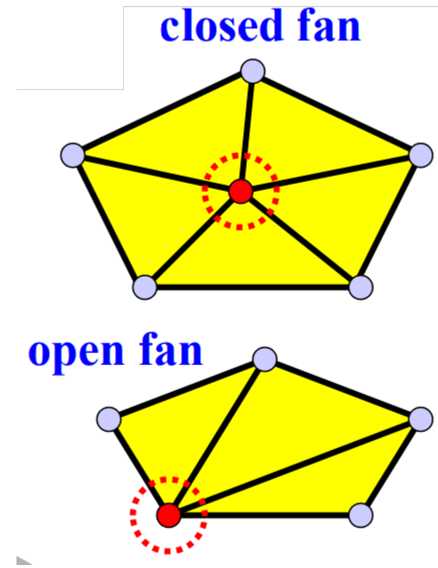


A polygon is called

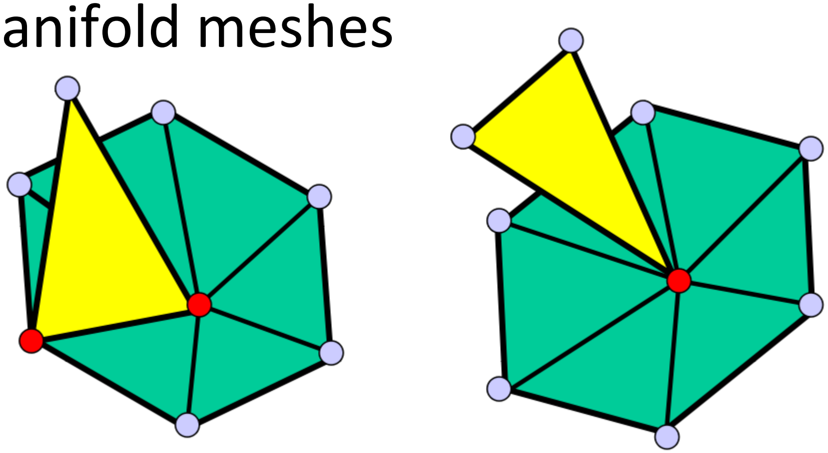
- flat, if all edges are on a plane
- closed, if $\mathbf{p}_0 = \mathbf{p}_{n-1}$

[illegible]

- A mesh is a manifold if
  - E[illegible] 2 [illegible]
  - [illegible]

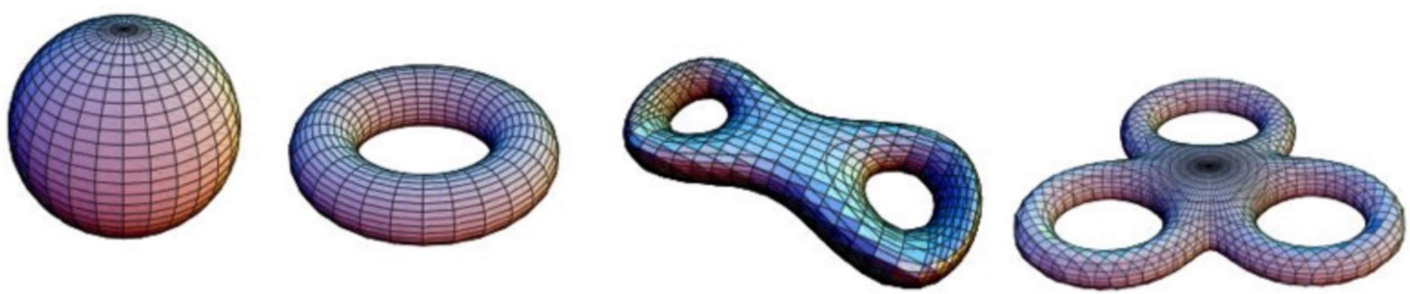


- Non-manifold meshes



[illegible]: G[illegible]

- **Genus:** Maximal number of closed simple cutting curves that do not disconnect the graph into multiple components.
- Or half the maximal number of closed paths that do no disconnect the mesh
- Informally, the number of **holes** or **handles**



Genus 0 Genus 1 Genus 2 Genus 3

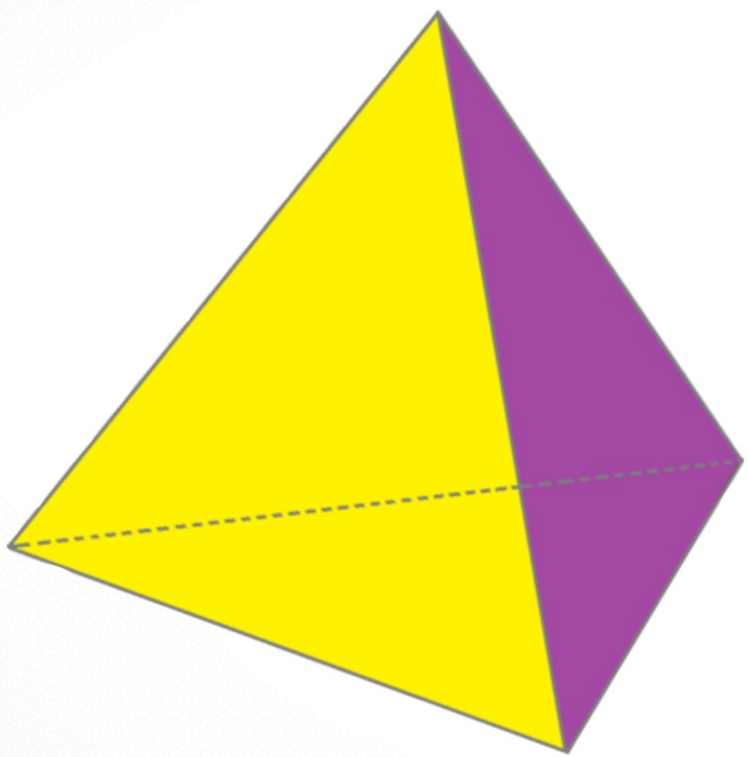
E[illegible] F[illegible]

- For a closed polygonal mesh of **genus** g , the relation of the number V of vertices, E of edges, and F of faces is given by **Euler's formula:**

$$V - E + F = 2(1 - g)$$

- The term $2(1 - g)$ is called the **Euler characteristic** χ

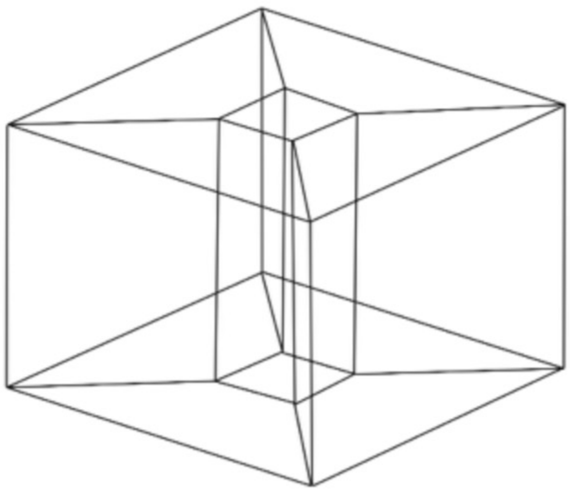
E[illegible] F[illegible]: [illegible]



$$V - E + F = 2(1 - g)$$

$$4 - 6 + 4 = 2(1 - 0)$$

E[illegible] F[illegible]



$$V - E + F = 2(1 - g)$$

$$16 - 32 + 16 = 2(1 - 1)$$

E□□□□□□□□

1. Let v, e, and f denote the numbers of vertices, edges and faces of a closed triangle mesh, respectively. Let's:
   - D□□□□□□= 2□– 4
   - D□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
2. Average vertex degree in a closed manifold

triangle mesh is ~6

G[illegible]

- Surface reconstruction
- Surface smoothing (noise removal)
- Surface simplification
- Remeshing for improving mesh quality

- Principal curvatures K_1 and K_2: the two principal curvatures at a given point of a surface are the maximum and minimum values of the curvature as expressed by the eigenvalues of the shape operator at that point.
- Gaussian curvature: $K = K_1 K_2$
- Mean curvature:
- Maximum curvature: $\max(K_1, K_2)$
- Minimum curvature: $\min(K_1, K_2)$